Basic Electrical and Electronics Engineering-II

Subject Code: ES-201

WBUT

Solved previous years' question papers

S. K. Bhattacharya

ALWAYS LEARNING

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Basic Electrical and Electronics Engineering-II

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Basic Electrical and Electronics Engineering-II

First Year
West Bengal University of Technology
Subject Code: ES-201

S. K. Bhattacharya
Former Director, National Institute of Technical Teachers' Training and Research, Government of India
Kolkata and Chandigarh

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Dedicated to my wife, Sumita,
without whose patience and encouragement
this work could not have been completed.

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Solved Question Paper S.1

Preface

The main objective of this book is to provide the students of Basic Electrical and Electronics Engineering-II of West Bengal University of Technology, Kolkata, with a clear and thorough understanding of the theory and applications of the subject. The contents of the book have been structured to follow the syllabus prescribed by West Bengal University of Technology. A roadmap to the syllabus has been included for the benefit of students. The utility of this book has also been enhanced by the inclusion of the latest university question paper with solutions.

This book has been developed on the basis of my long experience in teaching the subject to first-year B.E. students at a number of different engineering colleges. My experience as a technical teacher/trainer has helped me to prepare the text in a manner that is suitable for students in the first year of their B.E. course. This book covers all the important topics of basic electrical and electronics engineering. The chapters have been developed using the basic principles of learning and motivation. The easy explanation of topics and the large number of examples, illustrations and practice problems are the principle features of this book.

I am thankful to the publisher, Pearson Education, for bringing out this book on time and in such a presentable form. All suggestions to improve this edition are welcome.

S. K. Bhattacharya

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About the Author

Dr S. K. Bhattacharya is currently the principal of SUS Women's Engineering College, Mohali. Formerly, he was the principal of Technical Teachers' Training Institute, Chandigarh; Director of National Institute of Technical Teachers' Training and Research (NITTTR), Kolkata; and Director of Hindustan Institute of Technology, Greater Noida.

Dr Bhattacharya graduated in electrical engineering from Jadavpur University, obtained his M.Tech. degree from Calcutta University and his Ph.D. from Birla Institute of Technology and Science, Pilani. As a senior fellow of MHRD, Government of India, he attended the technical teacher's training programme at the Bengal Engineering College, Shibpur. On a Dutch Government fellowship programme, he attended a one-year teacher training programme at the Netherlands and the UK, and a fellowship programme of the British Council on educational technology at Sheffield, UK. Dr Bhattacharya has visited many institutions in India and in countries like the UK, the Netherlands, Australia, Japan, Korea, Philippines, Malaysia, etc. He is a fellow of the Institution of Engineers and the Institution of Electronics and Telecommunication Engineers.

A large number of popular books, technical papers, non-print type teaching and learning materials have been published by Dr Bhattacharya.

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Roadmap to the Syllabus

BASIC ELECTRICAL ENGINEERING-II

Unit I Electrostatics

Coulomb's law, electric field intensity, electric field due to a group of charges, continuous charge distribution, electric flux, flux density, electric potential, potential difference; Gauss's law, proof of Gauss's law, its applications to electric field and potential calculation; capacitor, capacitance of parallel plate capacitor, spherical capacitor, isolated spheres, concentric conductors, parallel conductors, energy stored in a capacitor.

Refer Chapter 1

Unit II DC Machines

Construction, basic concepts of winding (lap and wave); DC generator—principle of operation, EMF equation, characteristics (open circuit, load); DC motors—principle of operation, speed–torque characteristics (shunt and series machine), starting (by three-point starter), speed control (armature voltage and field control).

Refer Chapter 2

Unit III Single-phase Transformer

Core- and shell-type construction, EMF equation, no-load and on-load operation, phasor diagram and equivalent circuit, losses of a transformer, open- and short-circuit tests, regulation and efficiency calculation.

Refer Chapter 3

Unit IV Three-phase Induction Motor

Types, construction, production of rotating field, principle of operation, equivalent circuit and phasor diagram, rating, torque–speed characteristics (qualitative only); starter for squirrel-cage and wound-rotor induction motor; brief introduction of speed control of three-phase induction motor (voltage control, frequency control and resistance control).

Refer Chapter 4

Unit V Three-phase System

Voltages of three-phase balanced system, delta- and star-connection, relationship between line and phase quantities, phasor diagrams, power measurement by two-watt meters method.

Refer Chapter 5

Unit VI General Structure of Electrical Power System

Power generation to distribution through overhead lines and underground cables with single-line diagram.

Refer Chapter 6

BASIC ELECTRONICS ENGINEERING-II

Unit VII Field-effect Transistors

Concept of field-effect transistors (channel width modulation), gate isolation types, JFET structure and characteristics, MOSFET structure and characteristics, depletion and enhancement type; CS, CG, CD configurations; CMOS—basic principles.

Refer Chapter 7

Unit VIII Feedback Amplifiers, Oscillators and Operational Amplifiers

Concept (block diagram), properties, positive and negative feedback, loop gain, open-loop gain, feedback factors; topologies of feedback amplifier; effect of feedback on gain, output impedance, input impedance, sensitivities (qualitative), bandwidth stability; effect of positive feedback, instability and oscillation; condition of oscillation, Barkhausen criteria.

Introduction to integrated circuits, operation amplifiers and their terminal properties; applications of operational amplifiers; inverting and non-inverting mode of operation, adders, subtractors, constant-gain multiplier, voltage follower, comparator, integrator, differentiator.

Refer Chapters 8 and 9

Unit IX Digital Electronics

Introduction to binary number; basic Boolean algebra; logic gates and function realization with op-amps.

Refer Chapter 10

Students will attend three hours of lecture classes and one hour of tutorial per week. This course carries four credits.

Electrostatics

TOPICS DISCUSSED

- Electric charge
- Coulomb's law
- Electric field
- Types of dc machines
- Electric field intensity
- Electric flux and electric flux density
- Electric potential
- Potential difference
- Gauss's law
- Capacitors

1.1 INTRODUCTION

Matter is composed of a large number of atoms. Atoms in turn consist of a central nucleus and a number of tiny particles called electrons. The nucleus has protons and neutrons. The protons are considered to be positively charged while the electrons have an equal number of negative charge. The neutrons in the nucleus have no charge. The positive charge of protons and the negative charge of the electrons are equal and opposite each other. Hence, an atom as such carries no charge.

The deficiency or excess of electrons in an atom constitutes charge. An atom is said to be a negatively charged atom when it has one excess electron. Similarly, an atom is said to be positively charged when it loses an electron from its stable structure.

Electric current is defined as the rate of change of charge passing in a conducting material through a specified area. Mathematically, current, i, is expressed as

$$i = \frac{dq}{dt} \tag{1.1}$$

Charge in motion is called current, which can be termed as electricity in motion or current electricity.

When two charged bodies are separated by some insulating material, the charge remains stationary on the surface of the body. This is called static electricity. Electrostatics is the branch of science and engineering which deals with static charges. In this chapter we shall discuss the important laws and theorems of electrostatics, i.e. Coulomb's law and Gauss's law or theorem. The concepts of electric field, electric field intensity, electric potential and potential difference, capacitors and how a capacitor stores energy have also been thoroughly discussed.

1.2 ELECTRIC CHARGE

Any material is made up of a large number of tiny particles called atoms. Atoms are generally electrically neutral, i.e. the sum total of positive charge is equal to the negative charge in an atom. However, it has been found that when two bodies are rubbed together, some amount or charge is transferred from one body to the other due to frictional force. For example, when a glass rod is rubbed with a silk cloth, the glass rod attains positive charge (i.e. deficiency of electrons) and the silk cloth become negative charged (i.e. excess of electrons). The charges are retained in both glass and silk because they are insulators. These charges are, therefore, called static charges.

It has been found experimentally that when two similarly charged bodies, such as two glass rods rubbed with silk, are brought near each other they develop a force of repulsion. Positively charged bodies brought near negatively charged bodies attract each other.

A negatively charged body has an excess of electrons and a positively charged body has a deficiency of electrons. The net excess or deficiency of electrons in a body is called charge.

1.3 COULOMB'S LAW

Like charges repel and unlike charges attract each other. When charges exist on bodies, a force known as electrostatic force exists between them. Charles Coulomb, a French scientist, carried out a number of experiments to see the effect of placing charges near each other. A law of electrostatics known as Coulomb's law or inverse square law was framed by him on the basis of his experiments.

According to Coulomb's law, the force of repulsion or attraction between two charges is

(i) directly proportional to the product of the charges,
(ii) inversely proportional to the square of the distance between them, and
(iii) the nature of the medium in which the charges are placed.

Let us assume that two positive charges Q_1 and Q_2 are separated by a distance of d meters (Fig. 1.1). According to Coulomb's law, the magnitude of the force of repulsion is given by

$$F \propto \frac{Q_1 Q_2}{d^2} \quad \text{or,} \quad F = k \frac{Q_1 Q_2}{d^2} \text{ newton}$$

k is the constant of proportionality and depends upon the medium.

$$k = \frac{1}{4\pi\varepsilon} = \frac{1}{4\pi\varepsilon_0 \varepsilon_r}$$

where ε_0 is the permittivity of the vacuum and ε_r is called the relative permittivity.

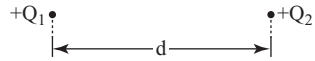


Figure 1.1 *Electrostatic force of repulsion by Coulomb's law*

Thus, the expression for force between two charges can be written as

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r d^2} \text{ newton} \tag{1.2}$$

where force, F is in newton (N); charges Q_1 and Q_2 are in coulomb (C); distance d is in meters; permittivity, ε in Farad/m (F/m); absolute permittivity, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m and is a constant.

$$\text{The value of } \frac{1}{4\pi\varepsilon_0} = \frac{1}{4 \times 3.14 \times 8.854 \times 10^{-12}} = 9 \times 10^9 \text{ (approx)}$$

Coulomb's law is usually referred to as Inverse Square law.

Example 1.1 Determine the force between two positively charged point charges of 1 coulomb when placed at a distance of 1m in the medium of free space (air).

Solution:

$$Q_1 = 1\,C,\ Q_2 = 1\,C$$

$$d = 1\ m\ \varepsilon_0 = 8.854 \times 10^{-12}\,F/m$$

The like charges will repel each other and the force of repulsion, F is

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 d^2} = \frac{1 \times 1}{4\pi \times 8.854 \times 10^{-12} \times 1^2}$$

$$= 9 \times 10^9 \text{ newton (N)}$$

This shows that the force is extremely high. This also means that 1 coulomb is a huge quantity of charge.

One coulomb charge is that amount of charge which when placed in free space at a distance of 1 m from an equal and similar charge produces a repulsive force of 9×10^9 N.

1.4 ELECTRIC FIELD

A charged body when placed in the vicinity of another charged body experiences a force of attraction or repulsion depending upon the kind of charges. *The region or space around a charged body in which a charge experiences a force of attraction or repulsion is called an electric field.*

The region around a charge Q_1 constitutes an electric field because any other charge Q_2 when placed in that region will experience a force. The electric field around a charge is represented by imaginary lines. These lines are called electric lines of force. An electric line of force can be defined as the path along which a unit positive charge which is free to move will tend to move in an electric field. The lines of force, theoretically, extend to infinity. The direction of the lines of force is determined by the direction along which a unit positive charge when placed at a point will tend to move. Figure 1.2 shows electric lines of force around an isolated positive and a negative charge, respectively. In one case, lines of force are emanating from the charged body and in the other case the lines of force are terminating at the surface of the charged body. Electric lines of force are supposed to originate from

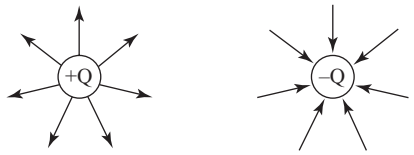


Figure 1.2 *Electric field around a positive and negative charge*

the surface of a positively charged body and terminate at the surface of a negatively charged body. The lines of force enter or leave the charged surface normally and they do not intersect with each other.

1.4.1 Types of Charge Distribution

Electric charge may exist in a point form, a line form, a surface form or a volume form. Accordingly, there are four types of charge distributions. They are (i) point charge, (ii) line charge, (iii) surface charge, and (iv) volume charge.

These are explained briefly as follows:

Point charge: A very small surface carrying a charge in a region or in a large surface area is called a point charge. Therefore, a point charge is simply a position with no dimensions.

Line charge: Electric charge distributed uniformly along a line or a wire is called a line charge. Charge density, ρ_L is the amount of charge per unit length of the line. It is expressed in Coulomb per meter (C/m). By integrating the charge $dQ = \rho_L dl$ for the entire length of the line, we can determine the total charge on the line. Such an integral is called line integral.

$$Q = \int_L \rho_L \cdot dl$$

where dl is an elementary length.

Surface charge: The charge is distributed uniformly over a surface is called surface charge. Surface charge density, ρ_s is the charge per unit area and is expressed as Coulomb per m^2 (C/m^2). To calculate the total charge on a surface, we have to integrate the charge $dQ = \rho_s \cdot ds$ over the given surface. Such an integral is called surface integral.

For the uniform charge distribution on a surface, S, the total charge existing on the surface is

$$Q = \int_s \rho_s \cdot ds$$

where ρ_s is the surface charge density and ds is an elementary surface.

Volume charge: If charge is distributed uniformly over the volume of the entire body, it is called volume charge. Volume charge density, ρ_v is the charge per unit volume. The total charge is calculated by taking the volume integral. The total charge, Q is found as

$$Q = \int_v \rho_v dv$$

where ρ_v is the volume charge density and dv is an elementary volume.

1.4.2 Electric Field due to a Group of Charges

The lines of forces around a number of charges can be represented as shown in Fig. 1.3. Two equal and opposite charges have been shown at a particular distance in Fig. 1.3 (a). Two similar charges placed at a distance have been shown in Fig. 1.3 (b).

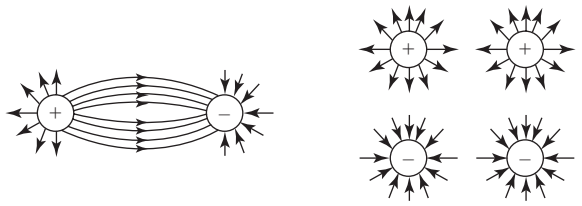


Figure 1.3 *Electric field due to a number of charges; (a) two equal and opposite charges; (b) two equal but similar charges*

1.5 ELECTRIC FIELD INTENSITY OR ELECTRIC FIELD STRENGTH

Electric field intensity is also called the Electric field strength. Electric field intensity, $\bar{E}$, is defined as the force experienced by a unit positive charge placed at that point in the electric field. Electric field is represented by $\bar{E}$ as this is a vector quantity. It has both magnitude and direction. The direction is that in which an isolated positive charge is forced to move.

Electric field intensity or electric field strength at a point in an electric field is the force acting on a unit charge, placed at that point.

Accordingly, the magnitude of electric field strength E is the force per unit charge at a point and is expressed as

$$E = \frac{F}{G} \text{ N/C} \tag{1.3}$$

The magnitude of force on a unit charge q, placed at a distance r in an electric field due to a charge Q is given by

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2} \text{ Newton}$$

The magnitude of electric field intensity, $\bar{E}$ due to a fixed charge Q is calculated from the expression of force,

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$$

E is equal to force per unit charge.

So,
$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \text{ newton/coulomb} \tag{1.4}$$

To calculate electrical field intensity due to number of point charges, we have to add the intensities due to each charge vectorially as

$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + \cdots$$

The magnitude and the direction of electric field at a point around a charge, Q, can be represented in vector form as $\bar{E}$ where

$$\bar{E} = \frac{Q}{4\pi\varepsilon r^2}\bar{a}_r$$

If the charge is placed in air, $\varepsilon = \varepsilon_0$.

For any other dielectric medium, $\varepsilon = \varepsilon_0\varepsilon_r$.

$\bar{a}_r$ is a unit vector pointing radially from or to the charge Q.

1.6 ELECTRIC FLUX AND ELECTRIC FLUX DENSITY

The total number of flux emanating from a positive charge is called electric flux. This is expressed as ψ (Psi). Faraday, on the basis of an experiment, showed that electric flux ψ = Q coulomb.

The electric flux crossing per unit area in an electric field is called electric flux density and represented by the letter D. If area, A, is expressed in m^2, and flux, ψ, in coulomb, then flux density D is

$$D = \frac{Q}{A} \text{ C/m}^2 \quad \text{or,} \quad D = \frac{\psi}{A} \text{ C/m}^2 \tag{1.5}$$

1.6.1 Relationship Between Electric Flux Density and Electric Field Intensity

Electric field, E, at a point at a distance, r, from a charge, Q, is expressed as

$$E = \frac{F}{Q} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2} = \frac{Q}{4\pi\varepsilon r^2}$$

or,
$$\varepsilon E = \frac{Q}{4\pi r^2} = D = \text{Charge per unit area}$$

Therefore, electric flux density, D, can be equaled with εE.

Therefore,

$$D = \varepsilon E = \frac{Q}{4\pi r^2} \tag{1.6}$$

1.6.2 Electric Field at a Point due to a Number of Charges

The force exerted on a point charge due to a number of charges is the vector sum of the individual force exerted by each charge.

The electric field at a point due to a number of charges is obtained by taking into account the effect of each charge separately and then taking the combined effect (using the law of superposition). Let $Q_1, Q_2, Q_3 \ldots Q_n$ be n number of charges as shown in Fig. 1.4. Let the combined electric field intensity, E, be obtained at point, P.

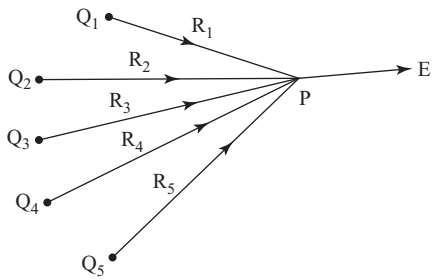


Figure 1.4 *Electric field at a point due to a number of charges*

As shown in Fig. 1.4, the distance of Q_1, Q_2, Q_3, etc. from the point P is R_1, R_2, R_3, … R_n respectively. The total electric field intensity at point P is the vector sum of the individual field intensities as produced by the charges at point P.

$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + ... + \bar{E}_n$$

$$= \frac{Q_1}{4\pi\varepsilon_0 R_1^{\,2}}\bar{a}R_1 + \frac{Q_2}{4\pi\varepsilon_0 R_2^{\,2}}\bar{a}R_2 + \frac{Q_3}{4\pi\varepsilon_0^{\,2} R_3^{\,2}}\bar{a}R_3 + \cdot\cdot$$

or,
$$\bar{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{Q_i}{R_i^{\,2}}\bar{a}R_i \qquad (1.7)$$

1.7 ELECTRIC POTENTIAL

Electric potential is the amount of work done in bringing a unit positive charge, i.e. 1 coulomb charge from infinity to a given point in an electric field.

An isolated positive charge +Q, fixed at a point P, is shown in Fig. 1.5. The force exerted by this charge on a unit positive charge if placed at infinity is zero. When the unit positive charge is moved towards the +Q charge from infinity some amount of work is done because a force of repulsion exists on the unit positive charge. When the unit positive charge has been moved from infinity to a point, the charge will attain some electric potential energy. The greater the distance moved, the greater is the potential energy attained. This way the closer the charge is brought towards the charge +Q, the greater the gain of potential energy by the unit positive charge. This potential energy is called *electric potential*.

Electric potential is the work done per unit charge.

$$\text{Electric potential} = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}\ \text{J/C} \qquad (1.8)$$

where W is the work done in joules in bringing a unit charge from infinity to that point.

If one joule of work is done in bringing a unit charge from infinity to a given point in an electric field, the potential at that point will be 1 volt. Therefore, $1\,\text{volt} = \frac{1\,\text{joule}}{1\,\text{coulomb}}$.

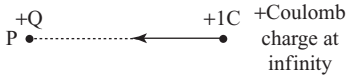


Figure 1.5 *Concept of electric potential*

Electric potential can be compared with the potential energy gained by a body when it is raised from the level of earth to a height. The amount of potential energy gained is equal to the amount of work done in raising the body to that height.

Potential at a point

Let us assume that a charge, q, is moved a distance, $\bar{dl}$, in an electric field, $\bar{E}$. The force experienced by the point charge, q, in an electric field, $\bar{E}$, is expressed as

Force experienced, $\bar{F}_e = q\bar{E}$

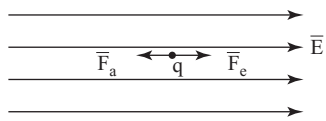


Figure 1.6 *Force experienced by a charge in an electric field*

An applied force, F_a, is required to bring the charged particle into equilibrium (otherwise the charged particle will accelerate in the direction of the electric field).

Thus,

$$F_a = -qE$$

The differential amount of work done, dw, is done by the applied force, F_a, when the charged particle is moved by a differential distance, $\overline{dl}$. Assuming $\overline{F}_a$ and $\overline{dl}$ makes an angle θ with each other,

$$dw = F_a\, dl \cos\theta = \overline{F}_a \cdot \overline{dl}$$

or,

$$dw = -q\overline{E} \cdot \overline{dl}$$

The work done in moving the charge from the initial to the final position can be calculated by integrating within that limit as

$$W = \int dw = -q\int_{initital}^{final} \overline{E} \cdot \overline{dl}$$

Potential difference is the work done by an external force in moving a unit positive charge from one point to the other in an electric field. The potential difference between two points A and B is calculated as

$$V = \frac{W}{q} = -\frac{q\int_i^f \overline{E} \cdot \overline{dl}}{q}$$

$$V_{AB} = -\int_B^A \overline{E} \cdot \overline{dl} \quad \text{(dot product)}$$

V_{AB} is the potential difference between points A and B. In the integral above, B is the initial point and A is the final point. The point B is usually taken at infinity. For example, let us assume that charge at a point is Q. Let a test charge, q, be moved from infinity to the point P at a distance R from the charge Q as shown in Figure 1.7.

The work done in moving the test charge, q, against a force, F, is given by

$$W = -\int_\infty^R F \cdot dr$$

$$F = \frac{Qq}{4\pi\varepsilon d^2}$$

$\therefore$

$$W = -\int_\infty^R \frac{Qq}{4\pi\varepsilon d^2}\, dr$$

$$= -\frac{Qq}{4\pi\varepsilon}\left[-\frac{1}{d}\right]_\infty^R$$

$$= \frac{Qq}{4\pi\varepsilon_0 R}$$

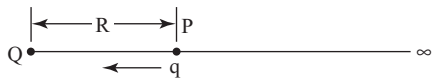


Figure 1.7 *Potential at a point P at a distance R from a charge Q*

The work done per unit charge is the potential V.

Therefore, potential at a point, $V = \dfrac{Q}{4\pi\varepsilon_0 R}$, (q = 1, a unit charge)

$$= 9 \cdot 10^9 \frac{Q}{R} \text{ volt. (when the medium is air)}$$

$$= 9 \times 10^9 \frac{Q}{\varepsilon_r R} \text{ volt (when in medium)}$$

1.8 POTENTIAL DIFFERENCE

Let a charge, Q_1, be located at a fixed point A. A charge, Q_2, is initially situated at infinity. At an infinite distance the force exerted by Q_1 on Q_2 is zero. Now let charge Q_2 be brought from infinity to position B by applying a force. Q_2 is now placed at B at a distance r_2 from charge + Q_1. Work has been done on Q_2 and Q_2 has gained potential energy. This energy will be returned as kinetic energy if the applied force is withdrawn. The charge will be pushed back to infinity by the kinetic energy.

The work done per unit charge is the potential of the charge at point B, which we may term as V_2.

$$V_2 = \frac{W_2}{Q_2} = -\int_d^{r_2} \frac{Q_1}{4\pi\varepsilon r_2^2}\, dr$$

$$= -\frac{Q_1}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_{\infty}^{r_2}$$

or, $$V_2 = \frac{Q_1}{4\pi\varepsilon r_2} \text{ J/C}$$

or, $$V_2 = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_r r_2} \text{ volts. (when in medium)}$$

Let us take another charge, Q_3, at infinity and move it towards the stationary charge, Q_1, either on the same line as Q_2 was brought or along a line from infinity to C towards A as has been shown in Fig. 1.8. The work done per unit charge here will be the potential at point C, which is at a distance, r_3, from $+Q_1$. Potential at point C, i.e. V_3 will be

$$V_3 = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_r r_3} \text{ volts. (when in medium)}$$

Since $r_3 < r_2$, V_3 will be greater than V_2. So, there exists a potential difference between point B and point C. Point C is at higher potential than point B.

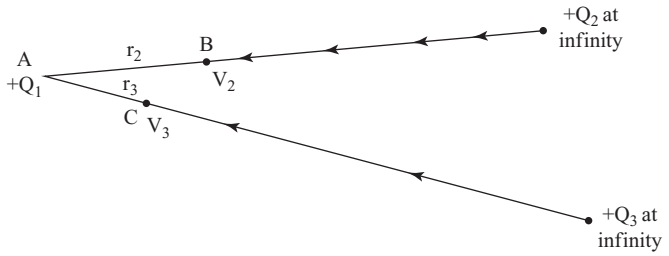


Figure 1.8 *Illustration of potential difference*

Example 1.2 Four charges Q_1, Q_2, Q_3 and Q_4 are placed on the four corners of a square of side 1m. Calculate the potential at the centre of the square. The values of the charges are given as $Q_1 = 1.0 \times 10^{-9}$ C, $Q_2 = 2 \times 10^{-9}$ C, $Q_3 = 3 \times 10^{-9}$ C and $Q_4 = 4 \times 10^{-9}$ C. The charges are placed in air.

Solution:

$$\text{Distance BD} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\text{Distance BX} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \text{ m}$$

The potential at point X is due to the combined effect of the four charges. The potential at X, i.e. V_X is

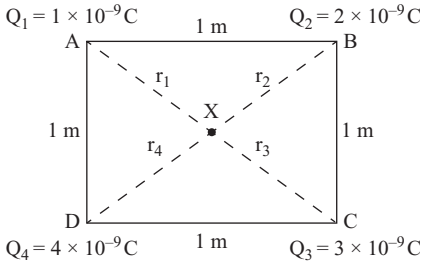


Figure 1.9

$$V_x = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} + \frac{q_3}{4\pi\varepsilon_0 r_3} + \frac{q_4}{4\pi\varepsilon_0 r_4}$$

Since $r_1 = r_2 = r_3 = r_4$, r = Distance BX = $\frac{1}{\sqrt{2}}$ m.

$$V_X = \frac{1}{4\pi\varepsilon_0 r}(q_1 + q_2 + q_3 + q_4)$$

$$= 9 \times 10^9 \times \sqrt{2}\ (1 \times 10^{-9} + 2 \times 10^{-9} + 3 \times 10^{-9} + 4 \times 10^{-9})$$

$$= 9 \times 10^9 \times \sqrt{2} \times 10^{-9} \times 10$$

$$= 90 \times 1.414$$

$$= 127.26 \text{ volts}$$

Example 1.3 Three charges of 3×10^{-9} C each are placed in the three corners of an equilateral triangle of side 3 m. Calculate the potential at the centroid of the triangle.

Solution:

The distance of point P at the centroid of the triangle from the corners of the triangle, A, B and C is equal.

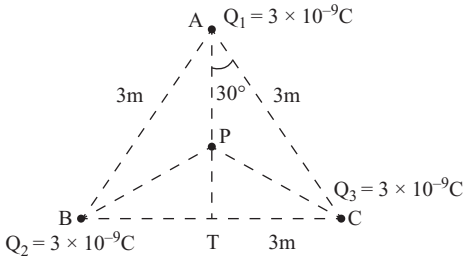


Figure 1.10

$$AP = BP = CP$$

$$AC \cos 30° = AT$$

$$AT = 3 \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$$

$$AP = \frac{2}{3} AT = \frac{2}{3} \cdot \frac{3\sqrt{3}}{2} = \sqrt{3}$$

Potential at P, V_P is calculated as

$$V_P = \frac{Q_1}{4\pi\varepsilon_0 \times \sqrt{3}} + \frac{Q_2}{4\pi\varepsilon_0 \sqrt{3}} + \frac{Q_3}{4\pi\varepsilon_0 \sqrt{3}}$$

$$= \frac{1}{4\pi\varepsilon_0 \sqrt{3}} (Q_1 + Q_2 + Q_3)$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{1}{\sqrt{3}} (3 \times 10^{-9} + 3 \times 10^{-9} + 3 \times 10^{-9})$$

$$= 9 \times 10^9 \frac{1}{\sqrt{3}} \times 10^{-9} \times 9$$

or,
$$V_P = \frac{81}{\sqrt{3}} = 46.7 \text{ volts.}$$

Example 1.4 Three charges of 5×10^{-9} C each are placed on the three corners of a rectangle ABCD. Calculate the potential at the fourth corner point of the rectangle. The sides of the rectangle are given as AB = CD = 4 m and BC = DA = 3 m. The medium is air.

Solution:

Let the charges be placed at A, B, and C as shown. We have to find the potential at point D. The distance DB is equal to $\sqrt{4^2 + 3^2} = 5$ m

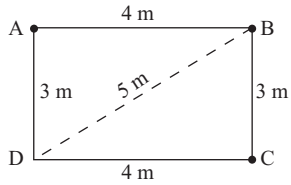


Figure 1.11

Potential, V, at point D due to the three charges is

$$V = \frac{Q_A}{4\pi\varepsilon_0 r_1} + \frac{Q_B}{4\pi\varepsilon_0 r_2} + \frac{Q_C}{4\pi\varepsilon_0 r_3}$$

or,
$$V = 9 \times 10^9 \left(\frac{5\times10^{-9}}{3} + \frac{5\times10^{-9}}{5} + \frac{5\times10^{-9}}{4} \right)$$

or,
$$V = 9 \times 10^9 \times 5 \times 10^{-9} \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{4} \right)$$

or,
$$V = 45 \cdot \frac{47}{60} = 35.25 \text{ volts.}$$

Similarities between Coulomb's law and Newton's law of gravitation

It is interesting to see the similarities between Coulomb's law and Newton's law.

Newton's law of gravitation is given as

$$F = G\frac{m_1 m_2}{r^2}$$

where m_1 and m_2 are the mass of the objects, r is the distance between them and G is the gravitational constant.

$$G = 6.67\times10^{-11}\,Nm^2/Kg^2$$

The force of gravity on any object on the surface of the earth is given as

$$F = G\frac{Mm}{r^2} \quad \text{or} \quad F = mg$$

Where M is the mass of the earth and $g = \frac{GM}{r^2} = 9.8 \text{ m/sec}^2$.

Here, r is the distance between the object, m, and the earth, i.e. between m and M, which is the radius of the earth.

$$F = mg \text{ N}$$

This force of earth's gravity acting on an object of mass m is also the weight of the body.

1.9 GAUSS'S LAW

A German scientist, Karl Fredrick Gauss, developed a law which states that "the total electric flux passing through a closed surface is equal to the total charge enclosed by the surface."

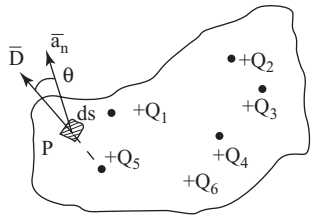


Figure 1.12 *A number of charges enclosed by a surface*

Figure 1.12 shows a closed surface surrounding a number of charges. According to Gauss's law, the total electric flux, ψ, passing through the closed surface is equal to

$$\psi = \text{Algebraic sum of all the charges inside the closed surface}$$
$$= Q_1 + Q_2 + Q_3 + Q_4 + \cdots + Q_n$$

or, $$\psi = \sum Q \text{ coulomb}$$

If the surface is irregular, the magnitude of flux density, D and its direction changes from point to point on the surface. A small surface area, $\overline{ds}$, on the surface can be represented by a vector. Its magnitude is represented by ds and its direction, which is normal to its surface is represented by $\bar{a}_n$.

Thus, $$\overline{ds} = ds\ \bar{a}_n$$

where $\bar{a}_n$ is a unit vector normal to the surface, ds, directed outwards at point P as shown.

The flux density at point P due to the charges is $\bar{D}$ and directed outward. Its direction is such that it makes an angle, θ, with $\bar{a}_n$.

The flux, dψ, passing out through the surface, ds, is

$$d\psi = |\bar{D}| \cos\theta\ ds$$
$$= |\bar{D}|\, ds \cos\theta$$

or, $$d\psi = \bar{D} \cdot \overline{ds} \tag{1.9}$$

i.e. dψ is the dot product of $\bar{D}$ and $\overline{ds}$ (remember dot product of $\bar{A}$ and $\bar{B}$ is $\bar{A} \cdot \bar{B} = |A||B| \cos\theta_{AB}$)

To calculate the total flux passing through the entire area we take the surface integral of Eq. (1.9) as

$$\psi = \int d\psi = \oint_s \bar{D} \cdot \overline{ds} \tag{1.10}$$

The integration sign $\oint$ indicates integration over a closed surface. The closed surface over which integration is carried out is called Gaussian surface.

The mathematical representation of Gauss's law is given as

$$\psi = \oint_s \bar{D} \cdot \overline{ds} = Q = \text{Charge enclosed} \tag{1.11}$$

The total flux passing through the closed surface is equal to the charge enclosed. This is irrespective of the shape and size of the enclosed area and charge distributions.

The charge enclosed may be a point charge, a line charge, a surface charge or a volume charge:

(i) If there are a number of point charges, $Q_1, Q_2, Q_3, \cdots + Q_n$ enclosed by the surface then, $\psi = Q = \Sigma Q_n$.
(ii) If there is a line charge with line charge density ρ_L then, $\psi = Q = \int_L \rho_L dl$.
(iii) If there is a surface charge with charge density ρ_S then, $\psi = Q = \int_S \rho_S \, ds$.
(iv) If there is a volume charge with volume charge density ρ_v then $\psi = Q = \int_v \rho_v \, dv$.

Gauss's law can be represented as

$$\psi = Q = \oint_S \overline{D} \cdot \overline{ds} = \int_v \rho_v dv \tag{1.12}$$

A Gaussian surface is a closed surface over which Gauss's law is applicable. The Gaussian surface will satisfy the following conditions:

(i) The surface must be closed.
(ii) The flux density vector, $\overline{D}$, must either be normal or tangential to the surface at each point on the surface.
(iii) The surface may be irregular but should be sufficiently large so as to enclose the entire charge.

1.9.1 Use of Gauss's Law

Gauss's law can be used to find electric field intensity, $\overline{E}$, or flux density, $\overline{D}$, for symmetrical charge distributions for point charges, line charges, surface charges or spherical distribution of charges. Gauss's law cannot be used to find $\overline{E}$ or $\overline{D}$ if the charge distribution is not symmetrical.

1.9.2 Proof of Gauss's Law

As per Gauss's law, irrespective of the shape of the surface and the charge distribution, the total flux passing through the surface is the total charge enclosed by the surface. Mathematically,

$$\psi = \oint_S \overline{D} \cdot \overline{ds} = Q \quad \text{(Charge enclosed)}$$

Proof

We will prove Gauss's law by considering a spherical surface, which encloses a charge, Q, at its centre as shown in Figure 1.13.

Let $\overline{ds}$ be a small surface area on the sphere. Its magnitude is ds and the direction normal to the surface ds is represented by $\overline{a}_n$ such that

$$\overline{ds} = ds \, \overline{a}_n$$

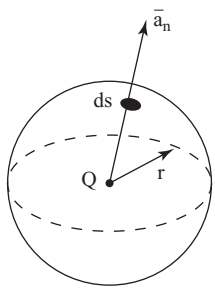


Figure 1.13 *A charge is enclosed by a spherical surface*

where $\bar{a}_n$ is a unit vector in the direction normal to the differential area ds.

The electric flux density, $\bar{D}$, at a point on the surface, which is at a distance r from the point charge Q is

$$|\bar{D}| = \frac{Q}{4\pi r^2}$$

$$D = \frac{Q}{4\pi r^2}\ \bar{a}_r$$

where $\bar{a}_r$ is a unit vector directed radially outward from the point charge and normal to the surface in the direction of $\bar{D}$.

The dot product of $\overline{D}$ and $\overline{ds}$ is

$$\overline{D} \cdot \overline{ds} = \frac{Q}{4\pi r^2}\,\overline{a_r}\ ds\ \overline{a_n}$$

For the spherical surface, as the direction of the unit vectors, $\bar{a}_n$ and $\bar{a}_r$, is the same, $\bar{a}_r\ \bar{a}_n = 1$
Therefore,

$$\overline{D} \cdot \overline{ds} = \frac{Q}{4\pi r^2}\ ds$$

Taking the surface integral of both sides,

$$\oint_s \overline{D} \cdot \overline{ds} = \int_s \frac{Q}{4\pi r^2} ds = \frac{Q}{4\pi r^2} \oint_s ds$$

For a sphere $\oint_s ds = s = 4\pi r^2$

Thus, $$\oint_s \overline{D} \cdot \overline{ds} = \frac{Q}{4\pi r^2} \times 4\pi r^2 = Q$$

The Gauss's law, $\oint_s \overline{D} \cdot \overline{ds} = Q$ is proved.

The proof of Gauss's law depends on the mathematical concept of the solid angle. For this it is best to use multidimensional integrals in spherical polar coordinates (r, θ, ϕ). Let us prove that $\oint_s ds$ for a sphere is $4\pi r^2$. We show ds as a part of a sphere of radius r and centre at the origin (Fig. 1.14).

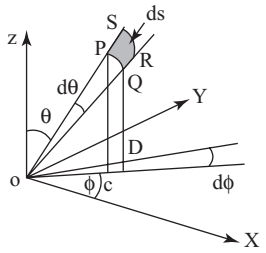


Figure 1.14 *The differential surface of sphere ds is the PQRS of radius r*

We know $$\theta = \frac{\text{Arc}}{\text{Radius}}$$

arc, PQ = r dθ

OC = r sin θ

arc, QR = r sin θ dθ

ds = spherical surface within PQRS = r dθ r sin θ dϕ = r^2 sin θ dθ dϕ

To calculate $\oint_s ds$ over all direction, the limits are from ϕ = 0 to 2π and θ = 0 to π.

The total solid angle subtended by a sphere centered on the origin is

Thus,

$$\oint_s ds = \int_{\theta=0}^{2\pi}\int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$= r^2 \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$= r^2 - [\,-1-1\,]\,2\pi$$

$$= r^2 \times 2 \times 2\pi$$

or, $$\oint_s ds = 4\pi r^2$$

This shows that a sphere subtends a solid angle of 4π steradians at its centre.

1.9.3 Point Form of Gauss's Law

The divergence of electric flux in a medium at a point is equal to the volume charge density at the same point. This is called the point form of Gauss's law and expressed as

$$\overline{V} \cdot \overline{D} = \rho_v \tag{1.13}$$

where $$\overline{V} \cdot \overline{D} = \frac{\partial Dx}{\partial x} + \frac{\partial Dy}{\partial y} + \frac{\partial Dz}{\partial z} \tag{1.14}$$

in the Cartesian coordinate system.

The divergence operation is represented by a mathematical operator called del, $\overline{V}$, which is a vector operator.

The del or divergence of vector flux density $\overline{D}$ at a particular point is a measure of how much the field represented by $\overline{D}$ diverges from or converges at that point.

1.9.4 Divergence Theorem

From Gauss's law, charge, Q, is represented as

$$Q = \oint_s \overline{D} \cdot \overline{ds} \tag{1.15}$$

The charge enclosed in a volume is expressed as

$$Q = \int_v \rho_v dv \tag{1.16}$$

According to the point form of Gauss's law as in Eq. (1.13)

$$\overline{\nabla} \cdot \overline{D} = \rho_v$$

From Eqs. (iii) and (i)

$$Q = \int_v \rho_v dv$$

or,
$$Q = \int_v \left(\overline{\nabla} \cdot \overline{D}\right) dv \tag{1.17}$$

From Eqs. (1.15) and (1.17),

$$\oint_s \overline{D} \cdot \overline{ds} = \int_v \left(\overline{\nabla} \cdot \overline{D}\right) dv \tag{1.18}$$

The above equation is called the divergence theorem. The surface integral can be converted into volume integral with the help of the divergence theorem provided the closed surface encloses some volume charge.

1.9.5 Application of Gauss's Law to Obtain $\overline{D}$ and $\overline{E}$

From the Gauss's law, $\oint_s \overline{D} \cdot \overline{ds} = Q$, let us see how we can obtain $\overline{D}$ and $\overline{E}$.

Field of a point charge

Let the Gaussian surface, i.e. a sphere of radius r enclose a point with charge Q.

Flux density $\overline{D}$ is in radial direction $\overline{a}_r$.

$$\left|\overline{D}\right| = \left|\overline{D}\right| \overline{a}_r$$

$$\overline{ds} = ds\ \overline{a}_r = r^2 \sin\theta\ d\theta\ d\phi\ \overline{a}_r \cdot \overline{a}_r$$

$$\overline{D} \cdot \overline{ds} = \left|\overline{D}\right| r^2 \sin\theta\ d\theta\ d\phi$$

$$\overline{D} \cdot \overline{ds} = \left|\overline{D}\right| r^2 \sin\theta\ d\theta\ d\phi$$

Now integrating over the whole surface $\oint_s \overline{D} \cdot \overline{ds} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left|\overline{D}\right| r^2 \sin\theta\ d\theta\ d\phi$

$$= \left|\overline{D}\right|\ r^2\ \left[-\cos\theta\right]_0^{\pi}\ \left[\phi\right]_0^{2\pi}$$

$$\oint_s \overline{D} \cdot \overline{ds} = 4\pi r^2 \left|\overline{D}\right| \tag{1.19}$$

$$\oint_s \overline{D} \cdot \overline{ds} = Q \tag{1.20}$$

From Eqs. (1.19) and (1.20),

$$4\pi r^2 \left|\overline{D}\right| = Q$$

$$|\overline{D}| = \frac{Q}{4\pi r^2} \quad \text{or} \quad \overline{D} = \frac{Q}{4\pi r^2}\overline{a_r} \tag{1.21}$$

$$\overline{E} = \frac{\overline{D}}{\varepsilon_0} = \frac{Q}{4\pi r^2 \varepsilon_0}\overline{a_r} \tag{1.22}$$

Coulomb's law also gave the same results. This is the field intensity at a radius r from a point charge Q. It can be concluded that the field outside the charge sphere is the same as if the charge Q were concentrated at the centre.

Potential at a radius r is $V = -\int_{\infty}^{r} E\ dl = -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r^2} dr = -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r}$

or, $V = \frac{Q}{4\pi\varepsilon_0 r}$ volts.

Therefore, the following steps can be used to obtain $\overline{D}$ and $\overline{E}$ using Gauss's law:

(i) Consider $|\overline{D}|$ and its direction

(ii) Consider $|\overline{ds}|$ and its direction

(iii) Take dot product, $\overline{D} \cdot \overline{ds}$

(iv) Choose a Gaussian surface and integrate over the surface

(v) Equate the charge enclosed with the integration obtained

(vi) Evaluate for $|\overline{D}|$ and express with its direction

(vii) Express $\overline{E}$ as $\overline{E} = \frac{\overline{D}}{\varepsilon_0}$

A charged wire of infinite length

Let the infinite line charge be along the z-axis as shown in Fig. 1.15. There cannot be any component of $\overline{D}$ along the z-axis. The $\overline{D}$ will have radial component. We will consider a Gaussian surface as a right circular cylinder of length L and radius r along the z-axis, as has been shown. The flux density at any point on the surface is directed radially at outward. $\overline{a_r}$ is the direction of $\overline{D}$.

$$\overline{D} = |\overline{D}|\ \overline{a_r}$$

$$\overline{ds} = r\ d\phi\ dz\ \overline{a_r} \quad \left(\therefore\ d\phi = \frac{\text{arc}}{\text{radius}} \text{ or arc} = r\ d\phi\right)$$

$\overline{a_r}$ is normal to ds.

$$\therefore \quad Q = \oint_s \overline{D} \cdot \overline{ds}$$

$$= \oint_s |\overline{D}|\, r\ d\phi\ dz\ (\overline{a_r} \cdot \overline{a_r})$$

$$= \oint_s |\overline{D}|\, r\ d\phi\ dz$$

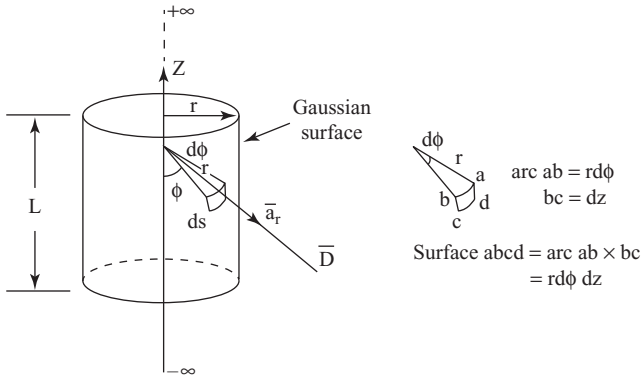


Figure 1.15 *Infinite line charge determination of $\bar{D}$ and $\bar{E}$*

$$Q = \int_{z=0}^{L}\int_{\phi=0}^{2\pi} |\bar{D}|\ r\ d\phi\ dz$$

$$= r\ |\bar{D}|\ [Z]_0^L\ \ [\phi]_0^{2\pi}$$

$$= 2\pi rDL$$

$$D = \frac{Q}{2\pi rL} = \frac{Q}{L}\times\frac{1}{2\pi r} = \frac{\rho_L}{2\pi r} \quad \left(\because \frac{Q}{L} = \rho_L \text{ C/m}\right) = \text{charge density}$$

$$\bar{D} = \frac{Q}{2\pi r}\bar{a}_r \text{ C/m}^2$$

or,

$$\bar{D} = \frac{\rho_L}{2\pi r}\bar{a}_r \text{ C/m}^2$$

$$\bar{E} = \frac{\bar{D}}{\varepsilon_0} = \frac{\rho_L}{2\pi r\varepsilon_0}\bar{a}_r \text{ V/m} \tag{1.23}$$

The potential difference between any two points, A and B, at a distance of a and b, respectively from the centre of a charged wire having a charge density ρ_L coulomb/m is calculated as

$$V_A - V_B = -\int_a^b \bar{E}\cdot\bar{dr}$$

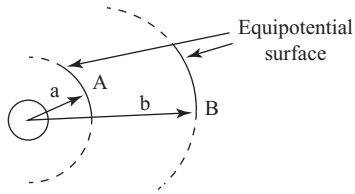


Figure 1.16 *Potential difference between two points*

The field is normal to the axis and directed away from the axis. The field is the same at equal distances from the axis, thus, forming equipotential surfaces as shown.

$$V_A - V_B = V_{AB} = -\int_b^a \frac{\rho_L \, dr}{2\pi\varepsilon_0 r} \, \bar{a}_r \cdot \bar{a}_r$$

or,
$$V_{AB} = \frac{\rho_L}{2\pi\varepsilon_0} \int_b^a \frac{dr}{r} = \frac{\rho_L}{2\pi\varepsilon_0} \log \frac{b}{a}$$

The potential difference between any point on the charged conductor and a point B at a distance b from the wire can be calculated by putting the value of the radius of the wire in place of 'a' in the above expression.

Infinite sheet with uniform charge distribution

Let us consider an infinite sheet on X–Y plane as shown in Fig. 1.17. The charge density is uniform and is designated as ρ_s. A differential surface on the infinite sheet is ds. Consider a long rectangular box as the Gaussian surface enclosing the area ds such that $ds = dx\ dy$. $\bar{D}$ is normal to the plane of the sheet. The unit vectors are $\bar{a}_n = \bar{a}_z$ and $-\bar{a}_n = -\bar{a}_z \cdot \bar{D}$ is perpendicular to the plane of the sheet having no component in X and Y direction.

Since $\bar{D}$ has no component in the X or Y direction, we will consider only the top and bottom surface of the rectangular box.

According to Gauss's law, Charge enclosed $= Q = \oint_s \bar{D} \cdot \bar{ds}$

$$Q = \oint \bar{D} \cdot \bar{ds} \text{ for the top surface} + \oint \bar{D} \cdot \bar{ds} \text{ for bottom surface.}$$

$$\left.\begin{aligned} \bar{ds} &= dx\ dy\ \bar{a}_z \\ \bar{D} &= D_z\ \bar{a}_z \end{aligned}\right\} \text{ for the top surface.}$$

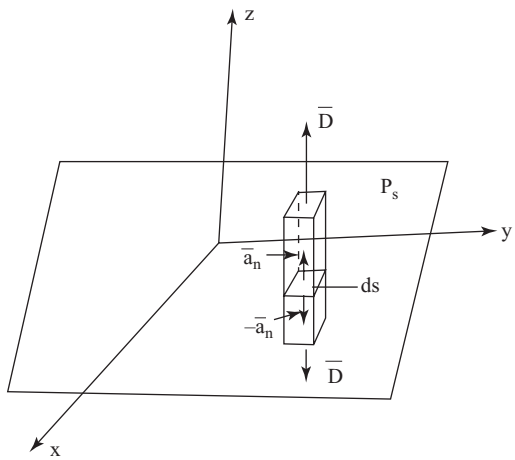


Figure 1.17 *Infinite sheet with uniform charge distribution*

$$\left.\begin{aligned}\overline{ds} &= dx\ dy\ (-\bar{a}_z)\\ \overline{D} &= D_z\ (-\bar{a}_z)\end{aligned}\right\}\ \text{for the bottom surface}$$

and

$$Q = \oint_{top} D_z\ dx\ dy\ \bar{a}_z . \bar{a}_z + \oint_{bottom} D_z\ dx\ dy\ \bar{a}_z . \bar{a}_z$$

$$= \oint D_z\ dx\ dy + \oint D_z\ dx\ dy$$

$$= 2\ D_z \oint dx\ dy$$

or,

$$Q = 2\ D_z A$$

where A is the total surface area.
Again, Charge = Charge density × Area

$$Q = \rho_s \times A$$

Surface,

$$\rho_s \times A = 2\ D_z\ A$$

$$D_z = \frac{\rho_s}{2}$$

$$\overline{D} = D_z\ \bar{a}_z = \frac{\rho_s}{2}\ \bar{a}_z \text{ coulomb/m}^2$$

$$\overline{E} = \frac{\overline{D}}{\varepsilon_0} = \frac{\rho_s}{2\varepsilon_0}\ \bar{a}_z \text{ V/m}$$

A spherical shell

Consider an imaginary thin spherical shell of radius, a, in air as shown in Fig.1.18(a). The charge is uniformly distributed over its surface. The charge density is ρ_s C/m^2. Using Gauss's law, we can calculate the flux density, $\overline{D}$, and field intensity, $\overline{E}$, on the surface, inside and outside the shell. Figure 1.18 (a) and (b) show a thin spherical shell with uniformly distributed charge.

The radius of the shell's surface is 'a.' The Gaussian surface shown by a dotted shell in Fig. 1.19 (a) is having a radius r < a.

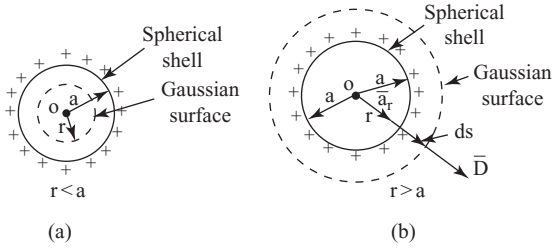


Figure 1.18 *Thin imaginary shell with uniformly distributed charge on the surface*

It can be seen from Fig. 1.18 (a) that the Gaussian surface does not enclose any charge. According to Gauss's law, when the Gaussian surface does not enclose any charge,

$$Q = \oint_s \overline{D} \cdot \overline{ds} = 0$$

In spherical co-ordinate,

$$\oint_s \overline{ds} = \oint r^2 \sin\theta d\theta d\phi$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$\oint_s \overline{ds} = r^2 [-\cos\theta]_0^{\pi} \; [\phi]_0^{2\pi}$$

$$= 4\pi r^2$$

We had

$$Q = \oint_s \overline{D} \cdot \overline{ds} = 0$$

We have seen $\oint_s \overline{ds}$ is not equal to zero.

Hence, $\overline{D} = 0$ and $\overline{D} = \varepsilon_0 \overline{E}$

So,

$$\overline{E} = \frac{\overline{D}}{\varepsilon_0} = 0 \tag{1.24}$$

The conclusion is that the electric flux density, $\overline{D}$, and electric field intensity, $\overline{E}$, at any point inside a spherical shell with uniform charge distribution on its surface is zero.

Now we will consider Fig. 1.18 (b) where the Gaussian surface is outside the sphere where r>a.

$$\overline{D} \cdot \overline{ds} = [D\overline{a}_r] \cdot [r^2 \sin\theta d\theta d\phi \overline{a}_r]$$

$$= Dr^2 \sin\theta d\theta d\phi \quad \because \; \overline{a}_r \cdot \overline{a}_r = 1$$

$$Q = \int_s Dr^2 \sin\theta d\theta d\phi$$

$$= Dr^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta d\theta d\phi$$

$$= Dr^2 \left[-\cos\theta\right]_0^{\pi} \; [\phi]_0^{2\pi}$$

$$= Dr^2 2 \times 2\pi$$

$$Q = 4\pi r^2 D$$

$$|D| = \frac{Q}{4\pi r^2}$$

$$\overline{D} = |D|\overline{a}_r = \frac{Q}{4\pi r^2}\overline{a}_r \text{ C/m}^2$$

$$\overline{D} = \varepsilon_0 \overline{E}$$

$$\overline{E} = \frac{\overline{D}}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2}\overline{a}_r \text{ V/m}$$

Thus, for r > a, the electric field is

$$\bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\bar{a}_r \text{ V/m} \tag{1.25}$$

Considering surface charge density as ρ_s C/m^2,
Charge Q = Charge density × Area

$$Q = \rho_s \times 4\pi a^2$$

$$\bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\bar{a}_r$$

Substituting,

$$= \frac{\rho_s \times 4\pi a^2}{4\pi\varepsilon_0 r^2}\bar{a}_r$$

or,
$$\bar{E} = \frac{\rho_s a^2}{\varepsilon_0 r^2}\bar{a}_r \text{ V/m}$$

and $\bar{D} = \varepsilon_0 \bar{E}$; So,
$$\bar{D} = \frac{\rho_s a^2}{r^2}\bar{a}_r \text{ V/m.}$$

Field intensity, $\bar{E}$, on the surface of the shell is calculated by putting r = a in Eq. (1.24) as

$$\bar{E} = \frac{Q}{4\pi\varepsilon_0 a^2}\bar{a}_r \text{ V/m} \tag{1.26}$$

Uniformly charged sphere or a charged cloud

Consider a sphere of radius 'a' having a uniform charge density ρ_v coulomb/m^3. We will calculate the field outside the sphere, i.e. at a radial distance, r > a. We will then calculate the field on the sphere, i.e. for r = a. Lastly, we will determine the electric field inside the sphere where r < a.

(i) Field outside the sphere (charged cloud).
For the differential area ds,

$$ds = r^2 \sin\theta \, d\theta d\phi$$

$$\overline{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \overline{a_r}$$

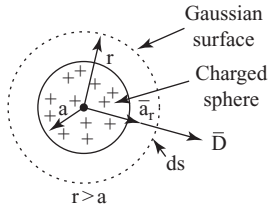


Figure 1.19 *A charged sphere having uniform charge density*

$$\overline{D} = |D|\,\overline{a}_r$$

$$\overline{D}\cdot\overline{ds} = |D|\,r^2\sin\theta d\theta d\phi\,\overline{a}_r\cdot\overline{a}_r$$

$$= Dr^2\sin\theta d\theta d\phi \quad (\because\ \overline{a}_r\cdot\overline{a}_r = 1)$$

According to Gauss's law,

$$\psi = Q = \oint_s \overline{D}\cdot\overline{ds}$$

$$= \int_{=0}^{2\pi}\int_{\theta=0}^{\pi} Dr^2\sin\theta d\theta d\phi$$

$$= Dr^2\left[-\cos\theta\right]_0^{\pi}\ \left[\phi\right]_0^{2\pi}$$

or, $$Q = Dr^2 4\pi$$

$\therefore$ $$D = \frac{Q}{4\pi r^2}$$

$$\overline{D} = \frac{Q}{4\pi r^2}\overline{a}_r\ \text{C/m}^2$$

$$\overline{D} = \varepsilon_0\overline{E}$$

So, $$\overline{E} = \frac{\overline{D}}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2}\overline{a}_r\ \text{V/m.}$$

Q is the total charge enclosed which is calculated as,

$$Q = \int_v \rho_v dv = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{a}\rho_v r^2\sin\theta dr d\theta d\phi$$

$$= \rho_v\left[\frac{r^3}{3}\right]_0^a\ \left[-\cos\theta\right]_0^{\pi}\ \left[\phi\right]_0^{2\pi}$$

$$= \rho_v\frac{a^3}{3}\left[-(-1-1)\right]2\pi$$

$$= \frac{4}{3}\pi a^3\rho_v\ \text{coulomb.}$$

Total charge = Volume × Volume charge density
Substituting the value of Q,

$$\overline{D} = \frac{Q}{4\pi r^2}\overline{a}_r = \frac{4\pi a^3\rho_v}{3\times 4\pi r^2}\overline{a}_r$$

$$\overline{D} = \frac{a^3\rho_v}{3r^2}\overline{a}_r \qquad (1.27)$$

and $$\bar{E} = \frac{\bar{D}}{\varepsilon_0} = \frac{a^3 \rho_v}{3\varepsilon_0 r^2} \bar{a}_r \tag{1.28}$$

(ii) Field on the surface of the sphere (charged cloud)

We will put r = a in Eqs. (1.27) and (1.28) so that

$$\bar{D} = \frac{a\rho_v}{3} \bar{a}_r \tag{1.29}$$

and $$\bar{E} = \frac{a\rho_v}{3\varepsilon_0} \bar{a}_r \tag{1.30}$$

(iii) Field inside the sphere (charged cloud)

For r>a, we had

$$\bar{E} = \frac{Q}{\varepsilon_0 4\pi r^2} \bar{a}$$

and $$\bar{D} = \frac{Q}{4\pi r^2} \bar{a}$$

where $$Q = \frac{4}{3} \pi a^3 \rho_v$$

For r < a, charge will be enclosed by the sphere of radius, r, only. The charge of area covered by radius beyond r and up to a will not affect $\bar{E}$ or $\bar{D}$.

Therefore, $$\bar{D} = \frac{\frac{4}{3}\pi r^3 \rho_v}{4\pi r^2} \bar{a}_r$$

or, $$\bar{D} = \frac{r}{3} \rho_v \bar{a}_r$$

and $$\bar{E} = \frac{\bar{D}}{\varepsilon_0} = \frac{r}{3\varepsilon_0} \rho_v \bar{a}_r \tag{1.31}$$

1.10 CAPACITORS

Capacitors are used to store energy. They consist of a pair of conductors with a potential difference maintained between them. Large capacitors are used in power systems for power factor improvement. On the other hand, memory chips incorporate millions of tiny capacitors. Such tiny capacitors represent the number 1 and 0 depending upon whether they are charged or uncharged. These memory chip capacitors operate with a few volts potential difference between the conductors separated by silicon oxide insulators (dielectric). Capacitors are also used in electrical ac circuits, e.g., in tuning circuits where a resonance condition is created.

In its simplest form, a capacitor is an electrical device constructed of two conducting plates separated by a dielectric material. Connecting leads are attached to the two plates. In general, a capacitor (also called a condenser) is an electrical device having two conductors separated by a dielectric material. The capacitance of a capacitor is defined as the ratio of the charge on one of its conductors to the potential difference between them. Symbolically, the capacitance of a capacitor, C, is expressed as

$$C = \frac{Q}{V} \text{ coulombs per volt (C/V)} \tag{1.32}$$

where Q is the charge on the conductor or the plate and V is the potential difference between the conductors or the plates.

If V = 1 volt and Q = 1 coulomb, then C = 1 farad.

The capacitance of a capacitor is 1 farad if the charge on the plate is 1 coulomb with a potential difference between the plates of 1 volt. Farad is a large unit. In practice, capacitors of micro-farad or pico-farad values are used (micro means 10^{-6} and pico means 10^{-12}).

The unit of capacitance is farad (F). One farad is the amount of capacitance when 1 coulomb of charge is stored with one volt applied across the plates.

1.10.1 Capacitor as a Charge Storing Device

The amount of charge that a capacitor can store per unit voltage across its plates is called its capacitance. Figure 1.20 shows two plates of a capacitor separated by a dielectric. The two plates, in their neutral state, have equal number of free electrons.

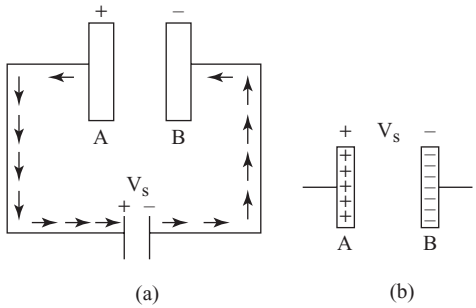


Figure 1.20 *(a) Capacitor storing charges, (b) capacitor retains charge when the source voltage is disconnected.*

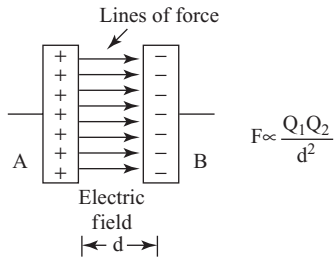


Figure 1.21 *Energy is stored in the electric field in a capacitor*

When the capacitor plates A and B are connected to a dc voltage source V_s, electrons from plate A are removed and deposited on plate B. Plate A loses electrons and the plate B gains electrons. This process continues. This electron flow takes place from one plate to the other via the source voltage. Electron flow does not take place through the dielectric because it is an insulator. The electron flow stops when the electric potential developed across the plates equals the dc voltage applied. At this stage, if the battery is disconnected, the capacitor plates will retain the charges and the voltage, V_s across the plates will exit for a long period of time. The capacitor is said to be in fully charged condition. A charged capacitor can work as a temporary battery. The capacitor stores energy in an electric field , which is established by opposite charges accumulated (stored) on the two plates as shown in Fig. 1.21.There exists a force between a positive and a negative charge on the plates. The force is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them.

1.10.2 Capacitance of a Parallel Plate Capacitor

Let us consider a parallel plate capacitor. Let A be the surface area of the plates and d the distance between them. Both the plates are separated by a dielectric material. If Q is the charge on a plate, then charge density,

$$\rho_s = \frac{Q}{A} \text{ or } Q = \rho_s A \tag{1.33}$$

Electric flux density, $\overline{D}$, in the dielectric medium is the same as ρ_s.

Therefore, $\overline{D} = \rho_s$

$$\overline{E} = \frac{\overline{D}}{\varepsilon} = \frac{\rho_s}{\varepsilon} = \frac{\rho_s}{\varepsilon_o \varepsilon_r} \tag{1.34}$$

where ε_o is the permittivity of free space and ε_r is the relative permittivity.

The value of $$\varepsilon_o = \frac{1}{36\pi \times 10^9} = 8.854 \times 10^{-12} \text{ F/m.}$$

The potential difference between the plates is the integral of electric field $\overline{E}$ over the distance d.

Voltage, $$V = \int_1^2 \overline{E} \cdot \overline{ds} = E\, d \tag{1.35}$$

From Eqs. (1.33), (1.34) and (1.35),

Capacitance, $$C = \frac{Q}{V} = \frac{\rho_s A}{Ed} = \frac{\rho_s A}{\frac{\rho_s}{\varepsilon} d} = \frac{\varepsilon A}{d}$$

$$\therefore C = \frac{\varepsilon A}{d} \text{ farad.} \tag{1.36}$$

Capacitance of a parallel plate capacitor with two or more dielectric mediums between the plates

Let a part of the distance between the plates of a parallel plate capacitor be filled with dielectric material of relative ε_r and the rest with air as has been shown in Figure 1.22. Let E_1 be the electric field intensity of the medium of permittivity ε_r and E_0 is the electric field intensity in air. In the dielectric medium, electric flux density is the same as the surface flux density on the plates.

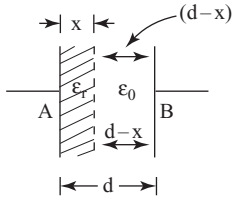


Figure 1.22 *Parallel plate capacitor with two dielectrics*

$$D_1 = \rho_s$$

$$E_1 = \frac{D_1}{\varepsilon} = \frac{\rho_s}{\varepsilon_o \varepsilon_r}$$

In air,
$$E_0 = \frac{D_0}{\varepsilon_0} = \frac{\rho_s}{\varepsilon_0}$$

Potential difference between the plates A and B

$$V = \int \overline{E} \cdot \overline{dl}$$

In this case,

$$V = E_1 x + E_0 (d - x)$$

$$= \frac{\rho_s}{\varepsilon_0 \varepsilon_r} x + \frac{\rho_s}{\varepsilon_0}(d - x) = \frac{\rho_s}{\varepsilon_0}\left[\frac{x}{\varepsilon_r} + d - x\right]$$

Change density,
$$\rho_s = \frac{\text{Charge}}{\text{Area}} = \frac{Q}{A}$$

$$C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s}{\varepsilon_0}\left[\frac{x}{\varepsilon_r} + d - x\right]}$$

or,
$$C = \frac{\varepsilon_0 A}{d + \frac{x}{\varepsilon_r} - x} = \frac{\varepsilon_0 A}{d + x\left(\frac{1}{\varepsilon_r} - 1\right)}$$

Thus, in case of two dielectrics, the capacitance is

$$C = \frac{\varepsilon_0 A}{d + x\left(\frac{1}{\varepsilon_r} - 1\right)} \text{ farad.} \tag{1.37}$$

When two dielectrics of permittivity ε_1 and ε_2 are used covering a distance of x_1 and x_2, respectively between the plates with a potential difference V between them, the capacitance is calculated as

$$V = E_1 x_1 + E_2 x_2$$

or,

$$V = \frac{\rho_s}{\varepsilon_{r_1}} x_1 + \frac{\rho_s}{\varepsilon_{r_2}} x_2$$

$$C = \frac{Q}{V} = \frac{\rho_s A}{\rho_s \frac{x_1}{\varepsilon_1} + \rho_s \frac{x_2}{\varepsilon_2}} = \frac{\rho_s A}{\rho_s \left[\frac{x_1}{\varepsilon_0 \varepsilon_{r_1}} + \frac{x_2}{\varepsilon_0 \varepsilon_{r_2}}\right]}$$

or

$$C = \frac{\varepsilon_0 A}{\left[\frac{x_1}{\varepsilon_{r_1}} + \frac{x_2}{\varepsilon_{r_2}}\right]} \text{ farad.}$$

In the same way it can be shown that the capacitance of a capacitor with multiple dielectrics is

$$C = \frac{\varepsilon_0 A}{\left(\frac{x_1}{\varepsilon_{r_1}} + \frac{x_2}{\varepsilon_{r_2}} + \frac{x_3}{\varepsilon_{r_3}} + ...\right)} \text{ farad.} \tag{1.38}$$

1.10.3 Capacitance of a Spherical Condenser

We have seen that a capacitor is formed between two parallel plate conductors separated by a dielectric.

Capacitors can be made using conducting plates of different shapes. For example, a capacitor can be made using two concentric spherical shells (spheres) of radius r_1 and r_2. When a is less them b, one shell will be inside another shell separated by a distance (b – a). Let a charge, Q, be uniformly distributed over the outer surface of the inner shell and the inner surface of the outer shell as shown in Fig. 1.23.

The potential difference between the surface of the spheres is

$$V_{ab} = -\int_b^a \overline{E}\ dr$$

For a spherical surface, flux density

$$\overline{D} = \frac{Q}{4\pi r^2}\,\overline{a}_r$$

$$\overline{E} = \frac{\overline{D}}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\overline{a}_r$$

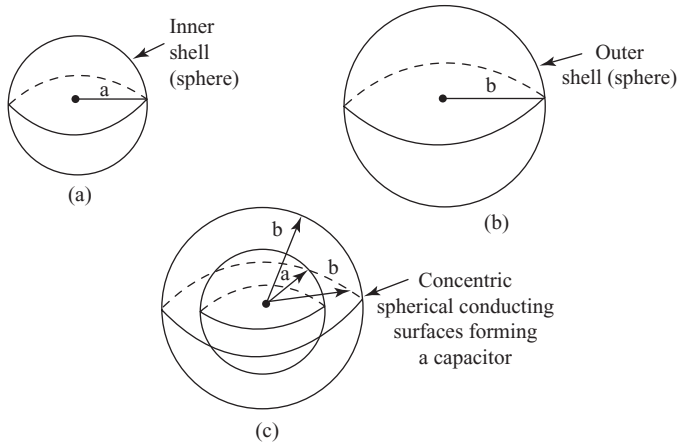


Figure 1.23 *Formation of a spherical capacitor*

$$V_{ab} = -\int_b^a \frac{Q}{4\pi\varepsilon_0}\frac{dr}{r^2} = -\frac{Q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_b^a$$

$$= +\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_b^a$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]$$

$$V_{ab} = \frac{Q}{4\pi\varepsilon_0}\frac{(b-a)}{ab}$$

$$C = \frac{Q}{V_{ab}} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\frac{(b-a)}{ab}} = 4\pi\varepsilon_0\frac{ab}{(b-a)}$$

The capacitance of a spherical capacitor with radius of concentric spheres a and b, respectively (b > a), separated by an air-gap is

$$C = \frac{4\pi\varepsilon_0 ab}{(b-a)} \text{ farad.} \tag{1.39}$$

1.10.4 Capacitance of an Isolated Sphere

When the outer shell of a spherical capacitor has an infinite radius, the inner sphere (shell) becomes an isolated sphere. We have already calculated the capacitance of two concentric spheres forming a spherical capacitor. The radii of the two spheres are a and b, respectively. Now, if we assume the radius

of the outer sphere to be infinity, then the capacitance of an isolated sphere can be obtained. Accordingly, the capacitance of a concentric sphere, $C = 4\pi\varepsilon_0 \dfrac{ab}{(b-a)}$ farad.

or,
$$C = 4\pi\varepsilon_0 \frac{a}{\left(1-\frac{a}{b}\right)}$$

If $b \to \infty$

$$C = \frac{4\pi\varepsilon_0 a}{1-0}$$

The capacitance of an isolated sphere, therefore is

$$C = 4\pi\varepsilon_0 a \ \text{F}$$

where 'a' is the radius of the sphere. This is the capacitance between the isolated sphere with respect to infinity.

It is interesting to note that an isolated sphere with one conducting surface works as a capacitor. The question arises as to know the location of the other conducting surface. The other conducting surface is in fact the earth.

The capacitance of an isolated sphere of radius, a, with respect to earth is

$$C = 4\pi\varepsilon_0 a \ \text{F} \tag{1.40}$$

1.10.5 Capacitance of Parallel Conductors

Overhead transmission and distribution lines used in the transmission and distribution of electricity form parallel conductors. The capacitance between any two lines or between a line and the earth may be required to be calculated.

Let us consider two parallel round conductors having charge density ρ_L C/m along their lengths. The two conductors are separated by a distance d. The wires have radius of r.

Let P be a point at a distance *x* from one conductor. The charge per unit length (see Eq.1.23) on the conductors are $+Q$ and $-Q$, respectively.

The electric field intensity at P due to charge on the two conductors is

$$E_{x_A} = \frac{+Q}{2\pi\varepsilon_0 x} \quad \text{and} \quad E_{x_B} = \frac{-Q}{2\pi\varepsilon_0 (d-x)}, \text{ respectively.}$$

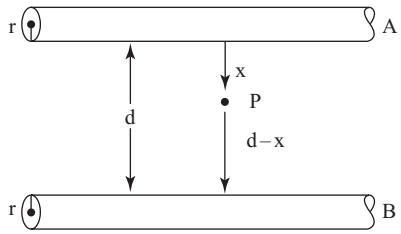


Figure 1.24 *Capacitance between two parallel conductors*

The potential difference between the two conductors, V is $V = \int_r^{d-r} E_x dx$

Integration of E_x is taken from r to d − r.

$$V = \int_r^{d-r} \left[\frac{Q}{2\pi\varepsilon_0 x} - \frac{(-Q)}{2\pi\varepsilon_0 (d-x)} \right] dx$$

$$= \frac{Q}{2\pi\varepsilon_0} \int_r^{d-r} \left[\frac{1}{x} + \frac{1}{d-x} \right] dx.$$

$$V = \frac{Q}{2\pi\varepsilon_0} \left[\log x - \log (d-x) \right]_r^{d-r}$$

$$= \frac{Q}{2\pi\varepsilon_0} \left[\log (d-r) - \log r - \log r + \log (d-r) \right]$$

$$= \frac{Q}{2\pi\varepsilon_0} \times 2 \log \left(\frac{d-r}{r} \right)$$

or,
$$V = \frac{Q}{\pi\varepsilon_0} \log \left(\frac{d-r}{r} \right) \tag{1.41}$$

If we consider the distance between the two conductors, d is much more than the radius of the conductor, r, i.e. d >> r, then (d − r) ≈ d.

Accordingly,

$$V = \frac{Q}{\pi\varepsilon_0} \log \frac{d}{r}$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{\pi\varepsilon_0} \log \frac{d}{r}} = \frac{\pi\varepsilon_0}{\log \frac{d}{r}} \text{ farad.}$$

Thus, the capacitance between two conducting lines of radius r and separated by a distance d is

$$C = \frac{\pi\varepsilon_0}{\log \frac{d}{r}} \text{ F.} \tag{1.42}$$

1.10.6 Capacitance of an Infinitely Long Wire Using the Method of Image Formation

A single wire parallel to the ground at a height of h is shown in Fig. 1.25. This single conductor creates an electric field, which is one half of the field created by a two wire system. The method of image formation can be used to calculate the capacitance of a single wire running parallel to the ground. A charge + Q is uniformly distributed on the conductor, which is at a height, h, from the ground.

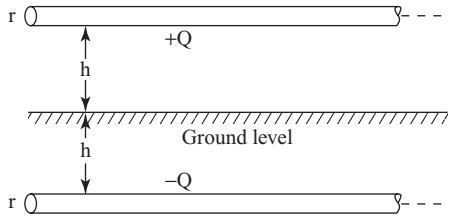


Figure 1.25 *A single wire at a height h from the ground*

A charge of − Q is assumed to exist in an imaginary image conductor below a distance of h from the ground. This is known as the method of image formation. The potential difference between the two conductors are calculated as (see Eq. 1.41)

$$V = \frac{Q}{\pi\varepsilon_0} \log \frac{d - r}{r}.$$

Here, the distance between the actual wire and its image is 2h. Instead of d we will now use d = 2h.

The potential between the actual line and the ground will be half of V.

In this case,

$$V = \frac{1}{2} \frac{Q}{\pi\varepsilon_0} \log \frac{(2h - r)}{r}$$

Considering 2h − r = 2h ($\because$ h >> r)

$$V = \frac{1}{2} \frac{Q}{\pi\varepsilon_0} \log \frac{2h}{r}$$

The capacitance of the long single conductor with the earth as the other conductor is

$$C = \frac{Q}{V} = \frac{Q}{\frac{1}{2} \frac{Q}{\pi\varepsilon_0} \log \frac{2h}{r}}$$

or,
$$C = \frac{2\pi\varepsilon_0}{\log \frac{2h}{r}} \text{ F} \qquad (1.43)$$

1.10.7 Capacitance of Concentric Conductors (Electric Cables)

Electric cables are used for the distribution of electricity. An electric cable has a central core of conducting material such as copper or aluminium. Adequate insulation is provided around the core. A lead sheath covering is provided over the insulation to prevent the entry of moisture. A cross-sectional view of a cable, in its simplest form, is shown in Fig. 1.26.

Let R be the radius of the lead sheath, i.e. the outer conductor and r be the radius of the inner conductor.

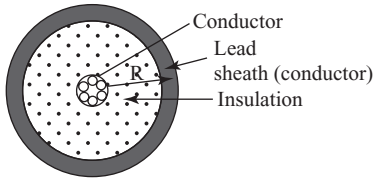


Figure 1.26 *Co-axial cable is equivalent to two concentric conducting materials separated by insulation*

The charge is uniformly distributed on the surfaces of the two concentric conductors. The charge density per unit length is ρ_L C/m.

The electric field intensity at a distance x is E_x.

The potential difference at a point P at a distance x from the centre of the core is

$$V = \int E_x dx$$

$$V = \int_r^R \frac{\rho_L}{2\pi\varepsilon_0 x} dx$$

$$= \frac{\rho_L}{2\pi\varepsilon_0} \int_r^R \frac{dx}{x}$$

$$= \frac{\rho_L}{2\pi\varepsilon_0} \left[\log x\right]_r^R$$

$$= \frac{\rho_L}{2\pi\varepsilon_0} \log \frac{R}{r}$$

Capacitance per unit length, $$C = \frac{\rho_L}{V} = \frac{\rho_L}{\frac{\rho_L}{2\pi\varepsilon_0} \log \frac{R}{r}}$$

or, $$C = \frac{2\pi\varepsilon_0}{\log \frac{R}{r}} \text{ F/m} \qquad (1.44)$$

If we have to calculate capacitance of a cable 1 km long, then

$$C = \frac{2\pi\varepsilon_0 \times 1000}{\log \frac{R}{r}} \text{ F}$$

$$= \frac{2 \times 3.14 \times 8.854 \times 10^{-12} \times 1000}{2.303 \log_{10} \frac{R}{r}} \text{ F}$$

$$C = \frac{24.14 \times 10^{-19}}{\log_{10} \frac{R}{r}} \text{ F}$$

Example 1.5 Calculate the capacitance of a co-axial cable 1 km long with the details shown in Fig. 1.27. A potential of 11 kV is applied between the inner conductor and the outer conductor, which is at earth's potential.

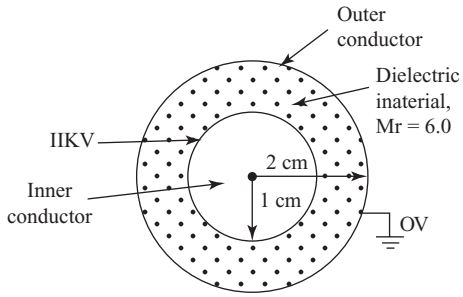


Figure 1.27

Solution:

This is a co-axial cable with inner radius r and outer radius R.

The capacitance per unit length is

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\log\frac{R}{r}}$$

For calculating the capacitance for 1 km length, we have to multiply the value of C by 1000 m. So, the capacitance of the cable of length 1 km is

$$C = \frac{2\pi\varepsilon_0\varepsilon_r \times 1000}{\log\frac{R}{r}}$$

Substituting values,

$$C = \frac{2 \times 3.14 \times 8.854 \times 10^{-12} \times 6}{\log_e \frac{2}{1}}$$

$$= \frac{6.28 \times 8.854 \times 10^{-12} \times 6}{2.303 \log_{10} 2}$$

Energy stored in a capacitor

Capacitors are used to store electric charge. Capacitance is a measure of the ability of a capacitor to store charge. The amount of charge that a capacitor can store per unit of voltage across its plates is called its capacitance. The capacitance is the charge per unit of voltage. Thus,

$$C = \frac{Q}{V}$$

where C is the capacitance, Q is the charge stored, and V is the voltage applied.

We know that opposite charges are distributed on the plates of a capacitor. Lines of force are established between the positive and negative charges. Many opposite charges distributed on the

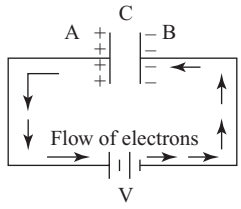


Figure 1.28 *Charging of a capacitor*

plates of a capacitor create many lines of force. An electric field is established between the opposite charges stored in the two plates or surfaces. A force exists between the charges on the opposite plates. Coulomb's law has defined that force. The greater the force between the charges on the plates, the more the energy stored. Let us consider a capacitor of C farads connected across a dc source of V volts as shown in Fig. 1.28. The electrons are transferred from plate A to plate B due to the application of this voltage as shown in the figure.

This involves expenditure of energy because electrons are moved against opposite forces—electrons are pulled from the plate A and are being pushed to plate B. During the charging process, work is being done in moving the electrons (i.e. charge) from one plate to the other. This work done is stored in the capacitor in the form of electrostatic field set up between the plates. Voltage builds up between the plates when electrons (charge) are being shifted from one plate to the other.

Let dq be the charge shifted at any point of time and v the potential across the plates, then work done dw is

$$dw = v\, dq$$

and

$$q = C\, v$$

so,

$$dw = v\, d\, q$$

$$= v\, d\, (C\, v)$$

$$dw = C\, v\, d\, v$$

The total work done during charging when the potential between the plates is raised from 0 V to V volts,

$$\int_0^V dw = \int_0^V C\, v dv = C\int_0^V v\, dv$$

or,

$$W = C\left[\frac{v^2}{2}\right]_0^V$$

or,

$$W = \frac{1}{2}CV^2 \text{ joules.}$$

$$= \frac{1}{2}\frac{Q}{V}V^2 = \frac{1}{2}QV \text{ joules.}$$

Energy stored in a capacitor is

$$W = \frac{1}{2}\, CV^2 \tag{1.45}$$

Energy stored in a capacitor (alternative method)

The energy stored in a capacitor can be found by calculating the work done in charging the capacitor. We know that potential is defined as the work done per unit charge. The work done in moving a small charge, dq, against a potential v is vdq. Again $v = \frac{q}{c}$

Work done, $$dw = v\, dq = \frac{q}{c}\, dq$$

The initial charge of the capacitor is zero. The work done in charging till a total charge of Q is attained is

$$\int dw = \int_0^Q \frac{q}{C} dq$$

$$= \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$= \frac{Q^2}{2C}$$

$$= \frac{C^2V^2}{2C} \quad (\therefore C = \frac{Q}{V})$$

$\therefore$ $$W = \frac{1}{2}CV^2 = \frac{1}{2}VQ$$

Energy is stored in the electric field between the plates. The energy stored in the electric field of a capacitor is

$$W = \frac{1}{2}\int \bar{D} \cdot \bar{E} dv$$

$$= \frac{1}{2}\int_a \varepsilon_0 \bar{E}.\bar{E} dv = \frac{1}{2}\varepsilon_0 E^2 \ J/m^3$$

where dv is a small cubical volume in the space between the plates.

If the dielectric material between the plates has permittivity ε_r,

$$\bar{D} = \varepsilon \bar{E} = \varepsilon_0 \varepsilon_r \bar{E}$$

If d is the distance between the plate, A the area of each plate and Q the total charge, then

$$\bar{D} = \frac{Q}{A}\bar{a}_x$$

and $$\bar{E} = \frac{V}{d}\bar{a}_x$$

$$W = \frac{1}{2}\int_v \bar{D} \cdot \bar{E} dv$$

$$= \frac{1}{2}\int_v \frac{Q}{A}\bar{a}_x \cdot \frac{V}{d}\bar{a}_x dv$$

$$= \frac{1}{2}\int \frac{CV}{A}\bar{a}_x \frac{V}{d}\bar{a}_x dv \qquad (\bar{a}_x.\bar{a}_x = 1)$$

$$= \frac{1}{2}cv^2 \int \frac{dv}{Ad}$$

$$= CV^2 \frac{v}{v} = \frac{1}{2}CV^2 \qquad (\because Ad = v)$$

Energy stored in an electric field in terms of $\bar{D}$ and $\bar{E}$

The amount of work done in bringing a unit positive charge from infinity to a given point in an electric field is called the potential of that point.

$$V = \frac{\text{Work}}{\text{Charge}} = \frac{W}{Q}$$

or, $$W = QV$$

The total energy stored in a system of n point charges is given as

$$W_t = \frac{1}{2}\sum_{i=1}^{n} Q_i V_i$$

If $Q_i = \int_v \rho_v dv$, the energy stored in an electric field, W_e is

$$W_e = \frac{1}{2}\int \rho_v v \, dv$$

According to Maxwell's first equation or point form of Gauss's equation,

$$\rho_v = \bar{V}.\,\bar{D}$$

and knowing $$\bar{D} = \varepsilon_0 \bar{E},$$

$$\bar{V}.\,\bar{E} = \frac{\rho_v}{\varepsilon_0}$$

So, $$W_e = \frac{1}{2}\int_v \varepsilon_0 \left(\bar{V}\cdot\bar{E}\right) V \, dv$$

From the standard vector identity we have,

$$\left(\bar{V}\cdot\bar{E}\right)V = \bar{V}\cdot\bar{V}\bar{E} - E\cdot\bar{V}V$$

Therefore,

$$W_e = \frac{1}{2}\varepsilon_0 \int_v \left(\overline{V} \cdot \overline{V}\overline{E} - E \cdot \overline{V}V\right) dv$$

$$= \frac{1}{2}\varepsilon_0 \int_v \overline{V} \cdot \overline{V}\overline{E}\ dv + \frac{1}{2}\varepsilon_0 \int_v \overline{E} \cdot \overline{E}\ dv \quad \left(\because\ \overline{E} = -\overline{V}\ V\right)$$

Applying the divergence theorem,

$$\int_v \overline{V} \cdot V\ E dv = \int_s \overline{V}\ \overline{E} \cdot ds$$

$$\frac{1}{2}\varepsilon_0 \int_s \overline{V}\overline{E} \cdot \overline{ds} = 0 \qquad \text{since closed surface integral is zero.}$$

So, $$W_e = +\frac{1}{2}\varepsilon_0 \int_s \overline{E} \cdot \overline{E}\ dv$$

or, $$W_e = \frac{1}{2}\int_s \varepsilon_0 E^2\ dv \quad \left(\because\ \overline{E} \cdot \overline{E} = E^2\right)$$

The expression for total energy stored, W_e

$$W_e = \frac{1}{2}\int_v \varepsilon_0 E^2\ dv \ \text{ joules}$$

or $$W_e = \frac{1}{2}\int_v \frac{D^2}{\varepsilon_0}\ dv \ \text{ joules}$$

$$W_e = \frac{1}{2}\int_v \varepsilon_0 \frac{\overline{D}}{\varepsilon_0} \cdot \overline{E}\ dv \quad \text{or} \quad W_e = \frac{1}{2}\int_v \overline{D} \cdot \overline{E}\ dv$$

Energy density, $$\frac{dW_e}{dv} = \frac{1}{2}\overline{D} \cdot \overline{E} \ \text{ J/m}^3$$

Example 1.6 Three positive charges of 2×10^{-9} C are placed on the three corners of a square of side 0.25 m^2. Calculate the magnitude and direction of the electric field intensity in the fourth corner of the square.

Solution:

Each side of the square is 0.5 m. The positions of the charges are shown.

Repelling forces will be developed at point D. The electric field at D due to charge at A is EAD. The electric field at D due to charge at C is ECD and the electric field at D due to charge at B is E_{BD}. Their directions have been shown in Fig. 1.29 (b).

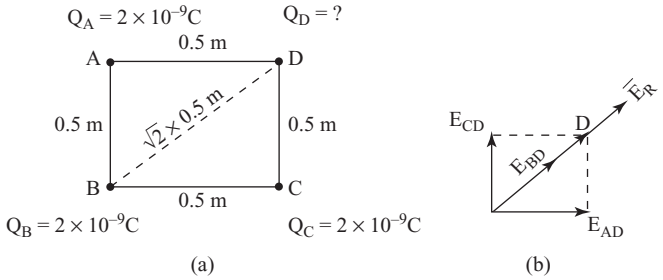


Figure 1.29

$$E_{AD} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-9}}{(0.5)^2}$$

$$= \frac{18}{10^{-2} \times 25} = 4 \times 18 = 72 \text{ V/M}$$

$$E_{CD} = E_{AD} = 72 \text{ V/m}$$

$$E_{BD} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\sqrt{2} \times 0.5\right)^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-9}}{2 \times 0.25}$$

$$= 36 \text{ V/m}$$

Resultant of E_{AD} and $E_{CD} = \sqrt{2} \cdot 72$

$$= 101.8 \text{ V/m}$$

This resultant and E_{BD} are in the same direction. Hence, the electric field intensity at

$$E_R = 101.8 + 36$$
$$= 137.8 \text{ V/m.}$$

Example 1.7 A parallel plate capacitor consists of two sheets of copper foil separated by a 1 mm thick plastic insulating sheet. The area of each copper foil is 0.2 m^2. The relative permittivity of plastic used is 5.2. Calculate the capacitance. What will be the energy stored when the capacitor is connected across a 100 V dc supply?

Solution:

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

Given, $\varepsilon_r = 5.2,\ A = 0.2\,m^2,\ d = 1 \times 10^{-3}\,m,\ \varepsilon_0 = 8.854 \times 10^{-12}$

$$C = \frac{8.854 \times 10^{-12} \times 5.2 \times 0.2}{1 \times 10^{-3}}$$

$$C = 8.854 \times 5.2 \times 0.2 \times 10^{-9}\ F$$

or, $$C = 9.208 \times 10^{-9}\ F$$

or, $$C = 9.208 \times 10^{-6}\ F$$

$$\text{Energy stored} = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 9208 \times 10^{-6} \times 100^2 \text{ joules}$$

$$= 4604 \times 10^{-2} \text{ joules}$$

$$= 46.04 \text{ joules.}$$

Example 1.8 The radii of the two spheres of a spherical condenser differ by 0.5 m. The space between the two spheres is filled with a dielectric material of relative permittivity 2.5. The capacitance of the capacitor is one nanofarad. Calculate the radius of each sphere.

Solution:

Given, $C = 1 \times 10^{-9}$ F, $\propto_r = 2.5$ $\mu_0 = 8.854 \times 10^{-12}$, $b - a = 0.5$ M, where a and b are the radii of the two concentric spheres. The capacitance of a spherical condenser is

$$C = 4\pi\varepsilon_0\varepsilon_r \frac{ab}{b-a}$$

(b − a) is given as 0.5

$$ab = \frac{C\,(b-a)}{4\pi\varepsilon_0\varepsilon_r} = \frac{C}{4\pi\varepsilon_0} \times \frac{b-a}{\varepsilon_r}$$

$$= 9 \times 10^9 \times 1 \times 10^{-9} \times \frac{0.5}{2.5}$$

$$= 1.8$$

We know, ab = 1.8 and (b − a) = 0.5
We have to calculate a and b.
We write

$$b + a = \sqrt{(b-a)^2 + 4ab}$$

Substituting values,

$$b + a = \sqrt{(0.5)^2 + 4 \cdot 1.8}$$

$$= \sqrt{0.25 + 7.2}$$

$$= \sqrt{7.45}$$

$$b - a = 0.5$$

or $\quad b + a = 2.73 \quad$ and $\quad b + a = 2.73$

$$2b = 3.23$$

$$b = 1.615 \text{ m} \quad \text{and} \quad a = 1.115 \text{ m}$$

Example 1.9 A ring of inner diameter R and outer diameter R + 2 in the x−y plane with $z = 0$, has uniform surface charge density ρ_s. Calculate the potential at the centre of the ring.

Solution:

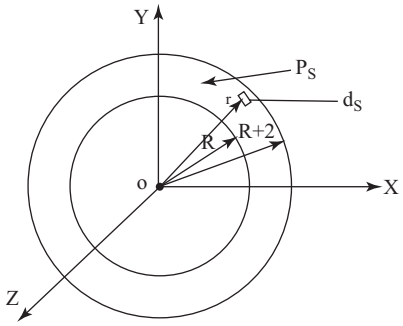


Figure 1.30

Let ds be a differential surface area on the ring. The differential area is at a distance r from the centre.

$$dQ = \rho_s \, ds$$

or, $$dQ = \rho_s r \, d\phi \, dr$$

$$dv = \frac{dQ}{4\pi\varepsilon_0 r} = \frac{\rho_s \, r \, d\phi \, dr}{4\pi\varepsilon_0 r} = \frac{\rho_s}{4\pi\varepsilon_0} d\phi \, dr$$

or, $$\int dv = \frac{\rho_s}{4\pi\varepsilon_0} \int\int d\phi \, dr$$

$$V = \frac{\rho_s}{4\pi\varepsilon_0} \int_{\phi=0}^{2\pi} \int_{R}^{R+2} d\phi \, dr$$

$$= \frac{\rho_s}{4\pi\varepsilon_0} [\, Q \,]_0^{2\pi} \; [r]_R^{R+2} = \frac{\rho_s}{\varepsilon_0} \text{ volts}$$

Example 1.10 Two point charges of 2 microfarad and −4 μF each are placed at a distance of 1m from each other on a straight line. Determine at what point the potential will be zero.

Solution:

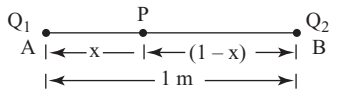


Figure 1.31

Let p be the point at which the potential is zero.
The potential at P due to the charge at A, i.e. V_A is given as

$$V_A = \frac{Q_1}{4\pi\varepsilon_0 x}$$

The potential F at P due to charge at B, i.e. V_B is given as

$$V_B = \frac{-Q_2}{4\pi\varepsilon_0 (1-x)}$$

Since the potential at P should be zero,

$$V_A + V_B = 0$$

$$\frac{Q_1}{4\pi\varepsilon_0 x} + \frac{-Q_2}{4\pi\varepsilon_0 (1-x)} = 0$$

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{x} + \frac{-Q_2}{(1-x)}\right] = 0$$

$$\frac{2\times10^{-6}}{x} - \frac{4\times10^{-6}}{1-x} = 0 \quad \text{or} \quad 2\times10^{-6}\left[\frac{1}{x} - \frac{2}{1-x}\right] = 0$$

or,
$$\frac{1}{x} = \frac{2}{1-x}, \qquad \begin{aligned} 1-x &= 2x \\ 3x &= 1,\ x = \frac{1}{3}\ \text{m} \end{aligned}$$

Example 1.11 A co-axial cable with a central core of 2 mm diameter passes through the centre of a cylinder made of dielectric material diameter 5 mm. The dielectric cylinder is coated from outside with a conducting sheath. The outer conducting sheath is at ground potential. Calculate the capacitance of the co-axial cable of 1 m length. The relative permeability of the dielectric material is 2.5.

Solution:
The electric field at a point with a distance r from the axis of the cable is

$$E_r = \frac{Q}{2\pi\varepsilon_0\varepsilon_r r}$$

where Q is the positive charge per unit length of the inner wire and −Q is the charge per unit length of the outer wire.

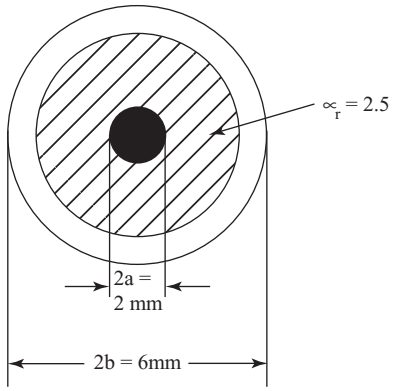


Figure 1.32

The potential difference between radii a and b is

$$V = -\int_a^b \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\frac{dr}{r} = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\log\frac{b}{a}$$

Capacitance of a meter length, $C = \frac{Q}{V} = \frac{2\pi\varepsilon_0\varepsilon_r}{\log\left(\frac{b}{a}\right)}$

Given, $2a = 2$ mm or $a = 1$ mm

$2b = 6$ mm or $b = 3$ mm

$\mu_r = 2.5 \quad \mu_0 = 8.854 \times 10^{-12}$

Substituting values,

$$C = \frac{2 \times 3.14 \times 8.854 \times 10^{-12} \times 2.5}{2.303 \log_{10}\frac{3}{1}} F$$

$$C = \frac{6.28 \cdot 8.854 \cdot 2.5}{2.303 \quad \log_{10} 3} \text{ pico farad} = \frac{60.3}{\log_{10} 3} = \frac{60.3}{0.477} = 126.4 \text{ PF.}$$

Example 1.12 Calculate the energy stored in a parallel plate capacitor, which consists of two metal plates of 64 cm^2 separated by a dielectric 1 mm thick with $\mu_r = 4.0$. A potential difference of 1 kV is applied across it. What amount of heat will be produced when the charged capacitor is discharged with a resistor?

Solution:

Given, $A = 64\ cm^2 = 64 \times 10^{-4}\ m^2$
$d = 1\ mm = 1 \times 10^{-3}\ m$
$\varepsilon_0 = 8.854 \times 10^{-12}$
$\varepsilon_r = 4.0$

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 4.0 \times 64 \times 10^{-4}}{1 \times 10^{-3}}$$

$$= 8.854 \times 4 \times 6.4 \times 10^{-12} = 230 \times 10^{-12} \text{ F}$$

$$\text{Heat energy dissipated} = \frac{\text{Energy stored}}{4.18} = \frac{\frac{1}{2}CV^2}{4.18} = \frac{\frac{1}{2} \times 230 \times 10^{-12} \times (1,500)^2}{4.18}$$

$$= 115 \times 10^{-6} \text{ joules}$$

$$= 27.4 \times 10^{-6} \text{ calories}$$

(since, 1 calorie = 4.18 joules.)

Example 1.13 The plates of a parallel plate capacitor have an area 640 cm^2 each. The relative permittivity of the dielectric used is 5.0 and the distance between the plates is 1 mm. The surface charge density on the capacitor plates is 1×10^{-8} coulomb per cm^2. Calculate the electric flux density and the potential difference between the plates.

Solution:

Distance between the plates, d = 1 mm = 1×10^{-3} m

Area of a plate, A = 640 cm^2 = 0.064 m^2 relative permittivity, $\mu_r = 5.0$

Charge density of the plates, $\rho_s = 1 \times 10^{-8}$ C/cm^2

$$= 1 \times 10^{-4} \text{ C/m}^2$$

Electric flux density, $D = \frac{\Psi}{A} = \frac{Q}{A} = 1 \times 10^{-4}$ C/m^2

Flux density, D, and electric field intensity, E, are related as

$$D = \varepsilon E = \varepsilon_0 \varepsilon_r E$$

so,
$$E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{1 \times 10^{-4}}{8.854 \times 10^{-12} \times 5.0} = 2.24 \times 10^6 \text{ V/m}$$

or,

Potential difference between the charged plates, V is

$$V = E \times d = \text{volts} = 2.24 \times 10^6 \times 1 \times 10^{-3} = 2{,}240 \text{ V.}$$

Example 1.14 A parallel plate capacitor has a distance of 3 cm between the plates. Half the distance is filled with a dielectric slab of relative permittivity 6.0. The rest of the space is filled with air. What will be the electric field intensity in air and in the dielectric medium inside the capacitor when a voltage of 1100 V is applied across the capacitor?

Solution:

The parallel plate capacitor with air and with a dielectric material in the space between the plates has been shown in Fig. 1.33. Let V_1 and V_2 be the potential difference across the air space and the dielectric space, respectively. Let E_1 and E_2 be the electric field intensities, respectively.

$$V_1 = E_1 d_1$$

$$V_2 = E_2 d_2$$

$$V = V_1 + V_2$$

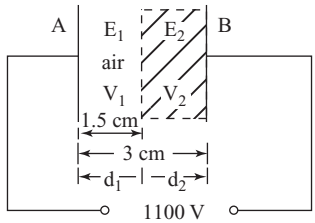


Figure 1.33

$$1100 = E_1 \times 1.5 \times 10^{-2} + E_2 \times 1.5 \times 10^{-2}$$

$$E_1 + E_2 = \frac{1100}{1.5 \times 10^{-2}} = 73333$$

The flux density, D, will be the same in the space between the plates irrespective of the material used.

Again, $D = \varepsilon E$

For air,

$$D_1 = \varepsilon_0 E_1$$

For the dielectric,

$$D_2 = \varepsilon_0 \varepsilon_r E_2$$

$$D_1 = D_2$$

or, $\varepsilon_0 E_1 = \varepsilon_0 \varepsilon_r E_2$

or, $E_1 = \varepsilon_r E_2$

or, $E_1 = 6E_2$

$$E_1 + E_2 = 73333 \quad (1.46)$$

and $E_1 = 6\, E_2 \quad (1.47)$

from Eqs. (1.46) and (1.47),

$$7\, E_2 = 73333$$

or $E_2 = 10476.2$ V/m

and $E_1 = 62856.8$ V/m

Example 1.15 A charge of 1.5×10^{-9} coulomb is at rest at a particular point in free space. Calculate the potential at point P, which is 20 cm away from the charge.

Solution:

Potential at $P = \dfrac{Q}{4\pi\varepsilon_0 r}$

Here, $Q = 1.5 \times 10^{-9}$ C

$r = 20$ cm $= 0.2$ m

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9$$

Substituting values,

$$\text{V at point P} = \frac{1.5\times10^{-9}\times9\times10^9}{0.2} = 67.5 \text{ volts.}$$

Example 1.16 The cross-sectional area of the plates of a parallel plate capacitor is 2000 cm^2. The distance between the plates is 2 mm. The relative permittivity of the dielectric used is 5.0. Each plate has a charge of 0.8×10^{-6} coulomb. Calculate (a) electric flux density in the dielectric medium (b) electric field intensity, and (c) potential difference between the plates.

Solution:

Electric flux density, $$D = \frac{\psi}{A} = \frac{Q}{A}$$

$$= \frac{0.8\times10^{-6}}{2000\times10^{-4}} = 4\times10^{-6} \text{ C/m}^2$$

We know,

$$D = \varepsilon E \quad \text{or} \quad D = \varepsilon_0\varepsilon_r E$$

Electric field intensity, $$E = \frac{D}{\varepsilon_0\varepsilon_r} = \frac{4\times10^{-6}}{8.854\times10^{-12}\times5}$$

$$= 9 \times 10^4 \text{ V/m}$$

Voltage across the plate, $$V = E \times d$$

$$= 9 \times 10^4 \times 2 \times 10^{-3}$$

$$= 180 \text{ Volt}$$

Example 1.17 Three point charges of magnitude $Q = 1 \times 10^{-6}$ coulombs are located in the three corners of an equilateral triangle of side 1 m. Find the force on one charge due to the other two charges.

Solution:

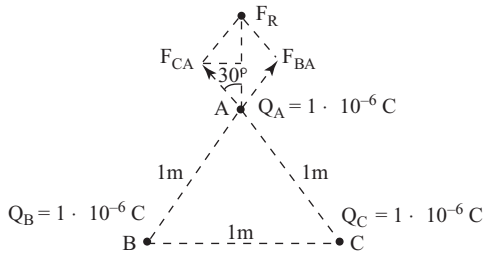


Figure 1.34

There will be a force of repulsion on charge Q_A by charge Q_B and Q_C in the directions shown. According to Coulomb's law,

$$F_{BA} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$$

$$= \frac{9\times10^9 \times 1\times10^{-6} \times 1\times10^{-6}}{1^2}$$

$$= 9 \times 10^{-3} \text{ N.}$$

$$F_{CA} = F_{BA} = 9 \times 10^{-3} \text{ N.}$$

The resultant of F_{BA} and F_{CA} will be a force on charge Q_A due to charge Q_B and Q_C.

$$F_R = 2\, F_{CA} \cos 30° = 2 \times 9 \times 10^{-3} \times 0.866$$

$$= 15.58 \times 10^{-3} \text{ N}$$

Example 1.18 Calculate the electric field strength at a distance of 50 cm from a point charge of $1 \times 10{-}^6$ coulomb placed in air.

Solution:

The electric field intensity or electric field strength at a point is the force per unit charge at that point.

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2} \text{ N}$$

If Q_2 is taken as the unity test charge then E in general can be written as

$$E = \frac{Q}{4\pi\varepsilon r^2} \text{ N/C}$$

If the medium is air then $\varepsilon = \varepsilon_0$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \text{ N/C}$$

Substituting values,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$= \frac{9\times10^9 \times 1\times10^{-6}}{(0.5)^2}$$

$$= 36 \times 10^3 \text{ N/C or V/m}$$

Unit of E is $\frac{N}{C}$

Unit of potential V is $\quad J/C\ V = J/C = NM/C$

$$J = Nm$$

$\therefore \quad E = \frac{N}{C} = \frac{V}{m}$

$$\frac{V}{m} = \frac{N}{C}$$

Example 1.19 Calculate the electric flux density and electric field intensity at a distance of 50 cm from a point charge of 1×10^{-6} coulomb.

Solution:
Electric field intensity is the force on a unit charge.

$F = \dfrac{Q \cdot q}{4\pi\varepsilon_0 r^2}$ where q is a unit charge.

$$E = \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \frac{9\times10^9 \times 1\times10^{-6}}{(0.5)^2}$$

$$= 36 \times 10^3 \text{ N/C or V/m}$$

$D = \varepsilon_0 E$ assuming free space as the medium.

$$= \frac{1}{36\pi\times10^9} \times 36\times10^3 \text{ C/m}^2$$

$$= \frac{10^{-9}}{3.14} \times 10^3$$

$$D = 0.315 \times 10^{-6} \text{ C/m}^2$$

$$= 315 \times 10^{-9} \text{ C/m}^2$$

Example 1.20 Two positive charges of magnitude 1×10^{-6} coulomb are placed in air at a distance of 2 m from each other. Calculate the potential at a point P which is 1 m above the middle of the distance between the charges.

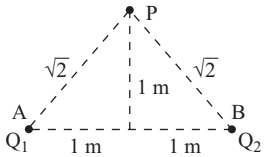


Figure 1.35

Solution:
Work done per unit charge is called the potential. The potential at P due to point charges Q_1 and Q_2 are

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0\sqrt{2}} \quad \text{and} \quad V_2 = \frac{Q_2}{4\pi\varepsilon_0\sqrt{2}}$$

Since both the charges are positive, the total potential will be due to the combined effect. Thus, the total potential V is

$$V = V_1 + V_2 = \frac{Q_1}{4\pi\varepsilon_0\sqrt{2}} + \frac{Q_2}{4\pi\varepsilon_0\sqrt{2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{\sqrt{2}} + \frac{Q_2}{\sqrt{2}}\right]$$

$$= 9 \times 10^9 \times 0.707 \times 2 \times 1 \times 10^{-6}$$

$$= 9 \times 1.414 \times 10^{-3} \text{ Volts.}$$

$$= 12726$$

$$= 0.012726 \text{ volts.}$$

Example 1.21 A ring of 2 m diameter has uniform charge distribution. The charge density is 1×10^{-8} C/m. Calculate the potential on the axis of the ring at a point 2 m from the centre of the ring.

Solution:

The distance of point P at 2 m from the centre of the ring of radius 1 m is equal to $\sqrt{2^2 + 1^2} = \sqrt{5}$ m

The total charge, Q, is equal to charge density multiplied by the length. So

$$Q = \rho_L L = 1 \times 10^{-8} \times 2\pi r$$

$$= 1 \times 10^{-8} \times 2 \times 3.14 \times 1$$

$$= 6.28 \times 10^{-8} \text{ C}$$

Potential, V, at point P is

$$V = \frac{Q}{4\pi\varepsilon_0\sqrt{5}} = \frac{6.28 \times 10^{-8}}{4\pi\varepsilon_0\sqrt{5}}$$

$$= \frac{9 \times 10^9 \times 6.28 \times 10^{-8}}{\sqrt{5}} \text{ volts.}$$

$$V = 250 \text{ volts}$$

Example 1.22 Two point charges $Q_1 = +2 \times 10^{-8}$ C and $Q_2 = -3 \times 10^{-8}$ C are separated from each other by 2 m. Determine the electric field at the mid-point between the two charges.

Solution:

The electric field at point P, which is at a distance of 1 m from the two charges is calculated by considering the individual charges as

$$E_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1^2} \quad \text{and} \quad E_2 = \frac{Q_2}{4\pi\varepsilon_0 r_2^2}$$

Here $r_1 = r_2 = 1\text{m}$

$$E_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{\theta_1}{r_1^2} = \frac{9 \times 10^9 \times 2 \times 10^{-8}}{1^2} = 180 \text{ N/c or V/m}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{\theta_2}{r_2^2} = \frac{9 \times 10^9 (-3 \times 10^{-8})}{1^2} = -270 \text{ N/C or V/m}$$

Resultant $$E = E_1 + E_2 = 180 - 270 = -90 \text{ v/m}$$

1.11 REVIEW QUESTIONS

A. Short Answer Type Questions

1. State Coulomb's law of force between any two point charges. State the unit of force.
2. Distinguish between electric flux density and electric field intensity. How are they related to each other?
3. Obtain an expression for electric field intensity at a point due to a number of point charges.
4. Distinguish between potential and potential difference.
5. Derive an expression for the capacitance of a parallel plate capacitor.
6. State Gauss's law. Show how surface integral can be converted into volume integral using the divergence theorem.
7. Explain the following terms—(i) electric field intensity (ii) electric flux density (iii) electric potential (iv) capacitance.
8. Derive an expression for the energy stored in an electric field.
9. Define and explain (i) permittivity of free space (ii) relative permittivity.
10. State and explain Gauss's law.
11. Obtain an expression for the energy stored in a capacitor.
12. Explain how a capacitor can store energy.
13. State Gauss's law and develop its mathematic form. Give two examples of its applications.
14. Explain the principle of electrical image as applied to the solution of problems related to electrostatics. Give a suitable example.
15. Deduce an expression for energy stored in an electrical field having point charges.
16. State Coulomb's law and explain the meaning of the factors that influence Coulomb's force.
17. Show that the electric field strength at any point due to an infinite sheet of charge is independent of the distance of the point from the sheet.
18. Establish the relationship between electric field, E, and electric potential, V.
19. State and explain (i) Gauss's law (ii) divergence theorem.
20. What is method of images? Explain with the help of a suitable example.
21. Prove that the total energy W in an electrostatic field is

$$W = \frac{1}{2}\int_v \overline{E} \cdot \overline{D}\ dv$$

where $\overline{E}$ is the electrical field intensity, $\overline{D}$ is the electric flux density and dv is a differential element of the volume.

22. Explain Gauss's law and derive an expression for the electric field, E, for a point outside the charge distribution and for inside the charge distribution.
23. Deduce an expression for the capacitance of concentric spheres of radii a and b (b > a) using Gauss's law.

B. Numerical Problems

24. Calculate the capacitance of an isolated sphere of 10 cm radius.

(Ans. $C = 11.1 \times 10^{-12}$ F)

25. The potential of two points A and B are 15 V and 20 V, respectively. If a charge of 10×10^{-6} C is moved from A to B, what is the work done?

(Ans. $W = 50 \times 10^{-6}$ joules)

26. The parallel plates of a capacitor of 0.04 m^2 area are separated by a distance of 1 cm. The permittivity of the dielectric used is 2.0. Calculate (a) the capacitance, (b) the charge on the plates when a pd of 100 V is applied, (c) the energy stored in the capacitor.

(Ans. 70.832×10^{-6} F, 70.832×10^{-10} C, 3.541×10^{-9} joules)

27. Three point charges each of magnitude 10^{-5} Coulomb are placed in the three corners of an equilateral triangle of side 100 cm. Calculate the resultant force on each of the charges.

(Ans. F = 1.56 N)

28. Calculate the electric field intensity at a distance of 0.2 m from a charge of 2×10^{-6} C in vacuum.

(Ans. $E = 4.5 \times 10^2$ kV/m)

C. Multiple Choice Questions

1. The proportionality constant in Coulomb's law is
   (a) $4\pi \times 10^{-7}$ (b) 9×10^{-9}
   (c) 9×10^{9} (d) 8.854×10^{-12}.
2. The unit of electric field intensity is
   (a) coulomb/newton (b) coulomb/meter
   (c) newton (d) newton/coulomb.
3. The point form of Gauss's law is
   (a) $\overline{\nabla} \cdot D = \rho_s$ (b) $\overline{V} \cdot D = \rho_v$
   (c) $\overline{\nabla} \cdot D = \frac{\rho_v}{\varepsilon_0}$ (d) $\overline{\nabla} \cdot D = Q$.
4. The magnitude of the force between $Q_1 = 1$ C and Q = 1 C when separated by 1m in free space is
   (a) 9×10^{-9} N (b) 9×10^{9} N
   (c) 8.854×10^{-12} N (d) 1 N.
5. The unit of electric flux is
   (a) coulomb (b) coulomb/m,
   (c) weber (d) weber/m^2.
6. The unit of electric field is
   (a) coulomb/newton (b) coulomb/meter
   (c) newton/coulomb (d) newton.
7. Potential has the unit of
   (a) joules/m^2 (b) joules/m^3
   (c) joules (d) joules/coulomb.
8. Which of the following statements is correct?
   (a) The constant of proportionality in Coulomb's law has no unit.
   (b) The direction of electric field is opposite to the direction of Coulomb's force.
   (c) Electric field at a point due to a point charge is proportional to $\frac{1}{r^2}$.
   (d) Electric field at a point due to a point charge is inversely proportional to r.
9. Which of the following statements is incorrect?
   (a) Electric field, $E = -\nabla V$
   (b) Unit of electric flux is coulomb
   (c) Unit of volume charge density is coulomb/m^3
   (d) Gauss's law is applicable even if the charge is outside the closed surface
10. Energy stored in an electrostatic field is
    (a) $W_e = \frac{1}{2}\varepsilon_0{}^2 E^2$ joules/m^3
    (b) $W_e = \frac{1}{2}\varepsilon_0{}^2 E$ joules/m^3
    (c) $W_e = \frac{1}{2}\varepsilon_0 E^2$ joules/m^3
    (d) $W_e = \frac{1}{2}\varepsilon_0 D_0 E$ joules/m.
11. The charge on a body is measured in
    (a) eV (b) coulomb
    (c) farad (d) gauss.
12. The unit of electric field intensity or electric field strength is
    (a) newton/coulomb (b) volt/meter
    (c) volt/coulomb (d) both as in (a) and (b).

Answers to Multiple Choice Questions

1. (c) 2. (d) 3. (b) 4 (b) 5. (a) 6. (c) 7. (d) 8. (c) 9. (d) 10. (c) 11. (b) 12. (d)

DC Machines

TOPICS DISCUSSED

- Principle of working of dc machines
- Construction details
- EMF equation of a dc machine
- Types of dc machines
- Characteristics of dc generators
- Applications of dc generators
- Operation of a dc machine as a motor
- Torque equation
- Starting a dc motor
- Speed control of dc motors
- Starter for a dc motor
- Types and characteristics of dc motors
- Losses and efficiency
- Applications of dc machines

2.1 INTRODUCTION

DC machines work either as a dc generator or as a dc motor. In a dc generator, a set of conductors or coils placed on a rotating body, called armature, are rotated continuously inside a magnetic field with the help of a prime mover (prime mover is another machine, may be a diesel engine or a turbine, which rotates the armature). The magnetic field is created by passing dc current through the windings of a set of field magnets. When conductors pass under alternate North and South poles, alternating EMF is induced in the armature winding. The ac generated gets converted into dc when the voltage is collected from the rotating armature through the brush and commutator arrangement. The brush and commutator arrangement, therefore, works like a full-wave rectifier which converts generated ac into dc for the output circuit. A dc generator, therefore, converts mechanical energy supplied through the prime mover to electrical energy to be supplied from the generator armature to an electrical load. We, of course, will need to create a magnetic field by a field system as shown in Fig. 2.1. It can be noticed from the figure that the magnetic poles are electromagnets fixed on a hollow cylindrical frame. Depending on the direction of the winding and the direction of current flow through the field windings, alternate North and South poles are formed, creating a magnetic field inside which the cylindrical rotor called armature is placed and is rotated.

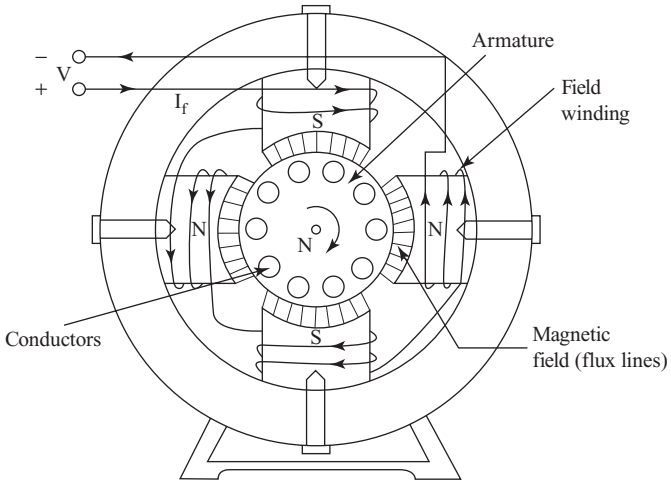


Figure 2.1 *Cross-sectional view of a four-pole dc machine*

The brush and commutator arrangement has not been shown in the figure. The armature conductors have been shown rotated in the magnetic field by a prime mover at N rpm (revolutions per minute).

All the field–pole windings have been shown connected in series. The direction of current flow in the pole windings are such that alternate North and South poles have been formed. The field system have been energized by passing current, I_f through their windings from a separate dc source. To understand the working of a dc generator we will consider, for simplicity, a two-pole construction having only one coil on the armature. The two terminals of the armature coil are to be brought out for connection to the load circuit. For this, two arrangements are possible, i.e., (i) through brush and slip-ring arrangement, and (ii) through brush and commutator arrangement. We will consider these two arrangements side by side and see the nature of output voltage and load current.

2.1.1 Nature of Load Current When Output Is Taken Out Through Brush and Slip-ring Arrangement

In brush and slip-ring arrangement the coil terminals are connected to two conducting slip rings which are fitted on one side of the rotating shaft. The carbon brushes are placed on the slip rings and are fixed. Connections from the brushes are taken to the load. When the shaft is rotated by a prime mover, the coil rotates. The slip rings are nothing but extensions of the coil-end connections. The slip rings also rotate as the coil rotates. The brushes sit on the slip rings and make sliping contact with the coil and are able to connect the EMF generated to the load for supply of current.

In Fig. 2.2 (a), the coilside a of coil a–a′ is under North pole and the coilside a′ is under South pole. After half revolution the positions of the coilsides change as shown in Fig. 2.2 (b). The direction of EMF induced in the coilsides have been shown. The EMF induced will cause current to flow through the load resistance as shown. It is noted that the direction of current through the load resistance has changed after half revolution of the coil.

In Fig. 2.2 (a), current flows from a to a′ and after half revolution of the coil, current in the coil flows from a′ to a. After every half revolution, current in the coil will get reversed.

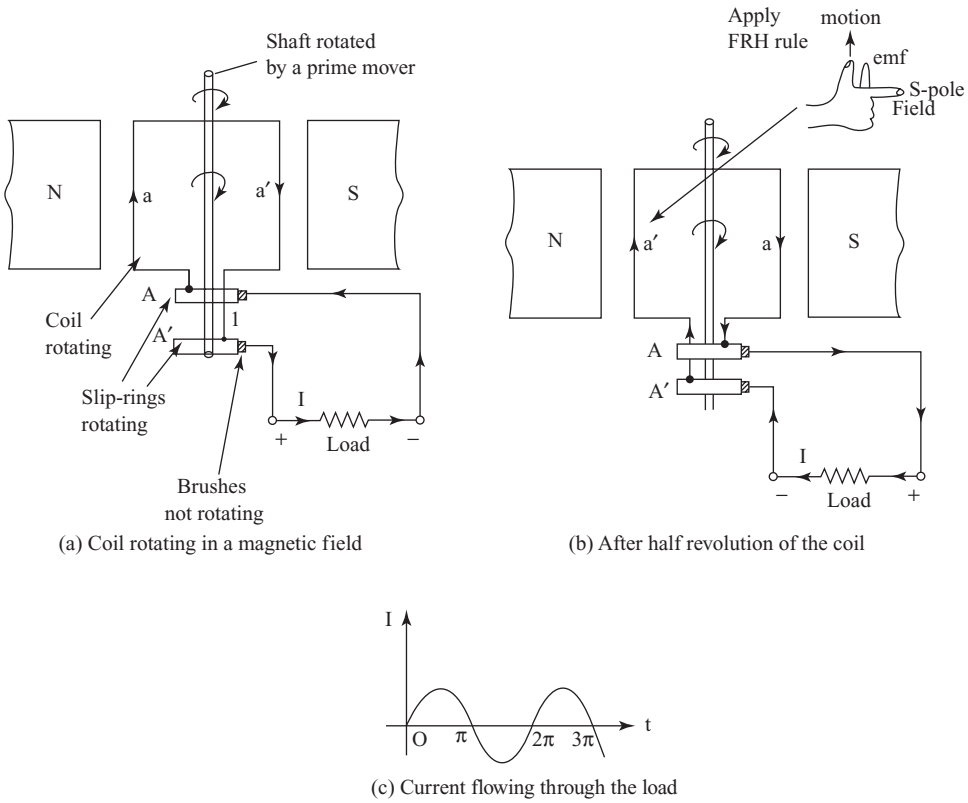


Figure 2.2 *AC generated in the coil causes ac to flow through the load when connection is through brush and slip-ring arrangement*

When the armature is rotated continuously by the prime mover, the EMF induced in the coil is alternating in nature as can be seen from Fig. 2.2 (a) and (b). The direction of EMF induced in the coil sides a and a′ has been determined by applying Fleming's Right-Hand rule. We can conclude that when a coil is rotating in a magnetic field, an alternating EMF is induced in it which will cause an alternating current to flow through the load resistance if connection is made through the brush and slip-ring arrangement.

2.1.2 Nature of Load Current When Output Is Taken Through Brush and Commutator Arrangement

Now we will consider a brush and commutator arrangement of connecting the rotating coil to the load. When a slip ring is cut into two pieces and joined together by putting insulation in the joints, a simple commutator assembly with two commutator segments are produced. The coil ends are connected permanently to the two commutator segments. The commutator is fixed on the shaft and rotates along with the coil when the shaft is driven by the prime mover as has been shown in Fig. 2.3.

The direction of the induced EMF in the coil sides is determined by applying Fleming's Right-Hand rule. When the coil is connected to the load, current will flow through the load as has been

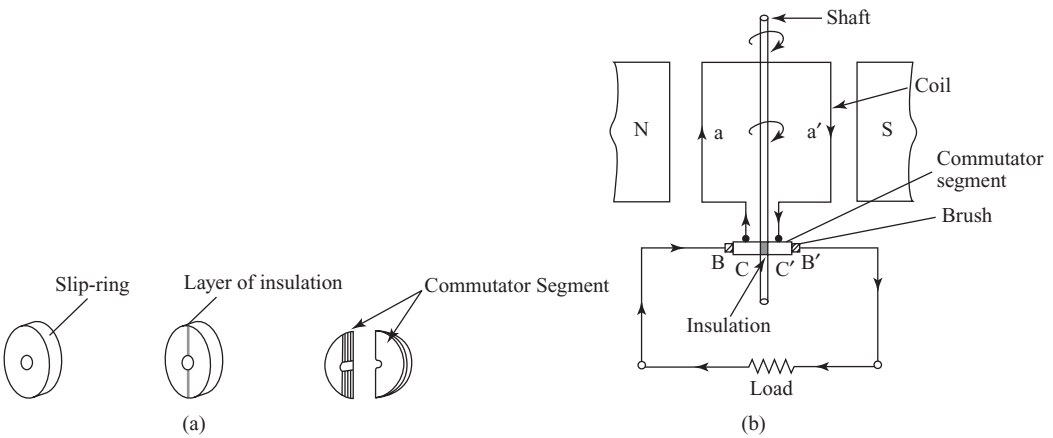
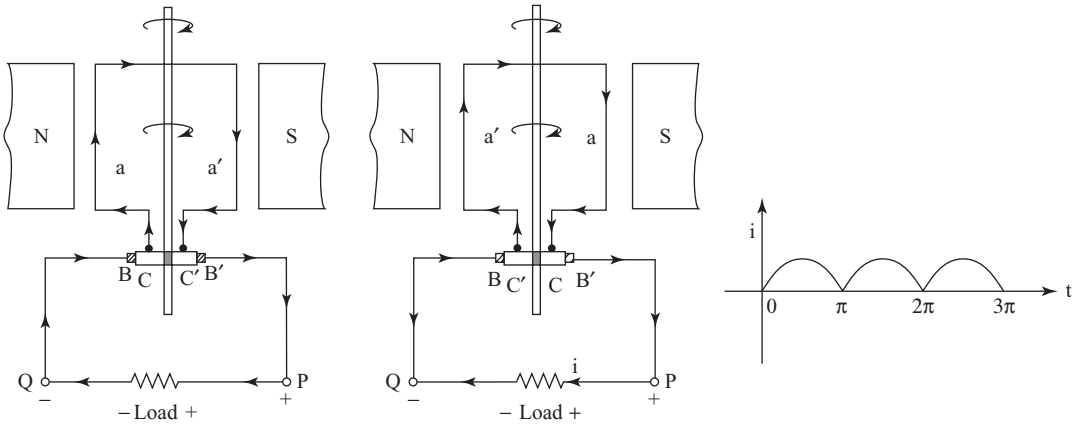


Figure 2.3 *(a) a slip ring cut into two pieces with a layer of insulation between the pieces; (b) a coil rotating in magnetic field has been connected to the load through brush and commutator arrangement*

shown. Current from coil side a′ flows to the load through commutator segment C′ and brush B′. From the load, the current returns to the coil through brush B, commutator segment C and then to the coil side a. No current can flow from commutator segment C to C′ or from C′ to C as there exits a layer of insulation between them. With such an arrangement when the coil rotates in the magnetic field, a unidirectional current will flow through the load as has been shown in Fig. 2.4 (a) and



(a) A coil is shown rotated in a magnetic field by a prime mover. The direction of current flowing through the load is from P to Q

(b) After half revolution the position of coil sides along with commutator segments change. The position of brushes are fixed. Load current flows from P to Q

(c) Output wave shape of the load current

Figure 2.4 *(a) A coil is shown rotated in a magnetic field; (b) after half revolution the position of the coil side along with commutator segments change, the position of brushes remain unchanged; (c) output dc wave shape of the load current*

(b). In Fig. 2.4 (a), current in the coil flows from coil side a to coil side a′. After half revolution the direction of current is changed from a′ to a. In every half revolution of the coil, this change in direction of current will take place. The current through the load resistance, however, will be unidirectional because the connections from the armature coil to the load have been taken through brush and commutator.

It is observed from the output current wave shape that we are getting a fluctuating dc and not a constant dc. In actual practice, in a dc generator, instead of using a single coil a large number of coils are placed on the armature slots so as to generate a considerable amount of voltage. Consequently, a large number of thin commutator segments are used to make the commutator assembly. The sum of the EMFs induced in the armature coils when connected to the load through the brush and commutator arrangement will be a steady dc current flowing through the load. The brush commutator assembly in a dc generator, in fact, works like a full-wave diode rectifier.

The function of brush and commutator in a dc machine working as a generator is to convert ac generated in the armature coils into dc at the output.

2.1.3 Function of Brush and Commutators in Motoring Action

We shall now examine the operation of an elementary dc motor by considering a single coil (for simplicity) on its armature. In a dc machine, operating as a motor, electrical energy is converted into mechanical energy. Fig. 2.5 shows the armature coil fed from a source of dc supply. The armature having the coil on it is placed in a magnetic field created by the field system as shown in Fig. 2.5 (a).

In Fig. 2.5 (b) is shown how current will flow from the positive polarity of the supply source through brush B′ and commutator segment C′ to coil side a′ and then to coil side a, returning through commutator segment C and brush B to the negative terminal of the supply source. By applying Fleming's Left Hand rule to coil-side a′ in Fig. 2.5 (b) we find that the conductor will experience an upward force whereas the coil side a will experience a downward force. These two forces would create a torque to rotate the armature in the anticlockwise direction.

After every half revolution, i.e., for every rotation of 180° mechanical, coil side a′ along with the connected commutator segment C′ will change positions with the coil side a and the connected

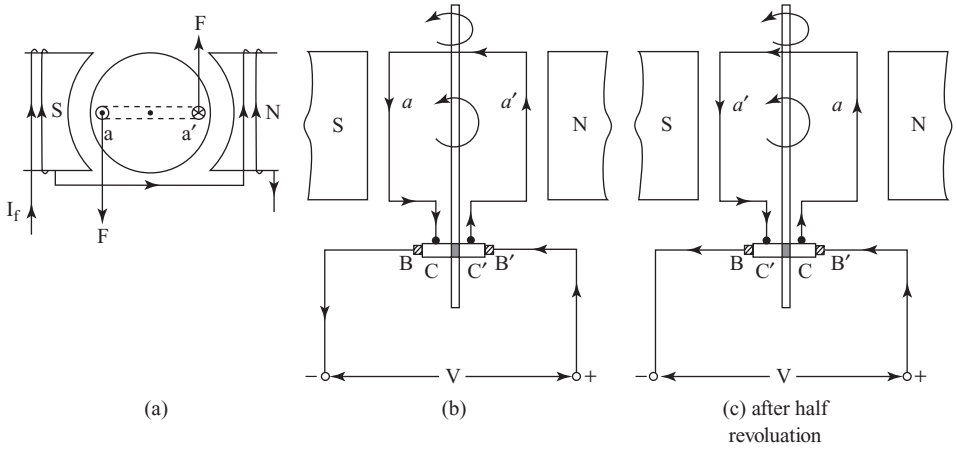


Figure 2.5 *Illustrates motoring operation. Supply is through brush and commutator arrangement for achieving continuous rotation*

commutator segment C. After half revolution it is seen that the direction of current in the coil has reversed. Earlier as in Fig. 2.5 (b), current was flowing from a′ to a and after half revolution current is flowing from a to a′ as shown in Fig. 2.5 (c).

Supply polarities remaining fixed, it is seen that current in the armature coil is alternating its direction. However, the direction of the rotation of the coil, as obtained, is unidirectional, i.e., in this case in the anti clockwise direction. The students are advised to check the nature of torque developed in the armature if the supply is given from a dc source but through the brush and slip-ring arrangement. It will be seen that the torque developed will be alternating in every half cycle thereby net torque being zero.

Thus, it can be concluded that the function of brush and commutator in a dc motor is to produce unidirectional torque, i.e., to cause rotation of the armature of the motor in a particular direction.

We shall now study the constructional details of a dc machine.

2.2 CONSTRUCTION DETAILS

A dc machine consists of a field system which produces the magnetic field, the armature which carries the armature conductors placed in slots, the brush and commutator arrangement, the shaft, and the bearings.

These are explained in brief in the following sections.

2.2.1 The Field System

The purpose of the field system is to produce a magnetic field inside which a set of conductors will be rotating. The field system consist of a set of electromagnets fixed on the inside periphery of a hollow cylindrical structure called the yoke as shown in Fig. 2.6. The field poles have field windings wound on a laminated iron core. The number of poles of a dc machine may be two or multiples of two. A dc current supplied from a dc source magnetizes the field system. Alternate North and South poles are formed on the basis of the direction of the current flowing through the field windings. Small poles, called interpoles are often fixed between two main poles, particularly in case of large dc machines.

The field windings are made of thin insulated copper wire of a large number of turns. The resistance of field winding is fairly high of the order of 100 Ω or so. The side view of such a field system has been shown in Fig. 2.6 (a).

2.2.2 The Armature

The armature of a dc machine is built by using circular laminated sheet steel to form a cylindrical structure with a shaft passing through its centre.

A simplified cross-sectional view of a dc machine with the armature placed inside the field system has been shown in Fig. 2.7.

In a dc machine the field system is stationary while the armature along with the commutator is the rotating part. The armature winding is made using a large number of coils connected in series and parallel to get the desired voltage and current. The coils are made of insulated copper wires and are placed in a large number of armature slots. The coil ends are connected to large number of commutator segments of the commutator. The commutator segments are insulated from each other using some good quality thin insulating sheets like mica sheets. Carbon brushes are placed on the commutator surface and terminals are brought out from the brushes.

The air-gap between the field poles and the armature is kept small, of the order of few mm. The commutator, like the armature, is cylindrical in shape and is made up of a large number of wedge-shaped segments of hard drawn copper. The fixed carbon brushes sitting on the commutator surface

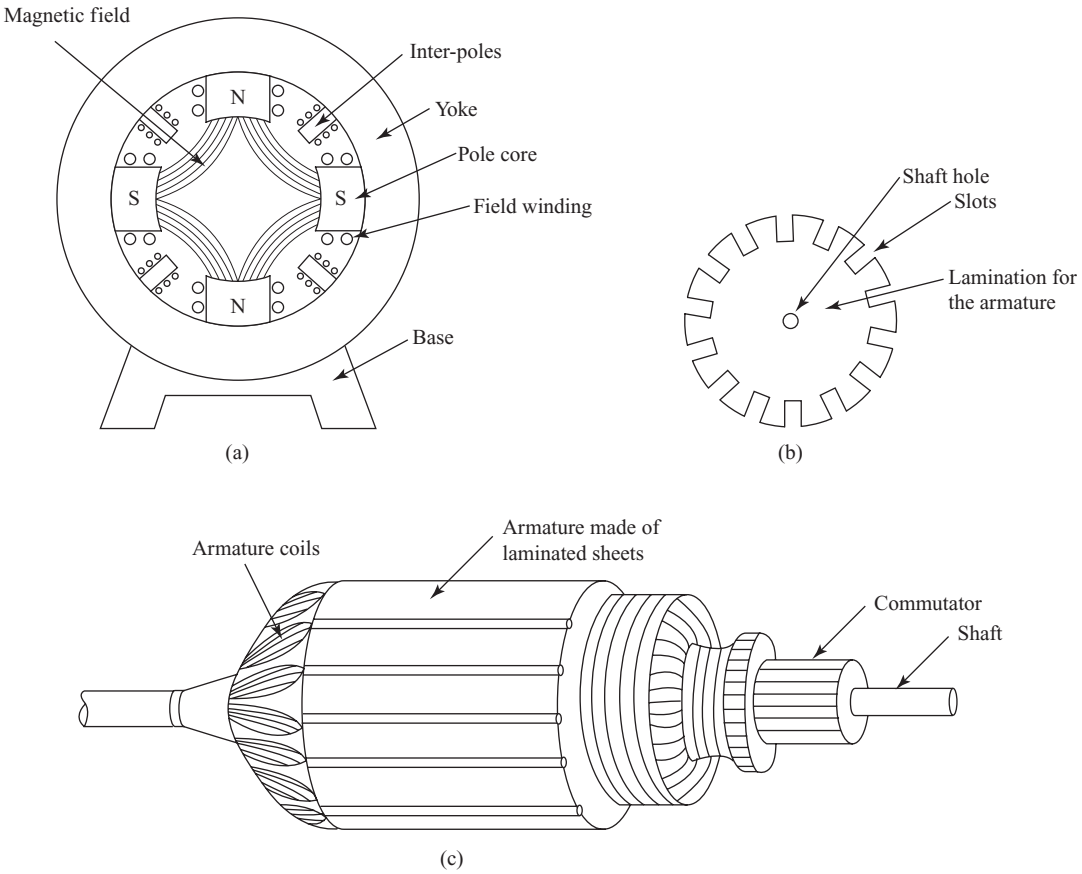


Figure 2.6 *Parts of a dc machine: (a) the field system creating a magnetic field when current will flow through the field windings; (b) laminated sheets used to make the cylindrical armature; (c) the armature made of laminated sheets and the commutator assembly*

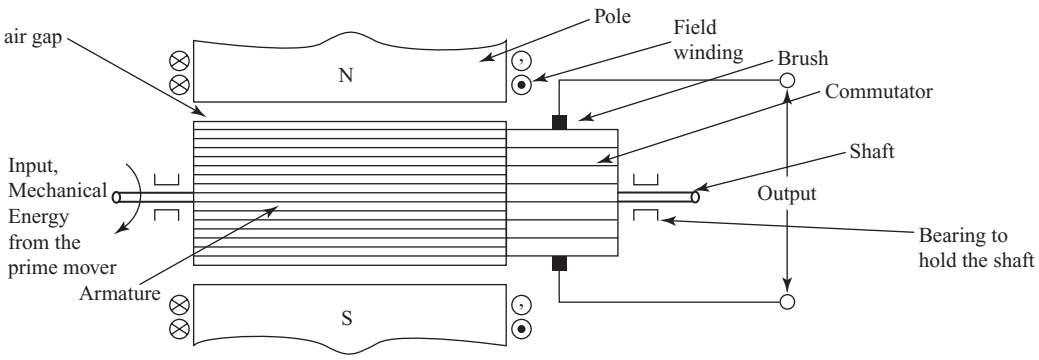


Figure 2.7 *Simplified diagram of a dc machine working as a generator*

make slipping contacts with the armature coils via the commutator segments either to collect current or to supply current from an external source (in case of motoring operation). Brushes made of carbon are conducting material but softer than the hard drawn copper used in the commutator. Brushes are placed in brush holders which are fixed with the stationary part of the machine. When the armature rotates, the brushes and the commutator surface make constant smooth rubbing, which over a period of time reduces the length of the brushes (due to wear and tear). There is no deterioration of the surface of the commutator due to this rubbing action. When required, the set of brushes can be replaced by new ones.

Armature and the commutator together is made into one unit. A shaft made of mild steel runs through the armature and comes out from both sides. Two sets of bearings are used to support the whole of the revolving system. The shaft extension on one end is used to connect the prime mover while at the commutator end the shaft is extended for use of the bearing. The shaft is held in position inside the stator with the help of end shields.

2.2.3 Armature Winding

When the armature coils rotate in the magnetic field, EMF is induced in each coil. If all the coils are connected in series, the total EMF available will be the sum of all the EMFs in all the coils. But the current that this winding would be able to deliver to the load will be governed by the current-carrying capacity of each of the armature coils. If higher current to desired, the coils have to be connected in series and parallel. Thus, the armature winding will have a set of coils connected in series in each of its parallel paths as shown in Fig. 2.8 (a) and (b). The arrangement is exactly similar to series–parallel connection of cells used to make a battery of certain voltage and ampere rating.

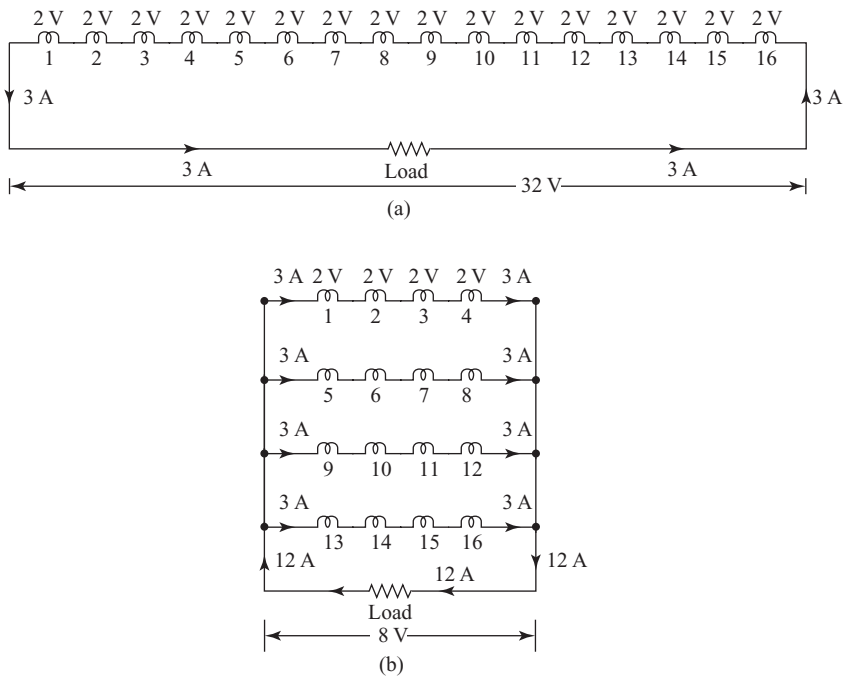


Figure 2.8 *Series–parallel connection of armature coils to form the armature winding: (a) series connection of all the armature coils; (b) series–parallel connection of the coils*

Let the armature winding be made of 16 coils. When the armature is rotated in the magnetic field at a certain speed, let EMF induced in each coil be 2 V. Let the current-carrying capacity of each coil be 3 A. If all the coils are connected in series to form the total armature winding, we will get 32 V across the armature terminals and a maximum current of 3 A can be delivered to the load. If the current increases beyond 3A, the winding will get heated up excessively, which is not desirable. If four coils are connected in series, there will be four series circuits which will induce an EMF of 8 V in each circuit. If the four series circuits are connected in parallel to supply current to the load, a maximum of 12 A can be supplied to the load although the rated current of 3A will flow through each coil as shown in Fig. 2.8 (a) and (b). Here the parallel paths of the armature winding is four. However, depending on the voltage and current requirements, the number of parallel paths could be different, say 2, 4, 6, etc.

2.2.4 Types of Armature Winding

All the coils placed in armature slots are connected together in a particular manner to form the armature winding. Two basic types of winding connections are made. They are (i) lap winding and (ii) wave winding. In all cases, the coil ends are connected to commutator segments. The commutator segments are, in fact, extension of coil end connections. Brushes placed on commutator touching the commutator segments make sliping contact with the coils. The two types of armature windings are explained below.

(a) Lap winding

Here, the end of one coil is connected to the beginning of the next coil. If connections are made this way the coils look as if they are super imposed on each other and then given a push in one direction as shown in Fig. 2.9 (a). Fig. 2.9 (b) shows a wave winding.

$1 - 1'$, $2 - 2'$, $3 - 3'$, etc. are the armature coils. In lap winding coil side $1'$ is connected with coil side 2, coil side $2'$ is connected to coil side 3 and this way the whole winding is completed. In lap winding the number of parallel paths formed in the armature winding is equal to the number of poles of the machine.

(b) Wave winding

Here the winding connections are made as show in Fig. 2.9 (b). The end of one coil is connected to the next coil side under the next similar poles. The winding so formed looks like a wave, and hence the name. The number of parallel paths formed is always equal to 2. Note that the coils are connected to commutator segments in both types of windings.

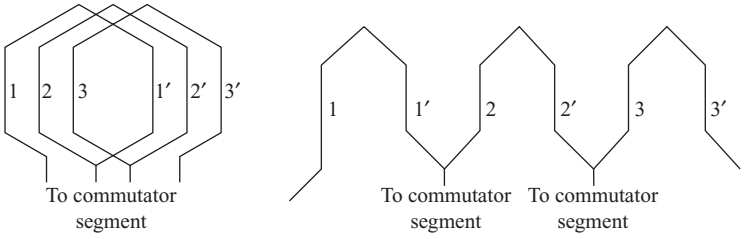


Figure 2.9 *Lap and wave windings illustrated*

2.3 EMF EQUATION OF A DC MACHINE

The total induced EMF in a dc machine, as can be visualized should depend on the speed at which the conductors are rotated in the magnetic field, the total number of conductors connected in series, and the total magnetic field flux being cut by the conductors. The exact equation in now being developed as under.

Fig. 2.10 shows a single conductor rotated by a prime mover in a magnetic field.
Let us assume the following:
No. of poles = P.
Flux per pole = ϕ Wb.
Speed of the driving prime mover = N rpm.
Actual number of armature conductors = Z.
Number of parallel paths of the armature winding = A.
Induced EMF in the conductors will be due to relative velocity between the conductor and the flux produced by the field poles.
When the conductor in Fig. 2.10 makes 1 revolution, the flux cut by the conductor = $P\phi$ Wbs.

(The students will appreciate that if there were two poles and flux per pole was ϕ Wbs, the conductor would cut 2ϕ flux, for a four-pole system, the flux cut per revolution would be 4 ϕ Wbs, and so on). Thus, if ϕ is the flux per pole, and P is the number of poles, flux cut by a conductor in 1 revolution will be $P\phi$ Wbs.

The conductor is rotating at a speed of N rpm or $\frac{N}{60}$ rps (revolutions per second)

Time taken by the conductor to make 1 revolution = $\frac{60}{N}$ seconds.

2.3.1 Induced EMF Is Equated to Flux Cut Per Second

Therefore, induced EMF in 1 conductor

$$= \frac{\text{Flux cut in 1 revolution in Wbs}}{\text{Time taken in making 1 revolution in secs}}$$

$$= \frac{P\phi}{60/N} = \frac{P\phi N}{60} \text{ V}$$

Z is the total number of armature conductors and they are connected in A number of parallel paths.

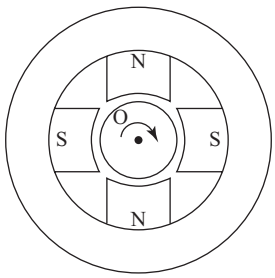


Figure 2.10 *Development of equation for EMF induced*

The total number of conductors per parallel path would be equal to $\frac{Z}{A}$.

The induced EMF available across the output terminals will be equal to the induced EMF per parallel path.

Thus the total induced EMF, E is

$$E = \frac{P\phi N}{60}\left(\frac{Z}{A}\right)V$$

or,
$$E = \frac{\phi ZNP}{60A}V \tag{2.1}$$

When the dc machine is also working as a motor, the current-carrying conductors placed in the magnetic field will develop torque and rotate in a particular direction. When they would rotate, EMF will also be induced in them.

The equation for induced EMF will be the same both in generating and motoring mode of operation of the machine. The induced EMF in the armature of a dc motor is often called back EMF as it opposes the applied voltage.

2.4 TYPES OF DC MACHINES

In the dc generator shown in Fig. 2.11, the armature has to be rotated by some prime mover and the field windings have to be excited by giving a dc supply to them. We, therefore, would need a separate dc source of supply for the field winding. Such a generator where the field winding is supplied from a separate dc source for its excitation is called a *separately excited dc generator*.

EMF E will be induced in the armature when it is rotated by a prime mover and the field windings are excited from a separate dc source.

Now, let as consider what would happen when the armature is rotated by a prime mover without exciting the field windings, i.e., when field winding current, $I_f = 0$.

Even when the field windings are not excited, there is some residual magnetizm in the field poles due to their earlier excitation.

A feeble magnetic field will, therefore, exist due to which a very small amount of voltage will be induced in the armature winding when rotated. Let us assume that this induced EMF is 5 V. If the field winding having a resistance of say 100 Ω is connected across the armature, a small amount of current, 5 V/100 Ω = .05 A will flow through the field windings, which will produce some more flux, and as a

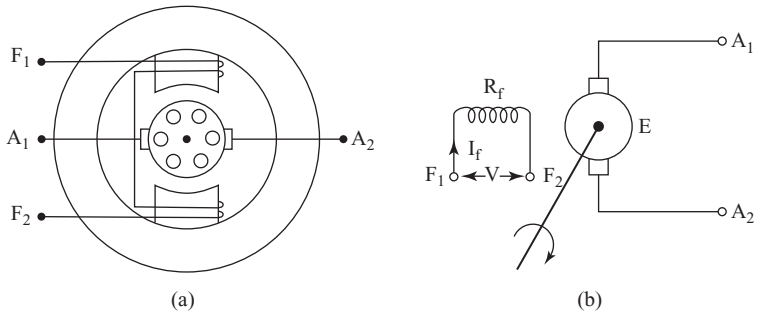


Figure 2.11 *(a) A dc generator with field and armature winding terminals brought out; (b) the field windings of the generator are excited from a dc supply source*

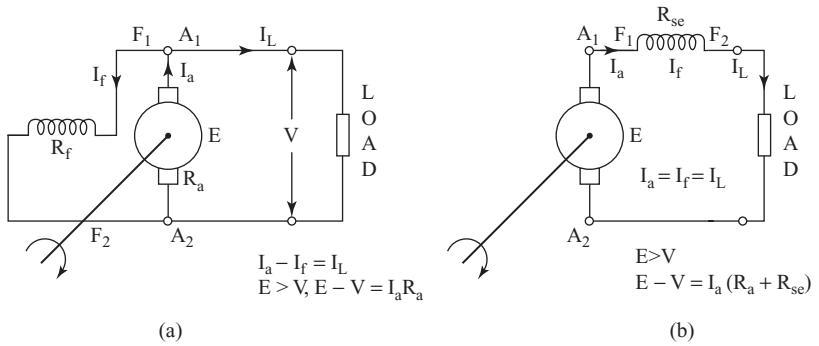


Figure 2.12 *(a) Shunt generator; (b) series generator*

consequence more EMF will be induced in the armature. This way voltage will be built up across the armature terminals when the field windings are connected in parallel with the armature as shown in Fig. 2.12 (a). Such a generator is called a shunt generator or a *self-excited generator*.

Fig. 2.12 (b) shows the connection diagram of field and armature windings for a series generator. The field current I_f is equal to $I_{a,}$ i.e., a very high current will now flow through the field windings. For a series generator, therefore, field windings are made of thick wires of a few turns to provide the required ampere turns needed for production of magnetic field of a particular strength.

A compound generator will have both shunt field winding and series field winding. Both the field windings are wound around the pole core. Shunt field winding is connected in parallel with the armature while the series field winding is connected in series with the armature. The resultant field produced will be equal to the sum of the field produced by the two field ampere turns. Such a generator is called a *cumulative compound generator.*

When the flux produced by the series field opposes the flux produced by the shunt field, the generator is called a *differential compound generator.*

2.5 CHARACTERISTICS OF DC GENERATORS

The characteristic of dc shunt, series, and compound generators will be different because of the way the field and armature windings are connected.

Let us examine the no-load and load characteristics of dc generators.

2.5.1 No-load Characteristics

Fig. 2.13 (a) shows a separately excited dc generator. The generator armature is rotated at a constant speed by a prime mover. When no field current is there, a small amount of EMF will be induced due to residual magnetism. By increasing the field current, I_f gradually, we will get the no-load or open-circuit characteristic as shown in Fig. 2.13 (a).

In the case of a shunt generator, the voltage gets built up due to residual magnetism as has been shown in Fig. 2.13 (b). The induced voltage will be the value at which the OCC and field resistance line cross each other. By adjusting the value of the field-circuit resistance, i.e., by adding an extra resistance in the field circuit, the value of E can be adjusted. The speed of the prime mover is assumed constant. A self-excited dc shunt generator will fail to build up its voltage if there is no residual magnetism in the field poles and if the value of the field resistance in higher than the critical field

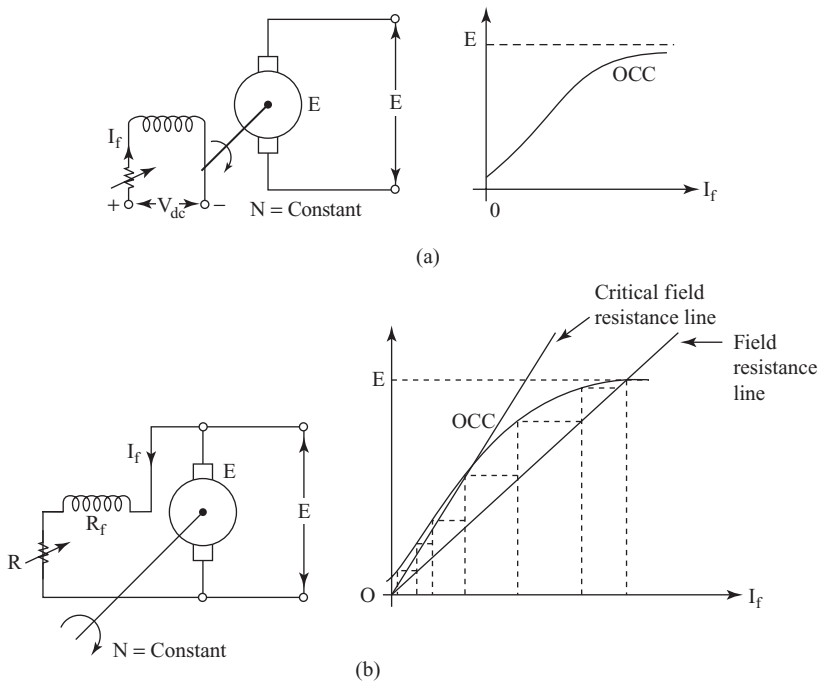


Figure 2.13 *No-load or open-circuit characteristic (OCC) of dc generators: (a) separately excited generator; (b) shunt generator*

resistance. The value of the critical field resistance can be determined by drawing a line tangent to the OCC and finding its slope. The students should notice that the OCC is initially linear, but later becomes somewhat horizontal. This shows that with increase of field current, I_f the induced EMF increases linearly but later the core saturates. Further increase of I_f does not give rise to much increase in induced EMF.

2.5.2 Load Characteristics

The load characteristic is drawn between the voltage available across the output terminals, V when the generator is loaded, against the load current I_L. The equation relating E, V, and I_a for a dc generator is given as

$$V = E - I_a R_a \tag{2.2}$$

Where R_a is the armature winding resistance and I_a is the current flowing through the armature winding.

Thus, as the generator is loaded, the voltage available across the load will be somewhat reduced due to $I_a R_a$ drop. It may be noted that R_a is very small and is of the order of less than 1 Ω. Therefore, $I_a R_a$ drop although small will reduce the terminal voltage with increase of load current or armature current. In addition, some more amount of voltage drop will be due to the reduction of the magnetic field strength when current flows through the armature winding which is called armature current reaction or simply armature reaction. This is explained as follows.

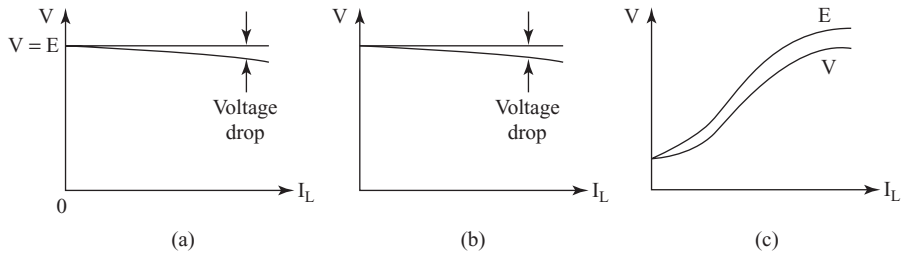


Figure 2.14 *Load characteristics of dc generators: (a) separately excited dc generator; (b) dc shunt generator; (c) dc series generator*

Effect of armature reaction

When the generator is loaded the armature current will produce a certain amount of flux that would come into existance in the air gap. This would create a demagnetization and cross-magnetization effect. The demagnetization component of the armature flux will work in opposition to the main field flux, thereby reducing the EMF induced, E and as a consequence reducing V. This effect of armature flux on the main field flux is called *armature reaction*. It may be noted that armature reaction occurs only when current flows through the armature winding i.e., only when the generator is loaded. Because of reduction of E due to *armature reaction, the terminal voltage E will further get reduced a little in case of shunt* generators. To *reduce the effect of armature reaction, compensating windings and inter poles are used,* which produces the same amount of flux as produced by the armature current but in opposite direction so as to eliminate the effect of armature flux on the main field flux. The load characteristics of dc generators have been shown in Fig. 2.14. The voltage drop from E to V is very small as compared to E. The shunt generators can be considered as constant voltage output generators for practical purposes.

In case of series generators, the field current is the same as the load current. Initially there is some induced EMF due to residual magnetizm of the field poles. As the generator is loaded, current through the field increases and E and V increase and later saturation effect takes place. Further loading increases the armature reaction effect and voltage starts falling.

2.6 APPLICATIONS OF DC GENERATORS

Shunt generators are used in applications, such as for battery charging, dc excitation in ac generators, lighting applications etc. A series generator's load characteristic is a rising one. That is, as the load increases, voltage increases due to increase in field current. Such generators cannot be used for lighting applications as voltage variation will effect illumination level.

However, dc series generators can be used to boost up voltage of an existing system to compensate for the voltage drop in the system. Series generators are also used as welding generators and in arc lamps. The characteristics of dc compound generators can be modified by the use of series winding either in circumlative or in differential mode. That is, either the series field flux will be aiding the main field flux or opposing it. The operation of a dc machine as a dc motor will now be dealt with.

2.7 OPERATION OF A DC MACHINE AS A MOTOR

Like dc generators, dc motors are also constructed to work as dc shunt, series, and compound motors. In all such motors, electrical power input, i.e., $V \times I$ gets converted into mechanical output. The

mechanical output is in the form of torque developed which enables the motor shaft to carry some mechanical load on it. For example, a dc motor will rotate the wheels of an electric train or a trolley bus. The constructional details of a dc motor is similar to that of a dc generator except for certain minor changes in the cooling system.

2.7.1 Working Principle of a DC Motor

A dc motor works on the basic principle that when a current-carrying conductor is placed in a magnetic field it experiences a force. In a dc motor, the armature carries a number of conductors placed in slots and the armature is placed inside the magnetic field created by field magnets as shown in Fig. 2.15. The field magnets are excited by field current passing through them. The field windings may either be excited from a separate dc source or could be excited from the source of supply provided to the armature. The armature is fed from a dc supply source of V Volts as shown. The direction of current flowing through the winding have been shown by cross ⊗ and dot ⊙ A cross indicates current flowing in the direction perpendicular to the plane of the paper downwards and a dot indicates that current is coming towords the observer looking into the paper.

There are six coils 1−1′, 2−2′, 3−3′, 4−4′, 5−5′, and 6−6′ placed in armature slots. Current in coil sides 1, 2, 3, etc. are shown by cross and current at the other coil sides, i.e., 1′, 2′, 3′, etc. are shown by dots. The flow of current through the coils has been shown diagrammatically as in Fig. 2.16. By applying Fleming's Left-Hand rule, it is seen that force developed on the upper half of the armature conductors in Fig. 2.15 is from right to left and for the lower half of the armature conductors it is from left to right. These forces lead to development of torque which causes rotation of the armature in the anticlockwise direction as shown in Fig. 2.15. It may be noted carefully from the figure that as the armature rotates in the anticlockwise direction, current in the conductors of a coil passing under the brushes would change. This will ensure that at any point of time any conductor on the upper half of the armature will have cross current and on the lower half will have dot current. There will be continuous rotation of the armature in one direction. The reversal of current in the coils passing under the brush is called commutation.

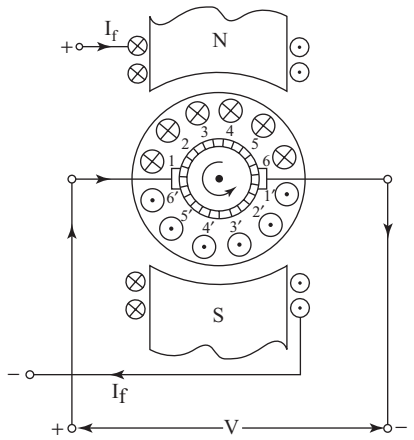


Figure 2.15 *Working principle of a dc motor illustrated*

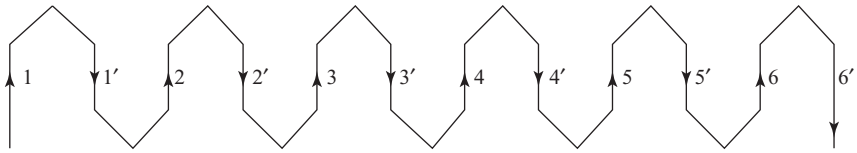


Figure 2.16 *Coils 1–1′, 2–2′, 3–3′, 4–4′, 5–5′, and 6–6′ of the armature are shown connected in series*

2.7.2 Changing the Direction of Rotation

The direction of rotation of the armature will change if the polarities of supply to either the armature or to the field windings are changed. If the polarities of supply to both the field and armature are changed, the direction of rotation will remain unchanged. This has been shown diagramatically as in Fig. 2.17.

Thus, to change the direction of rotation, we have either to change the polarities of supply to the armature or to the field.

2.7.3 Energy Conversion Equation

When the armature of the dc motor starts rotating because of the interaction between the field flux and the current-carrying armature conductors, EMF will be induced in the armature as the conductors are cutting the field flux. This EMF and the EMF induced in the armature when the machine is working as a generator is the same. However, now this EMF will oppose the supply voltage, V. Since the EMF induced opposes the supply voltage, it is also known as back EMF, E_b such that

$$V - E_b = I_a R_a \tag{2.3}$$

or,
$$E_b = V - I_a R_a \tag{2.4}$$

or,
$$I_a = \frac{V - E_b}{R_a} \tag{2.5}$$

and
$$VI_a = E_b I_a + I_a^2 R_a \tag{2.6}$$

From equation (2.6) we can write for a dc motor,
Electrical Input power = Electrical equivalent of mechanical power developed + Armature copper loss
Electrical equivalent of mechanical power developed – Rotationed losses = Mechanical power output

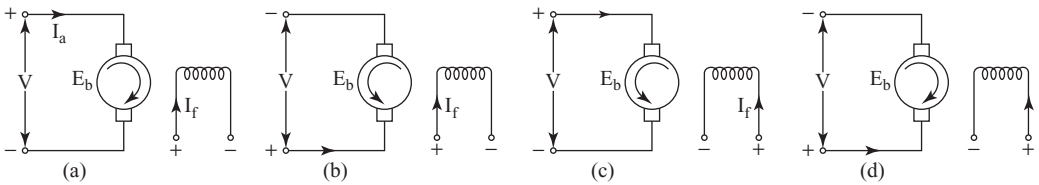


Figure 2.17 *Method of changing the direction of rotation of a dc motor: (a) polarities of both armature and field positive, leading to clockwise rotation; (b) polarities of armature changed, leading to anticlockwise rotation; (c) polarities of supply to field changed, leading to anticlockwise rotation; (d) polarities of both armature and field reversed, no change in direction of rotation*

2.8 TORQUE EQUATION

Torque developed T, angular velocity ω, and mechanical power, P are related as

$$P = T\omega$$

where T is in Nm and ω is in rad/sec.

In a dc motor, electrical power converted into mechanical power can be expressed as

$$E_b I_a = T\omega = T\frac{2\pi N}{60}$$

or,
$$T = \frac{60}{2\pi N} E_b I_a$$

$$= \frac{60}{2\pi N}\frac{\phi ZNP}{60A} I_a = \left(\frac{ZP}{2\pi A}\right)\phi I_a$$

Torque,
$$T = K\phi I_a \tag{2.7}$$

where
$$K = \frac{ZP}{2\pi A}$$

Thus, we can say that torque developed is proportional to the magnetic field strength or the magnetic flux ϕ and the magnitude of current I_a flowing through the conductors placed in the magnetic field.

2.9 STARTING A DC MOTOR

For a dc motor we can express armature current as (see Eq. 2.5)

$$I_a = \frac{V - E_b}{R_a} \quad \text{and} \quad E_b = E = \frac{\phi ZNP}{60A}$$

where R_a is the armature winding resistance and E_b is the back EMF or induced EMF in the armature.

At the moment of start, the speed N of the motor is zero. If N = 0, $E_b = 0$.

Thus,
$$I_a = \frac{V}{R_a}$$

The value of armature resistance for a dc motor is very small. Let us assume that V = 220 V and $R_a = 0.5\ \Omega$.Then at start,

$$I_a = \frac{220}{0.5} = 440\ \text{A}$$

This is a huge current to be allowed to flow through the armature in a small dc motor. To restrict this high amount of current to flow through the armature, a variable resistance can be connected in series with the armature so that Eq. (2.5) gets modified as

$$I_a = \frac{V - E_b}{R_a + R} \tag{2.8}$$

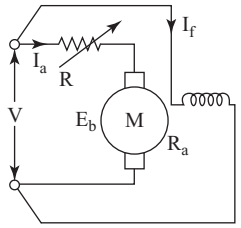


Figure 2.18 *Starting of a dc motor with a variable resistance connected in the armature circuit*

Once the motor starts rotating, back EMF E_b starts increasing and the numerator of the expression for I_a as in Eq. (2.8) gets reduced (Fig. 2.18). As the numerator goes on reducing, the denominator of expression (2.8) can be gradually reduced by reducing the value of variable resistance, R. This resistance is completely cut out of the circuit, once the motor picks up sufficient speed.

This variable resistance connected in the armature circuit is called a starter. *Thus, a starter is a variable resistance connected in series with the armature circuit to limit the initial current drawn by the motor.* Once the motor picks up speed, back EMF E_b comes into full existence, and automatically the armature current gets reduced even when the extra resistance is cut out. Removing the extra variable resistance from the circuit when it is not required is essential to avoid unnecessary wastage of energy as I^2R loss.

2.10 SPEED CONTROL OF DC MOTORS

The basic equations for a dc motor are

$$E_b = \frac{\phi ZNP}{60A} = K\phi N \tag{2.9}$$

$$V - E_b = I_a R_a \tag{2.10}$$

from Eqs. (2.9) and (2.10),

$$N = \frac{E_b}{K\phi} = \frac{V - I_a R_a}{K\phi} \tag{2.11}$$

From this expression for speed, we can say that the speed N of the motor can be changed by any or a combination of the following three methods.

2.10.1 Voltage Control Method

By changing the supply voltage V, the speed can be changed. As supply voltage can only be reduced, speed N can also be reduced from its rated value by this method.

2.10.2 Field Control Method

By varying the flux ϕ, speed can be changed. The field current produces the flux ϕ, and hence this method is called the field control method. By putting a variable resistance in the field circuit, field current can be reduced, and hence flux produced can be reduced. When flux is reduced, speed is increased. Thus, by the field control method the speed of the motor can only be increased. See Eq. (2.11).

By a combination of voltage control and field control methods, the speed of the motor can be increased and also decreased above and below its normal speed.

2.10.3 Armature Control Method

By putting an extra variable resistance in the armature circuit the speed can be reduced as

$$N = \frac{V - I_a(R_a + R')}{K\phi} \quad \text{where } R' \text{ is a variable resistance}$$

It can be seen here that the speed can only be reduced because the numerator will get reduced.

2.11 STARTER FOR A DC MOTOR

We have mentioned earlier that a starter is a variable resistance connected in series with the armature circuit during starting to reduce the starting current. This resistance is gradually cut out as the motor starts running. Two types of starters, namely a three-point starter and a four-point starter are described below.

2.11.1 Three-point Starter

A three-point starter circuit is described as follows. To start the motor, the starter arm, as shown in Fig. 2.19 (a) is moved in the clockwise direction. The arm will touch the point 1 of the starting resistance R. The whole of the resistance will appear in the armature circuit. The field winding will also get full supply through the coil of the NVR. The motor will develop torque and start rotating with full starting resistance in the armature circuit. The resistance will be cut in succession by moving the starter arm in the clockwise direction and will be brought to RUN position. In the RUN position, the soft iron piece fixed on the starter arm will face the NVR magnet piece and remain attracted. The starter arm, therefore, will stay in the RUN position against the spring tension, and the operator can remove his hand from the arm. In case of supply failure, the NVR electromagnet will get de-energized, and the starter arm will automatically return to OFF position due to the spring pressure. In case the motor is over loaded, the armature will draw excessive current which is not desirable. The coil of the OLR will remain energized when the motor is running on normal load and its armature will remain in the position shown. When the motor is drawing more current than its rated current the armature gets lifted and the contacts cc of the NVR get short-circuited thereby demagnetizing the NVR electromagnet. The starter arm will eventually return to OFF position stopping the motor.

Protecting devices like no-volt-release (NVR) and over-load-release (OLR) mechanisms have been added while designing a dc motor starter. The starter shown in Fig. 2.19 (a) is called a three-point starter. The three points or connection points are designated as L, A, and F. The connection of the starter terminals to the motor armature and field terminals and the supply terminals are as follows. Connect one supply line to L. Connect one armature terminal A to point A and connect starter terminal F to field terminal F as has been shown in Fig. 2.19 (a). The other ends of armature and field, i.e., AA and FF are joined together and are connected to the supply line L_2.

2.11.2 Four-point Starter

The disadvantage of a three-point starter is that when a large value resistance is connected in the field circuit to increase the speed of a motor, the field current gets reduced. Since the field winding and the coil of the NVR are connected in series, the current flowing through the coil of the NVR will also get

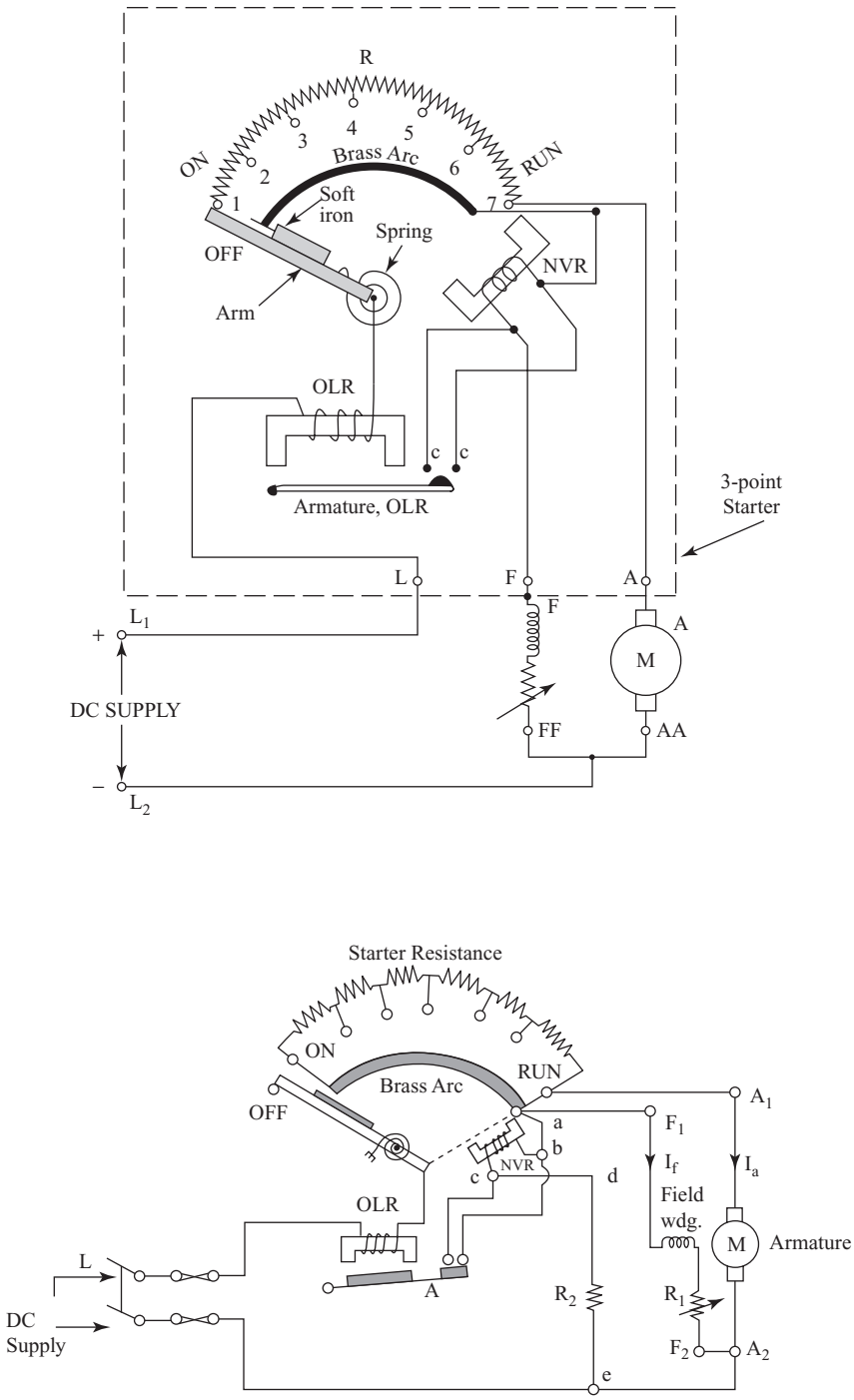


Figure 2.19 *(a) Three-point starter connections for starting a dc motor; (b) A four-point starter for a dc motor*

reduced. The attractive force of the NVR magnet to hold the starter arm in the RUN position against the spring tension may not be sufficient. The holding magnet may release the arm of the starter during normal running of the motor when current flowing through its coil becomes too small. The effect of this will be that the motor will stop, which may not be desirable.

In a four-point starter the NVR coil is connected independently across the supply voltage instead of connecting it in series with the motor field winding.

Thus, in a four-point starter there will be three parallel circuits connected across the supply voltage as has been shown in Fig. 2.19 (b). When the starter arm is brought to the ON position, current will flow through the armature circuit through the starter resistance. Current will flow from the supply via the starting arm and the starter resistance. This will limit the starting current to a large extent. Simultaneously, the field circuit will also get full supply through the brass arc. The variable resistance, R_1 can be used to control the field current to control the speed of the motor. Current will also flow through the NVR coil as supply will come through the starter arm, the brass arc, and following the path abcde as has been shown. Thus, change in field current, I_f will not have any effect on the current flowing through the NVR coil circuit. When the starter arm is brought to the RUN position, the armature will attain full speed and remain connected to the supply via the starter arm. The field circuit and the NVR circuit will get full voltage independently. The NVR will keep the starter arm in the RUN position even after the hand is released. The spring tension cannot bring back the arm to OFF position because of the attractive force of the NVR. In case of overload, the NVR terminals will be short circuited due to the attractive force of the OLR and the starter arm will get released.

2.12 TYPES AND CHARACTERISTICS OF DC MOTORS

DC machines are available as shunt, series, and compound machines. In motoring mode of operation, they are called dc shunt motors, dc series motors, and dc compound motors. The relationship between three variables, namely torque, speed, and load (load current) are studied to find the suitability of each type of motor for different applications. For example, if a mechanical load has to be rotated at a constant speed, we would need a motor as a drive whose speed will remain constant at all loads, i.e., there should not be any variation of its speed from no-load condition upto full-load condition. Again, if a set of dc motors are to drive an electric train, the starting torque developed by the motors should be very high. The motors have to develop sufficient torque so as to start the train from rest condition with a large number of passengers and other loads inside the train.

The characteristics of all types of motors are drawn as follows.

2.12.1 Characteristics of DC Shunt Motors

The basic equations of a dc motor are

$$E_b = K\phi N$$

$$E_b = V - I_a R_a$$

$$T = K\phi I_a$$

$$N = \frac{V - I_a R_a}{K\phi}$$

For a shunt motor flux produced by the field current is directly proportional to the field current. Hence,

$$\phi \propto I_f$$

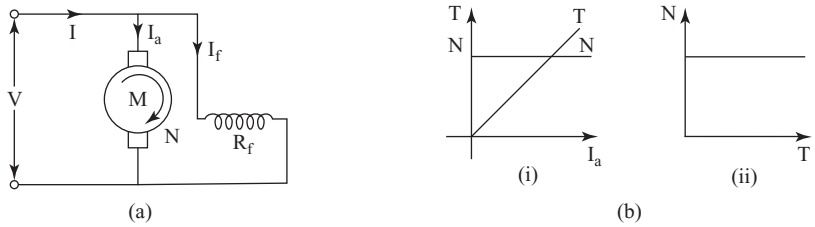


Figure 2.20 *(a) DC shunt motor; (b) characteristics of a dc shunt motor*

From Fig. 2.20 (a), $$I_f = \frac{V}{R_f}$$

As can be seen from Fig. 2.20 (a), when V and R_f are constant, I_f will be constant.

Since I_f is constant, ϕ is constant. Then

$$T = K\phi I_a = K_1 I_a \quad \text{where } K\phi = K_1$$

$$\therefore \quad T \propto I_a$$

Torque T versus current I_a characteristic has been shown in Fig. 2.20 (bi).

$$N = \frac{V - I_a R_a}{K_2 I_f} \quad \text{where } \phi = \frac{K_1}{K} = K_2$$

Since $I_a R_a$ is very small as compared to V, change of $(V - I_a R_a)$ with change of I_a will be very small.

Thus, N will remain constant as I_a, which is proportional to load, increases as has been shown in Fig. 2.20 (bi).

By knowing the variation of T and N against the load current, I_a, the relationship of T versus N can be drawn as shown in Fig. 2.20 (bii).

It can be seen that dc shunt motors are approximately constant speed motors and can be used in applications like lathe machines, drilling machines, milling machines, in printing press, paper mills, etc.

2.12.2 Characteristics of DC Series Motors

In a dc series motor the field winding is connected in series with the armature so that same current flows through the field and armature windings. The flux produced, ϕ is proportional to the field current which is equal to I_a. The relevant equations are written as

$$N = \frac{V - I_a R_a}{K\phi}$$

$$\phi \propto I_f \quad \text{and} \quad I_f \propto I_a$$

Therefore,

$$\phi \propto I_a$$

$$T \propto \phi I_a$$

$$T \propto I_a^2$$

Since $I_a R_a$ is very small as compared to V, $V - I_a R_a \phi V$ = constant, and $\phi \propto I_a$

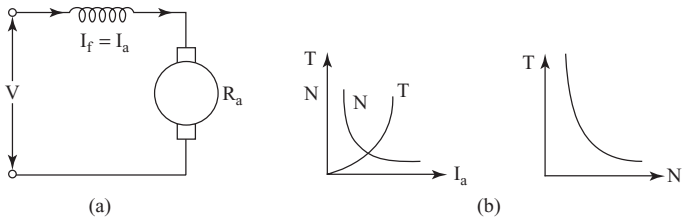


Figure 2.21 *(a) Circuit diagram of a series motor; (b) characteristics of a dc series motor*

Therefore, from $$N = \frac{V - I_a R_a}{K\phi}$$

we can write $$N \alpha \frac{1}{I_a}$$

i.e., $$N \times I_a = \text{constant}$$

Thus, the relations of N versus I_a and T versus I_a are drawn as shown in Fig. 2.21 (b). The relation between N and T is also drawn. From N versus T characteristics it is seen that at nee, i.e., at starting T is very high. That is, a series motor develops a very high torque at starting.

Therefore, a series motor is suitable for application as a drive motor in electric trains, cranes, hoists, trolley bus, etc., where the drive motor should develop very high starting torque.

From N versus I_a characteristic, it is observed that the motor will attain dangerously high speed when I_a is zero. That is, at no load the speed of the motor will be very high which may be dangerous. That is why a series motor is never allowed to run on no load. A load is always connected to its shaft before starting.

2.12.3 Characteristics of DC Compound Motors

In a compound motor two separate field windings are wound around each pole. One is shunt field winding and the other is series field winding. The shunt field winding is connected in parallel with the armature while the series field winding is connected in series with the armature as shown in Fig. 2.22. The flux produced due to the shunt field current remains constant but the flux produced by the series field current increases with the load current. The characteristic curves of a compound motor will be in between those of shunt and series motors.

The series field winding produces flux which is proportional to the armature current, i.e., the load on the motor. The flux produced by the series field either aids the shunt field or opposes the shunt field. (cummulative effect or differential effect). The characteristics relating T, N, I_a get modified from the shunt field characteristics as shown in Fig. 2.23.

In cummulative compound motors there is some drop in speed from no load to full load. For suddenly applied loads the motor speed gets reduced which may be advantageous in application like punching and shearing machines, rolling mills, lifts, mine hoists, etc. In differential compound motors, the resultant flux gets reduced as load increases, and hence the speed increases. This is seen from the expression for speed, N which is

$$N = \frac{V - I_a (R_a + R_{sc})}{K\phi}$$

From the above expression it can be seen that if flux ϕ is reduced, speed N will increase.

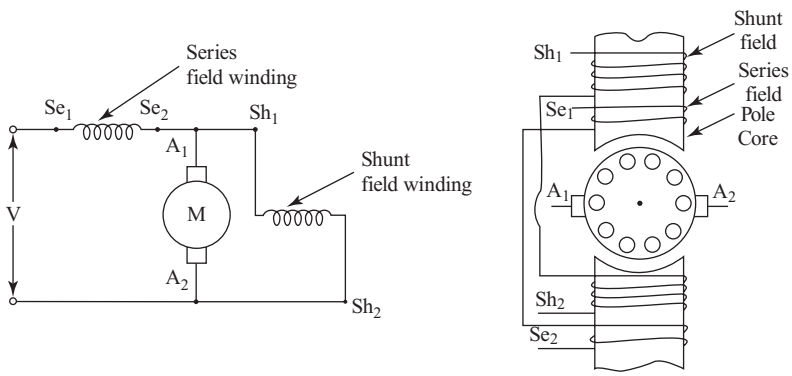


Figure 2.22 *Shows the connections of shunt and series field windings of a compound motor*

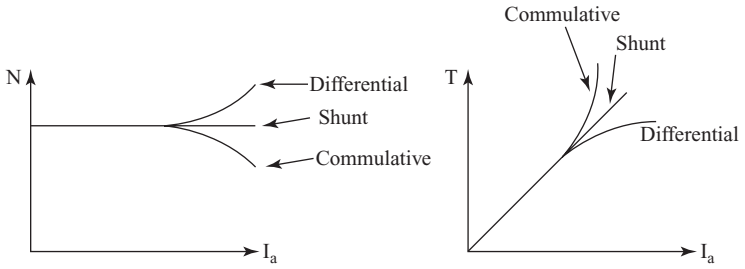


Figure 2.23 *Characteristics of dc compound motors*

2.13 LOSSES AND EFFICIENCY

The efficiency of a dc machine, like any other machine is the ratio of output power to the input power. The efficiency can never be 100 per cent because output is never equal to input. Some energy is lost in the machine during conversion of energy from mechanical to electrical or vice-versa. To achieve higher efficiency, the designer of the machine tries to keep the losses as low as possible.

2.13.1 Losses in a DC Machine

In a dc machine, like any other machines, the whole of input energy does not get converted into output energy. A portion of the input energy gets lost in the machine as shown in Fig. 2.24.

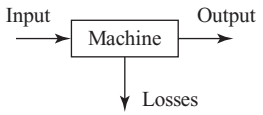
The various losses that take place in a dc machine are described as follows.

(a) I^2R loss in the armature winding

Due to current flow in the armature winding a good amount of power gets lost as $I_a^2 R_a$, where I_a is the armature current and R_a is the resistance of the armature circuit. As load on the machine changes, I_a also changes. Hence, $I_a^2 R_a$ is called the variable loss as this loss varies with the variation of load on the motor.

(b) Core loss or iron loss in the armature

Iron loss or core loss consists of hysteresis loss and eddy current loss. The core is made up of magnetic material and is subjected to variations in magnetic flux. When the armature rotates it comes under North and South poles alternately. *Hysteresis loss* occurs due to the alternate magnetization of the



Input = Output + Losses
or Output = Input – Losses

Figure 2.24 *Shows the relationship between input, output, and losses*

magnetic material. *Hysteresis loss depends upon the flux density, the frequency of variation of flux, and the volume of the core material.*

Eddy current loss is due to the presence of circulating current in the core material. When the armature rotates in the magnetic field EMF is induced in the armature core also. This EMF causes a circulating current i_c in the core which is wasted as $i_c^2\, r_c$ and produces heat. To reduce eddy current loss in the core, the core is made up of varnished, laminated steel sheets instead of a solid core. This causes increase of resistance, r_c through which the eddy current flows. *Eddy current loss depends upon flux density, frequency of alternation of flux, thickness of laminations used, and the volume of the core material.*

(c) Loss in the field windings

Losses take place in the field windings due to flow of current. This loss is equal to (VI_f) W where V is the applied voltage and I_f is the field current.

(d) Friction and windage losses

Due to rotation of the armature, air-friction loss which is also called windage loss, takes place. Frictional loss occurs due to brush and commutator rubbing and loss due to bearing friction.

2.13.2 Efficiency of DC Machine

The efficiency of a dc machine is expressed as

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

$$= \frac{VI_a}{VI_a + I_a^2 R_a + V I_f + C}$$

Where C is the sum of iron, friction, and windage losses.

2.13.3 Condition for Maximum Efficiency

To determine the condition for maximum efficiency we will differentiate the expression for η w.r.t. I_a and equate to zero as

$$\frac{d}{dI_a}\left[\frac{V}{V + I_a R_a + \frac{(VI_f + C)}{I_a}}\right] = 0$$

From which,

$$I_a^2 R_a = (VI_f + C)$$

that is, *variable loss = constant loss*.

Thus, the efficiency of a dc machine will be maximum at a load at which the variable loss becomes equal to the constant loss of the machine.

Testing of DC machines: Determination of efficiency

Efficiency of a dc machine can be determined by directly loading the machine. The output is measured and input is recorded. The ratio of output power to input power will give the value of efficiency. This method of determining efficiency is called direct loading method. The output and input are to be expressed in the same unit.

Efficiency of large machines are calculated by indirect method, i.e., by measuring the losses. Indirect method is preferred because for large machines, loading of the machine may be difficult in the laboratory. Further energy will be wasted during experimentation. A popular method, known as *Swinberne's method* of determining efficiency is described as follows.

Swinberne's method

In this method the dc machine is run as a motor. The applied voltage and the speed is adjusted to their rated values as shown in Fig. 2.25. There is no load connected to the motor shaft.

When the motor is running on no load, the input power is wasted as losses. The losses at no load are (i) iron loss; (ii) friction and windage loss, and (iii) $I_{ao}^2 R_a$ loss.

The armature current, I_{ao} at no load is small, and hence $I_{ao}^2 R_a$ will be very small. However, this value can be calculated. Iron loss, and friction and windage loss depend upon supply voltage and motor speed, respectively. The supply voltage is kept constant and speed of the motor is approximately constant at all loads. These losses are called constant losses as they remain constant at all loads. Thus, we can calculate the constant losses by subtracting $I_{ao}^2 R_a$ from the no-load input to the motor. By knowing the constant losses, efficiency of the machine can be calculated at any load current. For the sake of actual calculation of efficiency, let us take up one example.

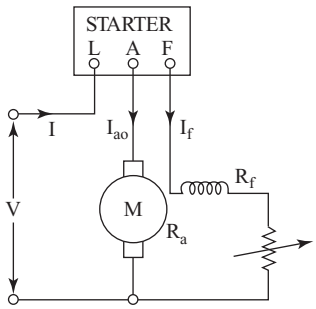


Figure 2.25 *Swineberne's test for determining efficiency*

Example 2.1 A 220 V, 50 kW dc shunt generator was run as a motor on no load at rated speed. The current drawn from the line was 8 A and the shunt field current was 2 A. The armature resistance of the machine is 0.1 Ω. Calculate the efficiency of the generator at full load.

Solution:

Input power at no load = VI

$= 220 \times 8$

$= 1760$ W

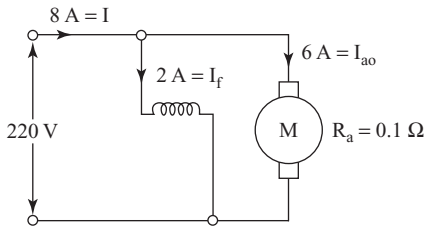


Figure 2.26

Iron, friction and windage, and field copper losses = No-load input – $I_{ao}^2 R_a$

$= 1760 - 6^2 \times 0.1$

$= 1760 - 3.6$

$= 1756.4$ W

These are constant losses.

Note that $I_{ao}^2 R_a$ loss in the armature at no load is very small.

The generator is rated at 50 kW and 220 V.

The generator output current at full load.

$$= \frac{50 \cdot 1000}{220} = 227.2 \text{ A}$$

Full-load armature Copper loss = $I_a^2 R_a$

$= (229.2)^2 \times 0.1$

$= 5253$ W

This is the variable loss.

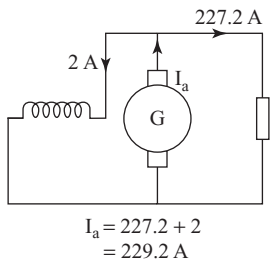


Figure 2.27

Efficiency of the generator in percentage = Output/(Output + Constant losses + Variable loss)

$$= \frac{50 \cdot 1000 \cdot 100}{50 \cdot 1000 + 1756.4 + 5253}$$

$= 89$ per cent

2.14 APPLICATIONS OF DC MACHINES

2.14.1 DC Generators

In earlier days dc generators were used to generate electricity and the power was supplied to consumers through dc distribution networks. At present, use of dc generators for generation and distribution of electricity is rare. All commercial generators are ac generators which are also called alternators. Generation and transmission of alternating current has a number of advantages. The use of dc generators is confined to supplying excitation current to ac generators and to convert ac to dc for industrial applications.

2.14.2 DC Motors

DC motors are available as dc shunt motors, dc series motors, and dc compound motors. DC shunt motors are more or less constant-speed motors. They may be used in driving a line shaft where speed has to be kept fairly constant between no load and full load. In situations where variable load has to be driven but at constant speed, such as driving a lathe, the speed change can be obtained using a shunt field regulator.

2.14.3 DC Series Motors

In applications where high starting torque is required, such as in driving hoists, cranes, electric trains, etc. series motors are used. Series motors are also used where the motor can be permanently coupled to the load, such as fans, where the torque requirement increases with speed.

Series motors attain very high speed at light load. That is why series motors should never be run on no load.

2.14.4 DC Compound Motors

DC compound motors are used in applications where large starting torque is required but there is a chance for the load to fall to a very low value. In such applications dc series motors cannot be used.

2.15 SOLVED NUMERICAL PROBLEMS

***Example* 2.2** A four-pole dc generator having wave-wound armature winding has 51 slots, each slot containing 20 conductors. Calculate the voltage generated in the armature when driven at 1500 rpm. Assume flux per pole to be 0.5 mWb.

Solution:

P = 4, A = 2 (because the armature winding is wave wound)

$$N = 1500 \text{ rpm}$$

Total number of armature conductors, $Z = 20 \times 51$

$$= 1020$$

Equation for induced EMF, $$E = \frac{\phi ZNP}{60A} V$$

Substituting values

$$E = \frac{0 \cdot 5 \times 10^{-3} \times 1020 \times 1500 \times 4}{60 \times 2}$$

$$= 255 \text{ V}$$

Example 2.3 A six-pole, lap-connected dc generator has a total of 650 conductors. The flux per pole is 0·05 Wb. Calculate the speed at which the armature is to be driven to generate an EMF of 220 V.

Solution:

P = 6, A = P = 6 (because the armature winding is lap connected)

$$\phi = 0.05 \text{ Wb; } E = 220 \text{ V}$$

$$Z = 650, N = ?$$

$$E = \frac{\phi ZNP}{60A}$$

Substituting values

$$220 = \frac{0.05 \cdot 650 \cdot N \cdot 6}{60 \cdot 6}$$

or,

$$N = \frac{220 \cdot 60}{650 \cdot 0.05} = 406 \text{ rpm}$$

Example 2.4 A four-pole 220 V dc shunt generator supplies a load of 3 kW at 220 V. The resistance of the armature winding is 0.1 Ω and that of the field winding is 110 Ω. Calculate the total armature current, the current flowing through armature conductors, and the EMF induced. Assume that the armature winding is wave wound.

Solution:

Output power = 3 kW = 3000 W

Power = output voltage, V × Output current, I_L

$$I_L = \frac{3000}{220} = 13.6 \text{ A}$$

From the Figure 2.28 it can be seen that

$$I_f = \frac{V}{R_f} = \frac{220}{110} = 2 \text{ A}$$

$$I_a = I_L + I_f$$

$$= 13.6 + 2 = 15.6 \text{ A}$$

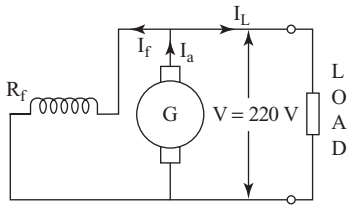


Figure 2.28

The armature winding is wave wound. The number of parallel paths is 2. That is, all the armature conductors are connected in such a way that half the armature current flows through each path. Thus, current flowing though each armature conductor will be $I_a/2$ i.e., $\frac{15.6}{2} = 7.8$ Amps. E is the EMF

induced in the armature. A voltage drop of I_aR_a takes place in the armature winding when it is supplying current. The remaining voltage, V is available across the load terminals.
Thus,

$$E - I_a R_a = V$$

or,
$$E = V + I_a R_a = 220 + 15.6 \times 0.1 = 218.44 \text{ V}$$

Example 2.5 A four-pole, 12 kW, 240 V dc generator has its armature coils wave connected. If the same machine is lap connected, all other things remaining constant, calculate the voltage, current, and power rating of the generator.

Solution:
In a wave winding all the armature coils are arranged in two parallel paths. The current-carrying capacity of each conductor, therefore, will be half of the total armature current.

$$I \times V = P$$

$$I = \frac{12 \cdot 1000}{240} = 50 \text{ A}$$

Current per path
$$= \frac{50}{2} = 25 \text{ A}$$

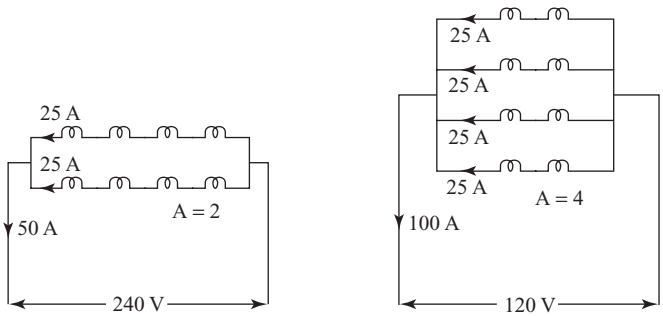


Figure 2.29

When there will be lap connection of windings, armature coils will be connected in P number of parallel paths. That is, in this case there will be four parallel paths. If each conductor or coil carries 25 A, the total output current will be 100 A. The number of coils in each path will be reduced to half, and hence the induced EMF per parallel path will be $\frac{240}{2} = 120$ V. The output power = 120×100 W $= \frac{120 \times 100}{1000}$ kW $= 12$ kW.
Thus, we observe that output power remains the same but the voltage and current ratings change.

Example 2.6 A dc shunt generator delivers 12 kW at 240 V while running at 1500 rpm. Calculate the speed of the machine when running as a shunt motor and taking 12 kW at 240 V. The armature resistance is 0.1 Ω and field resistance is 80 Ω.

Solution:
As a generator,

$$I_f = \frac{V}{R_f} = \frac{240}{80} = 3 \text{ A}$$

$$I_L = \frac{12 \cdot 1000}{240} = 50\ \text{A}$$

$$I_a = I_L + I_f = 50 + 3 = 53\ \text{A}$$

$$E_g = V + I_a R_a = 240 + 53 \times 0.1$$

$$= 245.3\ \text{V}$$

As a motor,

$$I_L = I_a + I_f$$

$$I_a = I_L - I_f$$

$$I_L = \frac{12 \cdot 1000}{240} = 50\ \text{A}, \quad I_f = \frac{V}{R_f} = \frac{240}{80} = 3\ \text{A}$$

$$I_a = 50 - 3 = 47\ \text{A}$$

$$E_m = V - I_a R_a = 240 - 47 \times 0.1$$

$$= 235.3\ \text{V}$$

Let the speed of the machine as generator be N_1 and as motor be N_2

$$E_g = \frac{\phi Z N_1 P}{60\,A} \text{ and } E_m = \frac{\phi Z N_2 P}{60\,A}$$

or,
$$\frac{Eg}{Em} = \frac{N1}{N2}$$

or,
$$N_2 = N_1 \frac{Em}{Eg} = 1500 \cdot \frac{235.3}{245.3} = 1439\ \text{rpm.}$$

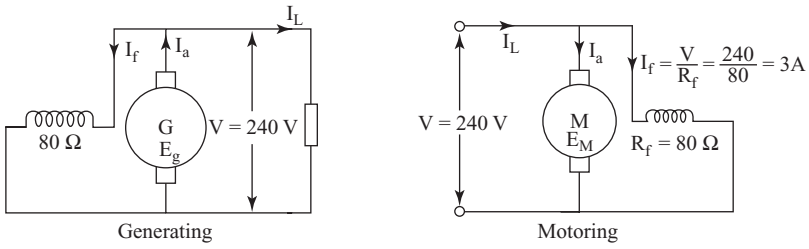


Figure 2.30

Example 2.7 A four-pole 220 V dc series motor has 240 slots in the armature and each slot has six conductors. The armature winding is wave connected. The flux per pole is 1.75×10^{-2} Wb when the motor takes 80 A. The field resistance is 0.05 Ω and the armature resistance is 0.1 Ω. The iron and friction losses 440 W. Calculate the speed of the motor. Also calculate the output horse power.

Solution:

In a series motor the armature winding and the field winding are connected in series across the supply voltage. Thus, the line current, field current, and the armature current are the same,

i.e., $I_a = I_f = I_L = 80$ A

The total member of armature conductors, $Z = 240 \times 6$

$= 1440$

Armature is wave connected, and hence A = 2

No. of poles = 4

$$V - E = I_a (R_a + R_{se})$$

or, $$E = V - I_a (R_a + R_{se})$$

$$= 220 - 80 (0.1 + 0.05)$$

$$= 208 \text{ V}$$

Again, $$E = \frac{\phi ZNP}{60\,A}$$

substituting values $$208 = \frac{1.75 \times 10^{-2} \times 1440 \times N \times 4}{60 \times 2}$$

or, $$N = \frac{208 \cdot 60 \cdot 2 \cdot 10^2}{1.75 \cdot 1440 \cdot 4} = 248 \text{ rpm}$$

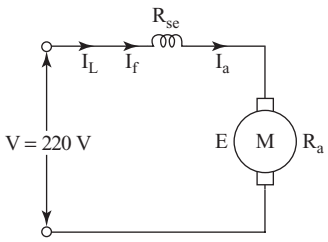


Figure 2.31

Power developed by the armature = $E \times I_a$

$$= \frac{248 \cdot 80}{1000} = 19.84 \text{ kW}$$

Power output = Power developed – Iron and Frictional losses

$$= 19.84 - 0.44$$

$$= 19.4 \text{ kW}$$

If we want to convert in horse power, we use the relation 1 kW = 0.735 hp.
Thus, power output = $19.4 \times 0.735 = 14.26$ hp.

Example 2.8 A 220 V dc shunt motor takes 5 A at no load. The armature resistance is 0.2 Ω and field resistance is 110 Ω. Calculate the efficiency of the motor when it takes 40 A on full load.

Solution:

$$I_f = \frac{V}{R_f} = \frac{220}{110} = 2 \text{ A}$$

$$I_L = I_a + I_f$$

or,
$$I_a = I_L - I_f$$

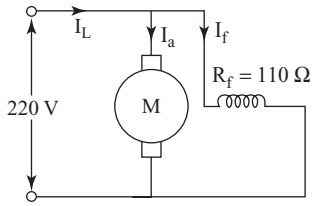


Figure 2.32

At no-load, $I_L = 5$ A

therefore, $I_a = 5 - 2 = 3$ A

At no-load, when the motor output is zero, the input = V I_L (No-load) = $220 \times 5 = 1100$ W.
The whole of input is lost as $I_a^2 R_a$ loss + $I_f^2 R_f$ loss + Iron loss + Friction and Windage loss.

$$I_a^2 R_a = 5^2 \times 0.2 = 5 \text{ W}$$

$$I_f^2 R_f = 2^2 \times 110 = 440 \text{ W}$$

Iron, friction, and windage losses $= 1100 - 5 - 440 = 655$ W

These losses are constant losses and are same at any load. This means, on full load these losses will remain at 655 W.

At full-load, $I_L = 40$ A

$$I_a = I_L - I_f = 40 - 2 = 38 \text{ A}$$

$$I_a^2 R_a = (38)^2 \times 0.2 = 289 \text{ W}$$

$$I_f^2 R_f = 2^2 \times 110 = 440 \text{ W}$$

Iron, friction, and windages losses $= 655$ W

Total losses $= 289 + 440 + 655 = 1384$ W

Efficiency
$$= \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{losses}}{\text{Input}}$$

$$= \frac{(220 \times 40 - 1384) \times 100}{220 \times 40} = 84.3 \text{ per cent}$$

Example 2.9 A four-pole dc generator has 1000 conductors. The flux per pole is 25 mWb. Calculate the EMF induced when the armature is lap connected and run at 1500 rpm. At what speed the generator must be driven to produce the same EMF with the armature winding wave connected?

Solution:
Case I

$$P = 4,\ Z = 1000,\ \phi = 25 \times 10^{-3} \text{ Wb}$$

$A = 4, N = 1500$ rpm

Induced EMF, $$E = \frac{\phi ZNp}{60A} = \frac{25 \times 10^{-3} \times 1000 \times 1500 \times 4}{60 \times 4}$$

For wave winding, $A = 2 = 625$ V

E to be the same,

$$625 = \frac{\phi ZNP}{60A} = \frac{25 \times 10^{-3} \times 1000 \times N \times 4}{60 \times 2}$$

or, $N = 750$ rpm

Example 2.10 Calculate the output power of a 12-pole separately excited having 1200 lap-connected conductors each carrying a current if 15 A. The armature is being driven at 300 rpm. The flux per pole is 60 mWb. Resistance if armature circuit is 0.1 Ω.

Solution:

Current flowing in each parallel path is the same as current flowing through each conductor in the path. Since there are 12 parallel paths in the armature (since A = P = 12), the total armature current is $15 \times 12 = 180$ A.

Induced EMF, $$E = \frac{\phi ZNP}{60\,A} = \frac{60 \times 10^{-3} \times 1200 \times 300 \times 12}{60 \times 12}$$

$$= 360 \text{ V}$$

$$V = E - I_a R_a$$

$$= 360 - 180 \times 0.1$$

$$= 342 \text{ V}$$

Power output $$= VI = \frac{342 \cdot 180}{1000} \text{kW} = 61.56 \text{ kW}$$

Example 2.11 A four-pole, 500 V, wave-wound dc shunt motor has 900 conductors on its armature. Calculate the speed of the motor if its armature current is 80 A, the flux per pole is 21 mWb and armature resistance is 0.1 Ω.

Solution:

The back EMF induced in the armature of the motor is E_b.

$$E_b = \frac{\phi ZNP}{60\,A} = \frac{21 \times 10^{-3} \times 900 \times N \times 4}{60 \times 2} \tag{2.12}$$

For motor

$$V - E_b = I_a R_a$$

or, $$E_b = V - I_a R_a$$

$$= 500 - 80 \times 0.1$$

$$= 492 \text{ V} \tag{2.13}$$

Equating (2.12) and (2.13)

$$\frac{21 \times 10^{-3} \times 900 \times N \times 4}{60 \times 2} = 492$$

or, $$N = \frac{60 \times 2 \times 492}{21 \times 10^{-3} \times 900 \times 4} = 780 \text{ rpm}$$

Example 2.12 A dc machine induces an EMF of 240 V at 1500 rpm. Find the developed torque for an armature current of 25 A.

Solution:

Power developed, $$P = E \times I_a$$
$$= 240 \times 25$$
$$= 6000 \text{ W}$$

Again, $P = T \times \omega$ where ω is the angular velocity in radians per second.

$$\omega = \frac{2\pi N}{60} \text{ where N is in rpm}$$

Therefore, $$T = \frac{P}{2\pi N/60} = \frac{60}{2\pi}\frac{P}{N} = 9.55 \times \frac{6000}{1500} = 38.2 \text{ Nm}$$

Example 2.13 A dc shunt machine connected to 220 V supply has armature resistance of 0.1 Ω and field resistance of 110 Ω. Find the ratio of the speed of the machine working as a generator to the speed of the machine when working as a motor when the line current is 100 A in both the cases.

Solution:

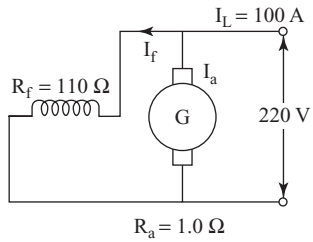


Figure 2.33

$$I_{sh} = \frac{V}{R_f} = \frac{220}{100} = 2\text{A}$$

As a generator, the machine will supply 100 A to the supply mains.

$$I_a = I_f + I_L = 2 + 100 = 102 \text{ A}$$

Output voltage = 220 V.

$$E_g = V + I_a R_a = 220 + 102 \times 0.1 = 230.2 \text{ V}$$

As a motor, the machine will draw 100 A from the supply out of which 2 A will go to the field circuit.

Therefore, $$I_a = 100 - 2 = 98 \text{ A}$$

$$E_m = V - I_a R_a = 220 - 98 \times 0.1 = 210.2 \text{ V}$$

$$\frac{\text{Speed of gen, } N_g}{\text{Speed of motor, } N_m} = \frac{E_g}{E_m}$$

or,
$$\frac{N_g}{N_m} = \frac{230.2}{210.2} = 1.095$$

2.16 REVIEW QUESTIONS

A. Short Answer Type Questions

1. Draw a next sketch of a dc machine and name the component parts.
2. What is the function of the following parts of a dc machine: (i) field poles; (ii) armature; (iii) brush and commutator; (iv) shaft?
3. Explain the function of brush and commutator in a dc machine for generating action.
4. With a simple example show how lap winding and wave windings are made.
5. Deduce the EMF equation for a dc machine.
6. Describe various methods of speed control of dc motors.
7. Explain why dc motors should require starters.
8. Draw the connection diagram of a dc motor starter.
9. Draw characteristics of dc series motors and mention applications.
10. State the various losses that occur in a dc machine.
11. What is hysteresis loss and eddy current loss?
12. Derive the torque equation for a dc motor.
13. Draw the connection diagrams for dc shunt, series, and compound motors.
14. Explain the working principle of a dc generator.
15. How can you determine the efficiency of a dc machine without actually loading the machine?
16. Why do we use laminated sheets for the armature and the field cores?
17. How can you change the direction of rotation of a dc motor?
18. Why is it advisable not to start a dc series motor without having any load on it?
19. Why do we connect the coils of the armature in series parallel?
20. What are the various losses in a dc machine. Which losses are called constant losses and why?
21. How can you determine the efficiency of a dc machine without actually loading the machine?

B. Numerical Problems

22. The wave-connected armature of a two-pole 200 V generator has 400 conductors and runs at 300 rpm. Calculate the useful flux per pole.

[Ans. ϕ = 10 mWb]

23. The induced EMF in a dc machine while running at 500 rpm is 180 V. Calculate the induced EMF when the machine is running at 600 rpm. Assume constant flux.

[Ans. E = 216 V]

24. A 250 V shunt motor draws 5 A while running on no load at 1000 rpm. Calculate the speed of the motor when it is loaded and draws a current of 50 A. The armature circuit resistance is 0.2 Ω and field circuit resistance in 250 Ω.

[Ans. N = 964 rpm]

25. A dc shunt machine has armature resistance of 0.5 Ω and field resistance of 750 Ω. When seen as a motor on no load at 500 V, the line current drawn is 3 A. Calculate the efficiency of the machine when it operates as a generator with an output of 2 kW at 500 V.

[Ans. η = 89.6 per cent]

26. A four-pole wave-connected dc armature has 50 slots with 10 conductors per slot. The armature is rotated at 1000 rpm of the useful flux per pole is 30 mwb, calculate the amount of EMF induced.

[Ans. 500 V]

27. A six-pole armature has 410 wave-connected conductors. The flux per pole is 0.02 wb. Calculate the speed at which the armature must be rotated so as to generate 400 V.

[Ans. 975 rpm]

28. A 200 V dc shunt motor having an armature resistance of 0.2 Ω and field resistance of 100 Ω draws a line current of 50 A at full-load at a speed of 1500 rpm. What will be its speed at half load?

[Ans. 1539 rpm]

29. A 500 V shunt motor takes a current of 5 A on no-load. Calculate the efficiency of the motor when it takes 100 A. Take R_a = 0.5 Ω and R_f = 250 Ω.

[Ans. 85.4 per cent]

30. A shunt motor takes 125 A at 400 V at 1000 rpm at a particular load. If the total torque remains unchanged, Calculate the speed and armature current when the magnetic field is reduced 80 per cent of its original value. Take R_a = 0.25 Ω.

[Ans. 1224 rpm, 156.25 A]

31. A dc shunt generator has a field resistance of 60 Ω and armature resistance of 0.03 Ω. As a generator, the machine delivers 40 kW at 240 V when driven at a speed of 450 rpm. Calculate the speed of the machine when running as a motor taking 40 kW of power input at 240 V.

[Ans. N=424 rpm]

C. Multiple Choice Questions

1. The expression for EMF induced in a dc machine is
   (a) $E = \frac{\phi ZNP}{60A}$ (b) $E = \frac{\phi ZNP}{60}$
   (c) $E = \frac{\phi ZNP}{60N}$ (d) $E = \frac{ZNPA}{60\phi}$.
2. Which of the following statements is not true for a dc machine?
   (a) EMF induced is directly proportional to air-gap flux
   (b) EMF induced is directly proportional to number of armature conductors
   (c) EMF induced in inversely proportional to number of parallel paths of the armature conductors
   (d) EMF induced is inversely proportional to the number of poles.
3. The poles and armature of a dc machine is made of laminated steel sheets to
   (a) reduce hysteresis loss
   (b) reduced eddy current loss
   (c) reduce I^2R loss
   (d) reduce humming noise of the core.

4. The EMF induced in the armature of a dc generator is alternating in nature but in the output circuit dc is made available by
   (a) Brush and slip-ring arrangement
   (b) Brush and commutator arrangement
   (c) diode rectifiers
   (d) converter circuit.
5. The windings of a dc machine are either
   (a) lap or wave connected
   (b) lap or spirally connected
   (c) wave or spirally connected
   (d) made of concentric coils or of short-pitched coils.
6. Critical field resistance of a dc generator is that value of the resistance at which
   (a) the field resistance line always lies below the OCC
   (b) the field resistance line is tangent to the OCC
   (c) the field resistance line crosses the OCC atleast at two points
   (d) the field resistance line does not touch the OCC at all.
7. The EMF induced in a four-pole dc generator having 1000 conductors, lap-connected windings, flux per pole of 10 mwb and rotated at 600 rpm is
   (a) 1000 V (b) 500 V
   (c) 250 V (d) 100 V.
8. The brush and commutator arrangement in a dc motor is used to achieve
   (a) unidirectional current in the armature
   (b) unidirectional torque to achieve continuous rotation of the armature
   (c) change in the direction of rotation of the armature
   (d) high starting torque.
9. The number of parallel paths of the armature winding of an eight-pole, 250 V, wave wound dc machine having 1500 armature conductors is
   (a) 4 (b) 2
   (c) 6 (d) 8.
10. Which of the following statements is not true for the EMF induced in a dc machine?
   (a) EMF induced is directly proportional to speed of the armature
   (b) EMF induced is inversely proportional to flux per pole
   (c) EMF induced is directly proportional to number of armature conductors
   (d) EMF is directly proportional to the flux per pole.
11. The direction of rotation of a dc motor can be changed
   (a) by reversing the polarities of the supply
   (b) by reversing either the polarities of the supply to armature or to the poles
   (c) by reversing only the polarities of the supply to the armature
   (d) by reversing only the polarities of the supply to the field poles.
12. The speed of a dc motor is
   (a) directly proportional to back EMF and inversely proportional to flux
   (b) inversely proportional to back EMF and directly proportional to flux
   (c) directly proportional to both back EMF and flux
   (d) inversely proportional to both back EMF and flux.
13. The nature of EMF induced in the armature coils in a dc machine is
   (a) dc (b) ac
   (c) pulsating dc (d) variable dc.
14. A dc machine is connected to 220 V supply mains. Its armature resistance is 0.2 Ω. What should the magnitude of EMF generated so that it may feed 100 A to the supply?
   (a) 200 V (b) 220 V
   (c) 240 V (d) 260 V.
15. Residual magnetizm of the field poles is necessary for the voltage built up in
   (a) dc shunt motor
   (b) dc shunt generator
   (c) dc series motor
   (d) dc separately excited generator.
16. A dc motor when connected directly to the supply would draw a very heavy current because
   (a) the back EMF at starting is zero
   (b) the back EMF at starting is maximum
   (c) the back EMF is opposing the supply voltage
   (d) torque required at starting is high.
17. The speed of a dc shunt motor can be reduced by
   (a) decreasing the field current
   (b) increasing the supply voltage to the motor
   (c) decreasing the supply voltage to the motor

(d) by decreasing the supply voltage to the motor and simultaneously decreasing the field current.

18. The relationship between torque, T and armature current, I_a for a dc series motor is
    (a) $T \propto I_a$ (b) $T \propto \frac{1}{I_a}$
    (c) $T \propto I_a^2$ (d) $T \propto \frac{1}{I_a^2}$.
19. The relationship between torque, T and armature current, I_a for a dc shunt motor is
    (a) $T \propto I_a$ (b) $T \propto \frac{1}{I_a}$
    (c) $T \propto I_a^2$ (d) $T \propto \frac{1}{I_a^2}$.
20. Efficiency of a dc machine is less than that of an equivalent transformer because
    (a) there is friction and windage losses in dc machines
    (b) core losses are more in dc machines than in transformers
    (c) copper losses are more in dc machines than in transformers
    (d) for all the reasons mentioned in (a), (b) and (c).

Answers to Multiple Choice Questions

1. (a)	2. (d)	3. (b)	4. (b)	5. (a)	6. (b)
7. (d)	8. (b)	9. (b)	10. (b)	11. (b)	12. (a)
13. (b)	14. (c)	15. (b)	16. (a)	17. (c)	18. (c)
19. (a)	20. (a)				

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Transformers

TOPICS DISCUSSED

- Applications of transformers
- Basic principle and construction details
- Core-type and shell-type transformers
- EMF equation
- Transformer on no-load
- Transformer on load
- Transformer circuit parameters and equivalent circuit
- Phasor diagram of a transformer
- Concept of voltage regulation
- Concept of an ideal transformer
- Transformer tests
- Efficiency of a transformer
- Condition for maximum efficiency
- All-day efficiency
- Calculation of regulation of a transformer
- Factors affecting losses in a transformer

3.1 INTRODUCTION

Electricity is generated in power houses. Power houses are situated at distant places wherefrom the generated power is to be taken to industries, commercial centres, residential colonies, and villages. The generators used in power houses generate electricity at a particular voltage. Normally the generation voltage is 11,000 V or 11 kV. Electricity is transmitted from the place of generation to the place of its use through electrical transmission lines which are taken overhead using transmission towers. The voltage level of the power to be transmitted is raised to higher values, say from 11 kV to 220 kV to reduce the cost of transmission. High-voltage transmission reduces the size of transmission line conductors thereby reducing the weight of the conducting material used. Since power is the product of voltage and current, for the same power if voltage is increased, the magnitude of current will decrease. For transmitting at this reduced current, the size of the line conductors will reduce, and hence the cost gets reduced. The power house wherefrom electricity is transmitted through transmission lines is called the 'sending end' whereas the other end of the transmission lines where the electricity is received for use is called the 'receiving end'.

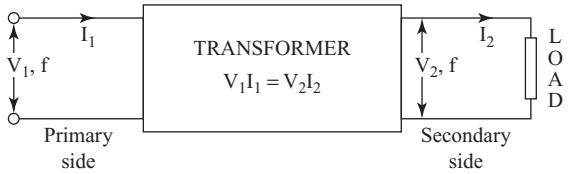


Figure 3.1 *Block diagramatic representation of a transformer*

At the sending end the voltage level is increased using step-up transformers before, connecting to the transmission lines, while at the receiving end the voltage level is lowered before the distribution of electricity for use. The voltage level is raised for reducing the cost of transmission while it is again lowered before supplying to consumers, for safety reasons. Consumers use electricity at 230 V or at 400 V.

A device known as transformer which either raises or lowers the voltage level of electrical power is always used at both the ends of the transmission lines. When voltage is raised from lower level to higher level, the device used is called a step-up transformer. When voltage is lowered from a high level to a low level, the transformer used is called a step-down transformer.

The frequency of the alternating voltage on both sides of the transformer will not change. Whatever the frequency of the input voltage is, the same will be the frequency of the output voltage.

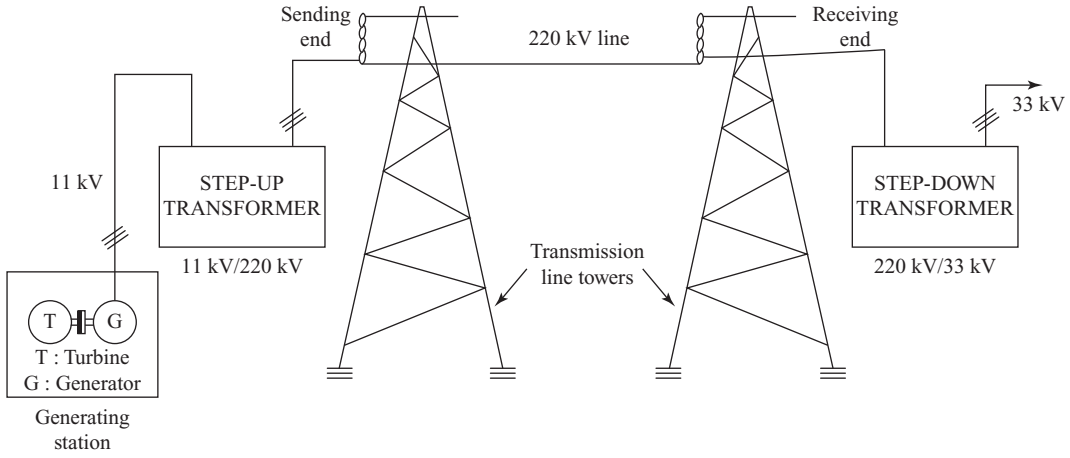
With this introduction, we may define a transformer as a device which raises or lowers the voltage level of any electrical power input without changing the frequency. The block diagramatic representation of a transformer has been shown in Fig. 3.1.

3.2 APPLICATIONS OF TRANSFORMERS

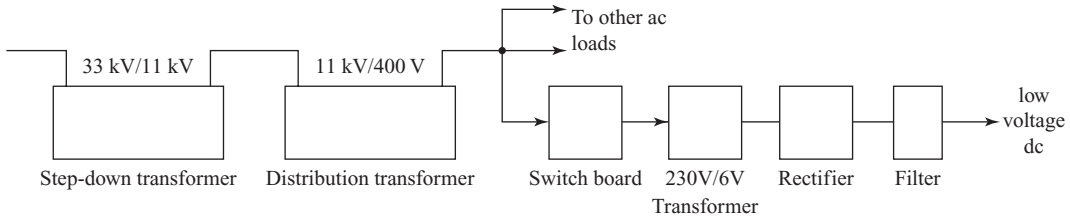
Volt–ampere rating of the transformer is the same whether calculated on the low-voltage side or at the high-voltage side. It must be noted that a transformer does not generate any electricity. It only transforms and transfers electrical power from one circuit to the other at different voltage levels. Depending upon the requirement, transformers are made for various voltage and current ratings. Transformers used to raise the voltage level at the sending end of the transmission lines, and to lower the voltage level at the receiving end are called *power transformers*. These are very big transformers rated at, say, 11 kV/220 kV and several MVAs (mega volt ampers) as their power ratings.

Smaller transformers are used in lowering the voltage level for the purpose of distribution of electricity to consumers. The transformer which feeds electricity to your house will have specifications like 11 kV/400 V, 50 Hz, 500 kVA. Here, the voltage is being reduced to 400 V or 230 V at the user end for safe supply to residences. These are called *distribution transformers*. Supply of power at high voltage to residences may lead to chances of fatal accidents and other problems. Equipment to be used also have to be manufactured for higher voltage ratings. Insulation of wires used in house wiring will have to be sustaining for higher voltages. Therefore, electricity is supplied at residences at 230 V for single-phase appliances and at 400 V for three-phase equipment. Very small transformers are used in many electrical and electronic equipment, and gadgets to lower the voltage level from 230 V on one side to, say, 6 V or 3 V, on the other side. For example, if you are to construct a battery eliminator for your transistor radio, or your tape recorder, you need to get 6 V dc supply from the available 230 V ac supply. A transformer is required to step down the voltage and then a diode rectifier and filter are required to get the steady 6 V dc output. Fig. 3.2 shows the use of transformers of different voltage ratings.

Electricity is generated in the power house when a turbine (T) rotates a generator (G). The generation voltage is 11 kV which is stepped up to 220 kV by a step-up transformer. The output of the transformer



(a) Single-line diagram of the power transmission system (only one line has been shown)



(b) Block-diagramatic representation of the distribution of electricity for end use

Figure 3.2 *Illustrates the use of transformers in stepping-up and stepping-down operations*

is connected to the high-voltage transmission line. At the receiving end of the transmission line, a step-down transformer is used to bring the voltage level again to a lower level. The power received is further stepped-down to lower voltage by use of distribution transformers and is connected to the load of the consumer, i.e., to industry, commercial buildings, and residential houses. Fig. 3.2 (b) shows the use of distribution transformers for supply to various kinds of electrical loads including low-voltage dc supply after rectification. Thus, we have seen that transformers of different voltage levels and power capacity are used for transmission, distribution, and utilization of electricity. Transformers are, therefore, seen as one of the most important components of the whole power system network.

3.3 BASIC PRINCIPLE AND CONSTRUCTION DETAILS

Transformers work on the principle of electromagnetic induction. To understand the principle of working of a transformer, one small experiment can be performed as explained below. We will need one battery, one single-way and one two-way switch, two coils, and a PMMC-type dc low-range center zero voltmeter. The connections are as shown in Fig. 3.3. In Fig. 3.3 (a), when the switch S is turned on and off quickly, the voltmeter neddle will get deflected. In Fig. 3.3 (b), a two-way switch is used. When the switch is quickly turned on and off, the current through coil 1 will flow in opposite directions every time the switch is operated. There will be voltage induced in the second coil as would be indicated by the deflection of the needle of the voltmeter in opposite directions.

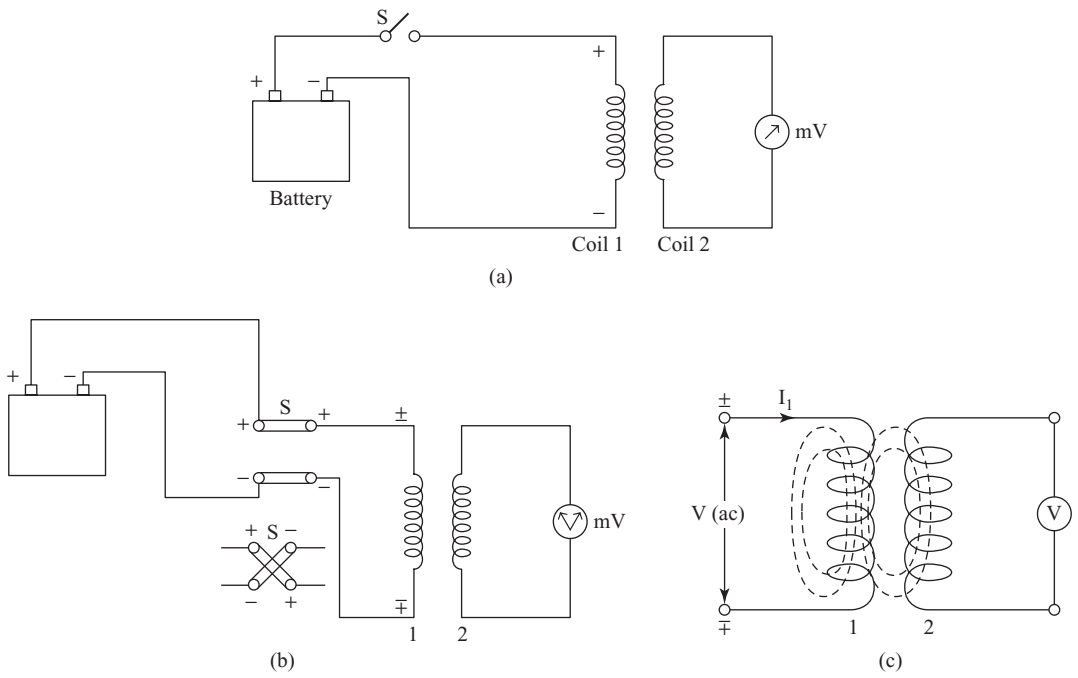


Figure 3.3 *Principle of electromagnetic induction: (a) changing current in coil 1 produces EMF in coil 2; (b) changing current flowing in reverse direction in coil 1 produces alternating voltage in the second coil; (c) alternating voltage applied to coil 1 induces alternating EMF in the second coil*

Operation of the two-way switch in opposite directions changes the polarity of supply voltage to coil 1. The frequency of change of polarity will depend upon how quickly the switch is repeatedly operated. This is equivalent to connecting an ac supply of certain frequency to coil 1, which has been shown in Fig. 3.3 (c).

Voltage is induced in the second coil due to changes of current flowing in the first coil. When current flows through the first coil a flux is produced around the coil. If current is changing, the flux produced will also change. If the second coil is placed near the first coil, there will be a changing linkage of the flux by the coils. This will induce EMF in both the coils. The magnitude of EMF induced will depend upon the rate of change of the flux linkage and the number of turns of the coil.

3.3.1 Basic Principle

The basic principle of the transformer is that EMF is induced in a coil due to the rate of change of flux linkage by it as has been shown in Fig. 3.3 (c).

In a transformer, two coils, which are also called windings wound on an iron core, are used. Coil 1 is called the primary winding and coil 2 is called the secondary winding. See Fig. 3.4 (b). These windings are made on an iron core which is made of a magnetic material. The magnetic material permits easy establishment of the flux through the core, and hence through the windings. Since it is necessary to produce more flux by using small current, the reluctance of the flux path must be low. Iron has high permeability, i.e., low reluctance. Use of iron as the core material for the windings improves the

magnetic coupling between the windings, which is essential for the transfer of power from one circuit to the other efficiently.

By changing the ratio of the number of turns of the coils the magnitude of the induced EMF in the second coil can be changed, e.g. if the number of turns of the second coil is less than the first coil, EMF induced will be less. The frequency of the induced EMF in the two coils will be the same as that of the frequency of power supply to the first coil. Now, it should be possible to connect an electrical load across the second coil, and power will be delivered to the load. Thus, when electrical supply is connected to the primary circuit or the winding, power gets transferred to the second circuit via the magnetic circuit. This device, called transformer, is based on the same principle of magnetic coupling of two coils. The constructional details of a transformer is described as follows.

3.3.2 Construction Details

Fig. 3.4 (a) shows the outside view of a transformer. It may be noted that the transformer is placed inside an iron tank filled with oil. The tank has some radiating tubes and fins so that oil inside the tank gets circulated and the heat from the transformer is radiated to the atmosphere. The transformer consists of a core made up of a magnetic material around which two coils are placed. One coil is connected to supply voltage as shown in Fig. 3.4 (b). This coil is called the primary winding. The other coil is called the secondary winding. Supply is taken from the secondary winding by connecting any electrical load like fan, tube light, electrical motors, etc.

The transformer core is made up of thin sheets called laminations. The laminated silicon steel sheets are cut into proper size from a big sheet and are placed one above the other to form the core of required width and cross section. The laminated sheets are tightly fastened to form the core. If the laminations are not tightly fastened, they will vibrate in the magnetic field and give rise to humming noise. This magnetic vibration of laminations is known as magnetostriction which is not desirable.

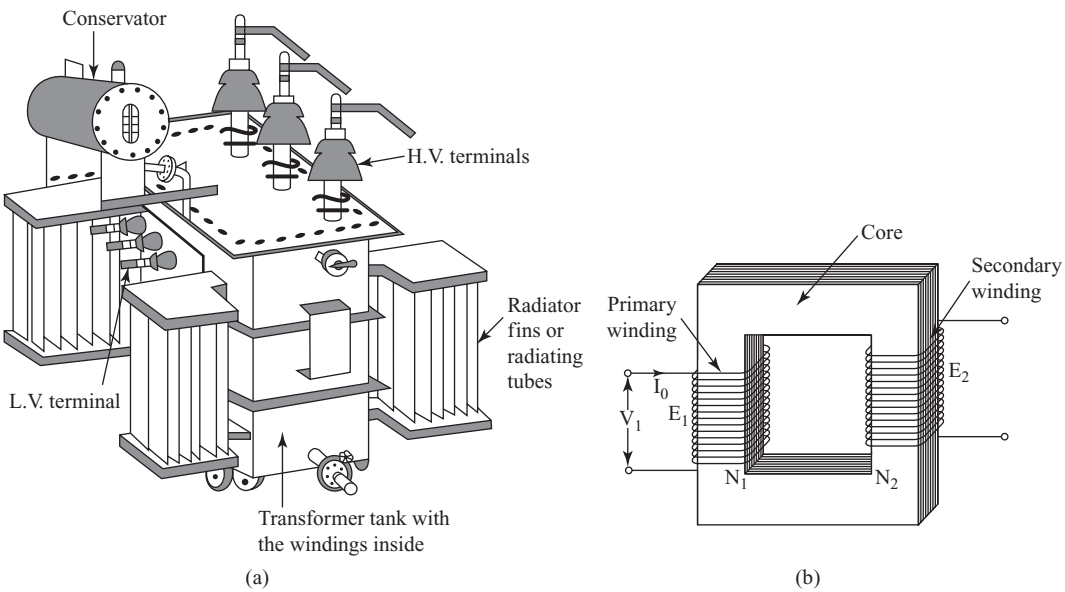


Figure 3.4 *Constructional details of a transformer: (a) Outside view of a transformer; (b) view of core and windings*

The core is made up of a magnetic material using thin laminated sheets instead of a solid one. This is done to reduce power loss in the core due to circulating current flowing in the core and producing undesirable heating of the core as well as of the windings which are wound on the core. The reason for circulating current in the core is explained below. When an alternating voltage is applied across the primary winding, an alternating current I_0 will flow through the winding. This current will produce an alternating flux which will link (i.e., pass through) both the primary and secondary windings. The flux will close their path through the magnetic material, i.e., the core material. EMFs will be induced in the both the windings due to change of flux linkage as induced EMF, e is expressed as

$$e = -N\frac{d\phi}{dt} \tag{3.1}$$

where, N is the number of turns of the winding, and ϕ is the flux produced.

EMF will also be induced in the core material when the material is being subjected to an alternating magnetic field. Due to this induced EMF in the core, a circulating current, called eddy current will flow across the core cross section. If the core is laminated and the laminated sheets are varnished with varnish insulation, the eddy current will get reduced, and hence there will be reduced eddy current loss. To reduce the eddy current loss in the core it is, therefore, made up of thin laminated silicon steel sheets.

Instead of iron, the core is made up of laminated silicon steel sheets. When a certain percentage of silicon is added to steel, the hysteresis loss in the core gets reduced. Hysteresis loss occurs due to orientation of the magnetic dipoles of the magnetic material when the material is subjected to the alternating magnetic field.

The windings of the transformer are made up of insulated copper wires. The cross-sectional area of the winding wires will depend upon the requirement of current-carrying capacity, and the number of turns are calculated according to the voltage ratio of primary and secondary windings. The core and the winding assembly are placed inside a tank filled with transformer oil for the purpose of providing insulation to the windings and also for cooling purpose. Transformer oil used is mineral oil having high dielectric strength. The tank is provided with radiating tubes so that heated oil gets circulated through the tubes and heat produced in the transformer is radiated to the atmosphere through the oil circulating from the tank through the radiating tubes.

Heat is produced in the transformer due to I^2R loss in the windings and hysteresis and eddy current loss in the core. The I^2R loss in the windings will depend upon the magnitude of current flow through the windings when the transformer is supplying some electrical load. The core loss which is the sum of hysteresis loss and eddy current loss remains constant at any load. Even when the transformer output circuit, i.e., the secondary winding, is not connected to any load, there will be core loss once an alternating voltage is applied to its primary winding. As long as the primary voltage is kept constant, the core loss will remain constant. That is why the core loss is called constant loss. The I^2R loss which is also called copper loss is a variable loss, as it varies with the magnitude of load current.

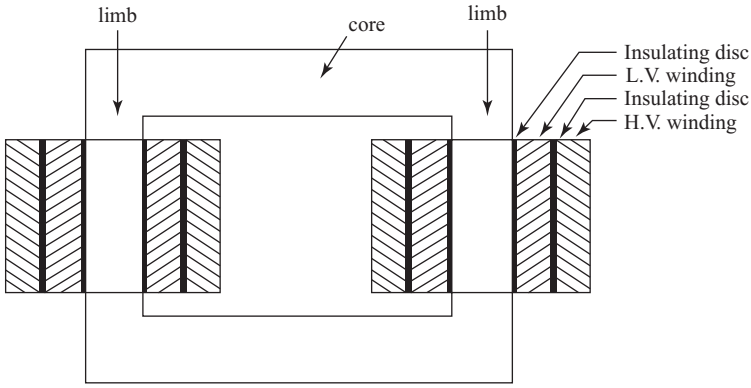
These losses will overheat the transformer unless the heat generated is radiated out to the atmosphere. If a transformer gets too overheated, its insulation strength will reduce and ultimately there may be short circuit inside the transformer damaging it completely.

Transformers are manufactured as single-phase transformers and as three-phase transformers. In three-phase transformers three separate windings are made for both primary and secondary sides. The windings are connected either in star or in delta. Terminal connections are brought out through low voltage (L.V.) terminals and high voltage (H.V.) terminals. A conservator tank fitted with a breather is placed above the tank. The conservator is connected to the transformer tank with a pipe and carries transformer oil. The empty space above the level of oil in the conservator is provided to allow the expansion of oil in the tank due to heating and for the removal of gas formed.

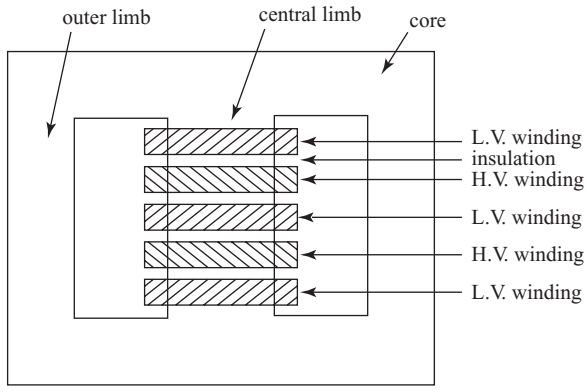
3.4 CORE-TYPE AND SHELL-TYPE TRANSFORMERS

There are two types of core constructions, viz core type and shell type. In core-type construction, the primary and secondary windings are placed around the limbs of the transformer core. The windings are made in cylindrical form and are placed around the core limbs. First, the low-voltage winding is placed around the limbs. Over the low-voltage windings are placed the high-voltage windings. The high-voltage winding is placed somewhat away from the core so as to reduce the insulation problem. The windings are insulated from the core through insulating cylindrical disc made of insulating material. The windings are made up of insulated copper wires in two sections or parts and are connected after they are placed in positions to form primary and secondary windings, respectively.

In a shell-type construction, the windings are placed in the central limb. The windings are wound in the form of a number of circular discs, and are placed one above the other. The extreme two discs on the central limb are low-voltage winding sections. These, l.υ sections and h.υ. sections are then connected to form low-voltage- and high-voltage windings. The width of the central limb is twice the width of the outer limbs so that the flux density is the same in all the limbs.



(a) Core-type construction



(b) Shell-type construction

Figure 3.5 *Core-type and shell-type transformers*

The choice of type of core to be used for transformer construction depends on a number of factors. In power transformers, in general, the core-type construction is preferred while for distribution transformers, the shell-type construction is preferred. The leakage flux and leakage reactance of a transformer depends upon the magnetic coupling between the primary and secondary windings. Voltage regulation and short-circuit impedance depend upon the leakage reactance of a transformer. In the shell-type construction the magnetic coupling is better than in the core-type construction.

3.4.1 Power Transformers and Distribution Transformers

Power transformers are connected at the two ends of the transmission lines to step up or to step down the voltage. A number of such transformers are connected in parallel depending upon the amount of power to be transmitted. They are rated for high voltages, e.g. 11 kV/220 kV, 100 MVA. The size of such transformers is very large. They are installed outdoors in a substation.

Distribution transformers feed electricity to the consumers. They are rated for voltages like 11 kV/400 V. These transformers are generally of pole-mounted type and always remain energized being ready all 24 hours to supply electricity to the consumers. Even if there is no consumption of electricity from a distribution transformer, it has to remain energized all the time. The core losses of such transformers must be low by design. Otherwise, their all-day operating efficiency will be low.

3.5 EMF EQUATION

We have known that EMFs are induced in the transformers primary and secondary windings when alternating flux link both of them. The frequency of alternation of the flux will depend upon the frequency of the primary supply voltage which is normally 50 Hz. The magnitude of the EMF induced in the two windings of a transformer will be different if they have different number of turns. Let us now derive the EMF equation. Referring to Fig. 3.6, and considering a sinusoidal input voltage, V_1 at a frequency, f, the flux produced due to current, I_0 is $\phi = \phi_m \sin \omega t$.

The general equation for the instantantaneous value of the EMF induced, e is expressed as

$$e = -N\frac{d\phi}{dt}$$

[Note that the minus sign indicates that the induced EMF opposes the supply voltage according to Lenz's law.]

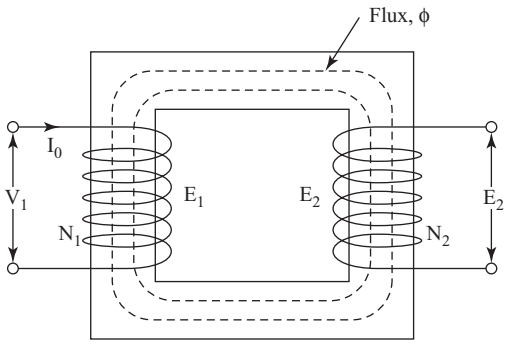


Figure 3.6 *Two windings of a transformer wound on a common core*

A sinusoidally varying flux is represented as

$$\phi = \phi_m \sin \omega t$$

where ϕ is the instantaneous value, and ϕ_m is the maximum value

Considering $\phi = \phi_m \sin \omega t$,

$$e = -N \frac{d(\phi_m \sin \omega t)}{dt}$$

we get

$$\begin{aligned} e &= -N \phi_m \omega \cos \omega t \\ &= -N \phi_m \omega [-\sin (\omega t - \pi/2)] \\ &= N \phi_m \omega \sin (\omega t - 90°) \end{aligned}$$

This equation is of the form

$$e = E_m \sin (\omega t - 90°)$$

where $E_m = N \phi_m \omega$

or, $E_m = N \phi_m 2\pi f$

We know for a sinusoidal voltage, the RMS value is $\frac{1}{\sqrt{2}}$ times its maximum value. If, E is the RMS value of the induced EMF, then

$$E = \frac{E_m}{\sqrt{2}} = \frac{N\phi_m 2\pi f}{\sqrt{2}} = \frac{N\phi_m 2 \times 3.14 f}{1.414}$$

or, $$E = 4.44 \phi_m f N \text{ V} \tag{3.2}$$

Primary winding has N_1 turns. The induced EMF in the primary winding E_1 is

$$E_1 = 4.44 \phi_m f N_1 \text{ V} \tag{3.3}$$

Since the same flux producing E_1 also links the secondary winding having N_2 number of turns, the induced EMF in the secondary winding, E_2 is

$$E_2 = 4.44 \phi_m f N_2 \text{ V} \tag{3.4}$$

Dividing Eq. (3.3) by Eq. (3.4),

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

or, $$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

K is called the ratio of voltage transformation. For a step-up transformer, $N_2 > N_1$, and hence, the value of K is more than 1. For a step-down transformer, N_2 is less than N_1, and hence $K < 1$.

3.6 TRANSFORMER ON NO-LOAD

Transformer on no-load means that the secondary winding is open and no electrical load (like a motor, a heater, a fan, an air-conditioner, etc.) is connected across its terminals for supply of electrical power. Since the transformer on no-load is not doing any useful work except that it remains energized and is ready to supply electricity when required, its output on no-load is considered to be zero, On the input side, the transformer will draw some small amount of current, I_0. Simplified representation of a transformer on no-load has been shown in Fig. 3.7 (a). Since there is no load connected across the secondary winding, the circuit is open, and hence no current will flow through the secondary winding as has been shown. The primary supply voltage is V_1 and the current flowing through this winding is I_0. What will be the phasor relationship between V_1 and I_0? If the winding is a purely inductive one, I_0 will lag the voltage V_1 by 90°. However, there will be hysteresis loss and eddy current loss in the core. Thus, I_0 should have a component I_C in phase with V_1. The core loss is equal to $V_1 I_C$ Watts. Therefore, I_0 will lag V_1 by an angle somewhat less than 90°. If V_1 is shown vertical, I_0 will lag V_1 by an angle ϕ_0 of say 85° as has been shown in Fig. 3.7 (b).

The induced EMFs E_1 and E_2 which are due to a time-varying flux ϕ will lag ϕ by 90°. This can be observed from the EMF equation where it was shown that when $\phi = \phi_m \sin \omega t$, the induced EMF $E = E_m \sin(\omega t - \pi/2)$. Again, according to Lenz's law, the induced EMF E_1 must oppose the cause from which it is due, i.e., V_1. The magnitude of E_1 will be somewhat less than V_1. This can be observed from Fig. 3.7 (a) where it is seen that current I_0 flows from a higher potential V_1 to a comparatively lower potential E_1. Thus E_1 opposes V_1, E_1 is created by ϕ and lags ϕ by 90°. The phasor relationship of V_1, I_0, ϕ, E_1, E_2 has been shown in Fig. 3.7 (b). It has also been shown that I_0 can be resolved into two components I_m and I_c, where I_m is in phase with ϕ and is responsible for producing ϕ. This I_m is also called the magnetizing current because this current magnetizes the core, i.e., produces the required flux in the core. I_m is equal to $I_0 \sin \phi_0$. I_c is equal to $I_0 \cos \phi_0$. Therefore, $I_0 = \sqrt{I_m^2 + I_c^2}$. The no-load power input W_0 is equal to $V_1 I_0 \cos \phi_0$ which equals $V_1 I_c$. The induced EMF in the secondary winding, i.e., E_2 has been shown lagging flux ϕ by 90°. It has been assumed that $N_2 > N_1$, and hence $E_2 > E_1$.

Here, $\cos \phi_0$ is the no-load power factor which is very low (may be 0.1 or so).

The no-load power input is wasted as a loss as there is no output.

$$\text{Input} = \text{output} + \text{losses}$$

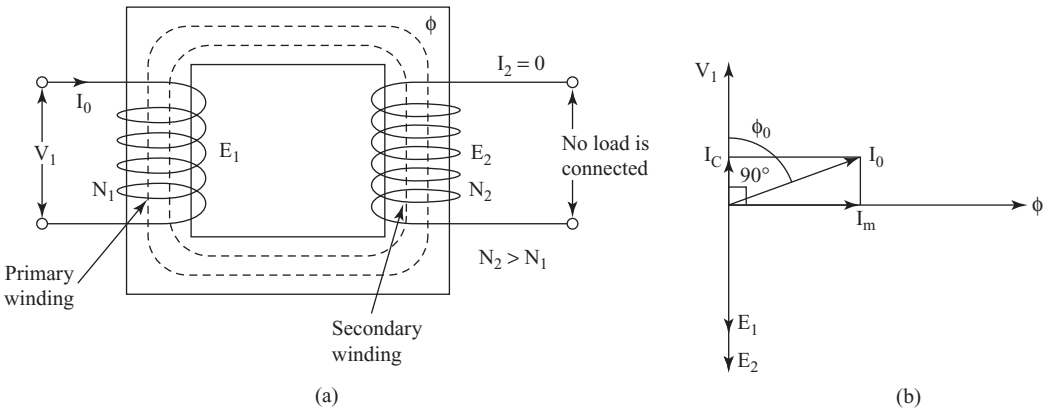


Figure 3.7 *Transformer on no-load: (a) core and the windings; (b) no-load phasor diagram*

If output is zero, input equals losses. Let us see what are the no-load losses. Since current I_0 is flowing through the primary winding which has a resistance of, say R_1, there will be some amount of copper loss in the winding as $I_0^2 R_1$ Watts. However, since I_0 is small, $I_0^2 R_1$ will also be small. Since the core is made up of a magnetic material there will be loss in the core. The core loss is due to two reasons. One is called hysteresis loss. Hysteresis loss is caused due to the magnetization of the magnetic material in alternate directions in every half cycle of the supply voltage. The magnetic dipoles of the magnetic core material align themselves in alternate directions producing alternating flux. The work done due to this is equivalent to the input energy spent and is called hysteresis loss.

The other loss component is due to eddy current. Large number of small eddy currents flow in the magnetic core material due to the EMF induced in the core, which is subjected to alternating magnetic field similar to the two windings. EMFs get induced in the core material for the same reason as for the coils. This EMF induced in the core creates current which continues to circulate in the core and heat up the core unnecessarily. This current is of no use to us and leads to only waste of input energy. This is called eddy current loss. The sum of hysteresis loss (W_h) and eddy current loss (W_e) is called core loss (W_c) or iron loss (W_i). The no-load input power is expressed as

$$W_0 = V_1 I_0 \cos\phi_0 = I_0^2 R_1 + W_h + W_e$$

If we neglect the small amount of $I_0^2 R_1$, then, no-load input power

$$W_0 = W_h + W_e = W_c, \text{ i.e., equal to core loss}$$

3.7 TRANSFORMER ON LOAD

When some electrical load is connected across the secondary terminals, power will be supplied to the load from the primary supply via the magnetic circuit. A current of I_2 will flow in the secondary circuit. The voltage available across the load, V_2 will be somewhat less than E_2.

When the transformer is loaded, the secondary current I_2 will create flux in the core in the opposite direction to that of the original core flux ϕ which was produced on no-load. Thus, the resultant flux will get reduced momentarily. This will reduce the induced EMF E_1 and E_2. As E_1 is reduced, the difference between V_1 and E_1 will increase and due to this more current of amount I_1' will flow from the supply mains through the primary winding. This current will produce an opposing flux to that produced by I_2 such that $I_2 N_2 = I_1' N_1$. Then, the two fluxes will balance each other, and hence the original flux ϕ will remain unchanged in the core. Irrespective of the magnitude of the load current, the net core flux remains practically constant at all load conditions.

Using ampere-turns balance equation

$$I_2 N_2 = I_1' N_1$$

or,

$$I_1' = \left(\frac{N_2}{N_1}\right) I_2 = K I_2$$

It may be noted that under load condition the primary current I_1 is equal to the sum of no-load current I_0 and I_1', which is K times I_2. I_1' is the additional current drawn by the primary winding due to the loading of the transformer.

Thus,

$$I_1 = I_0 + I_1' = I_0 + KI_2$$

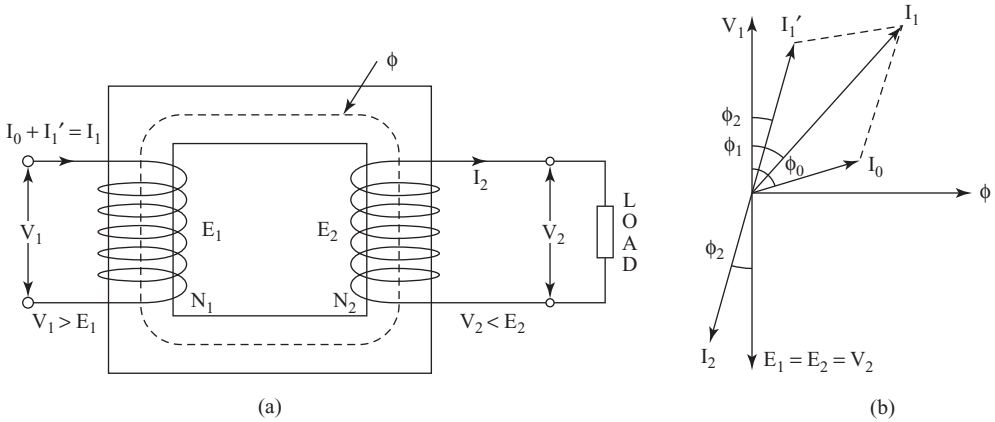


Figure 3.8 *(a) Transformer on load; (b) simplified phasor diagram of a transformer on resistive–inductive load*

The phasor diagram relating all the parameters under loading condition neglecting the voltage drop due to winding resistances and leakage reactances has been shown in Fig. 3.8 (b). The phasor diagram is for some resistive–inductive load when the load power factor angle is ϕ. That is why I_2 has been shown lagging the load voltage V_2 by an angle ϕ_2. I_1' is the additional primary current drawn from the supply source to counter balance the magnetizing effect of $I_2 (I_2 N_2 = I_1' N_1)$.

3.8 TRANSFORMER CIRCUIT PARAMETERS AND EQUIVALENT CIRCUIT

Earlier, while explaining transformer on load, we had neglected the winding resistances and leakage reactances of the transformer. Since the windings are made of copper wire of certain cross-sectional area, they will have some resistance. Thinner the wire, higher will be the resistance, for a particular length of the wire. The resistances of the primary and secondary windings are called R_1 and R_2 respectively. When current flows through the windings there will be voltage drop $I_1 R_1$ and $I_2 R_2$ in the primary and secondary windings, respectively. The resistances R_1 and R_2 can be shown carrying current I_1 and I_2 as in Fig. 3.9. When the resistances have been shown separately, the windings are considered made up of some number of turns but without having any resistances. In other words, we may say that when the resistances of the windings have been shown separately, the windings will be assumed as having no resistance. It may be noted that input voltage V_1 is higher than the induced EMF E_1 (current flows from

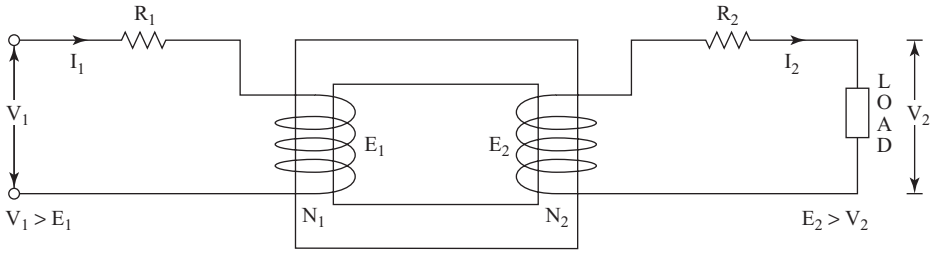


Figure 3.9 *Transformer on load*

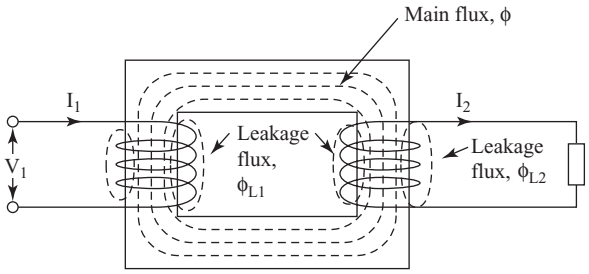


Figure 3.10 *Leakage flux and leakage reactance of a transformer*

higher potential V_1 to lower potential E_1). The induced EMF E_2 is greater than load terminal voltage V_2. Neglecting reactances of the windings, the voltage equation are

$$V_1 - I_1 R_1 = E_1$$

and

$$E_2 - I_2 R_2 = V_2$$

The power loss in the primary and secondary windings are respectively, $I_1^2 R_1$ and $I_2^2 R_2$. These are also called copper losses. Note that the above two voltage equations have been written considering only the resistances of the windings. Voltage drop due to reactances of the windings has been neglected.

In addition to the resistance of the windings, the windings will have leakage reactances due to the leakage flux in the core. The concept of leakage reactance due to leakage flux is explained below. Fig. 3.10 shows a transformer on load.

The main flux ϕ is common to both the windings. This flux links both the primary and secondary windings. In addition, some more flux, called leakage flux, ϕ_L is created due to current I_1 and I_2 flowing through the two windings. These fluxes will also create some induced EMFs in the two windings. Since ϕ_{L1} is proportional to I_1, ϕ_{L1} will be in phase with I_1. Similarly, ϕ_{L2} is proportional to I_2 and will be in phase with I_2. ϕ_{L1} will induce an EMF E_{L1}, and ϕ_{L2} will induce an EMF E_{L2} in the windings, respectively.

E_{L1} will lag I_1 by 90° and E_{L2} will lag I_2 by 90°. These induced voltages are balanced by the reactance voltage drop in the two windings, respectively. The primary and secondary winding leakage reactances are called, respectively, X_1 and X_2 so that $I_1 X_1$ is considered voltage drop in the primary winding due to leakage reactance X_1 and $I_2 X_2$ is considered voltage drop in secondary winding due to leakage reactance X_2. It is to be noted that EMFs E_1 and E_2 are induced in primary and secondary windings due to main flux ϕ whereas E_{L1} and E_{L2} are induced in these windings due to their leakage fluxes ϕ_{L1} and ϕ_{L2}. The effect of leakage flux and the resulting induced EMF in the two windings are represented by two reactances X_1 and X_2 creating voltage drops. The reactances X_1 and X_2 are in fact two fictitious (imaginary) reactances considered to represent the effect of leakage flux. The complete transformer circuit with its parameters has been shown as in Fig. 3.11. In this circuit diagram we have not considered the no-load current, I_0 drawn by the transformer. So, the circuit shown in Fig. 3.11 is an approximate equivalent circuit of the transformer.

The primary circuit impedance is Z_1 and the secondary winding impedance is Z_2.

$$Z_1 = \sqrt{R_1^2 + X_1^2} \text{ and } Z_2 = \sqrt{R_2^2 + X_2^2}.$$

Note that the transformer is a coupled circuit. For the sake of simplicity in calculation, we might like to convert it into a single circuit by transferring the circuit parameters of the primary circuit to the secondary circuit and vice versa.

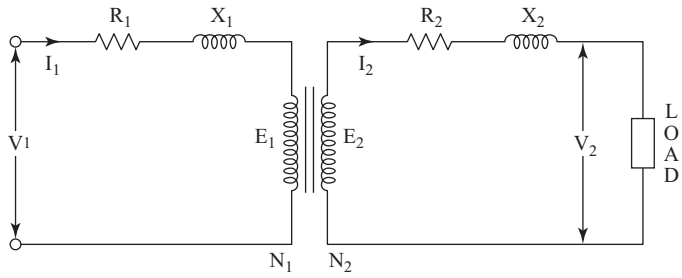


Figure 3.11 *Transformer approximate equivalent circuit*

Let us see how the secondary circuit parameters can be transferred to the primary side. Let R_2' be the value of R_2 when transferred to the primary side. By considering the same amount of losses in the resistance when transferred from one current level to the other, we can equate the copper losses as

$$I_2^2 R_2 = I_1^2 R_2'$$

or,
$$R_2' = R_2 \left(\frac{I_2}{I_1}\right)^2 = R_2 \left(\frac{N_1}{N_2}\right)^2$$

$\therefore$
$$R_2' = \frac{R_2}{K^2}, \quad \text{where} \quad K = \frac{N_2}{N_1}$$

Reactance $X = \omega L$ and $L = \dfrac{\propto N^2 A}{l}$

Therefore, $X \alpha N^2$

Let X_2, when transferred from the secondary circuit to the primary circuit, be X_2'

$$X_2' = X_2 \left(\frac{N_1}{N_2}\right)^2 = \frac{X_2}{K^2} \qquad \because \frac{N_2}{N_1} = K.$$

By transferring the circuit parameter on the primary side, the approximate equivalent circuit of the transformer can be represented as shown in Fig. 3.12.

This circuit can further be simplified by adding the resistances and the reactances for the sake of calculations.

Now we will consider the no-load current of the transformer along with the load currents I_2 and I_1 to draw the complete equivalent circuit.

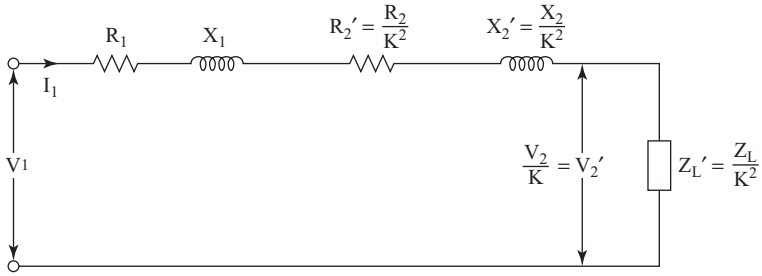


Figure 3.12 *Transformer approximate equivalent circuit having transferred the secondary parameters to the primary side*

It may be noted that I_1 is the sum of I_1' and I_0. I_0 has two components, I_m and I_c. I_m lags V_1 by 90° whereas I_c is in phase with V_1. I_m can be shown as a current flowing through an inductive reactance called the magnetizing reactance X_m whereas I_c can be shown as a current flowing through a resistance R_c as shown. The sum of I_m and I_c is I_0. Sum of I_0 and I_1' is I_1. The complete equivalent circuit representing all the parameters has been shown in Fig. 3.13.

The above circuit can be simplified by neglecting I_0 which is about three to five per cent of the rated current of the transformer. So by removing the parallel branch and adding the resistances and reactances we draw the approximate equivalent circuit as shown in Fig. 3.14. Thus, the circuit becomes the same as was drawn in Fig. 3.12 earlier.

Combining the resistances and reactances, the equivalent resistances R_e' and equivalent reactance X_e' are represented as Z_e' as has been shown in Fig. 3.15.

This simplified equivalent circuit of the transformer can be used to calculate the performance in terms of voltage regulation of the transformer under various loading conditions.

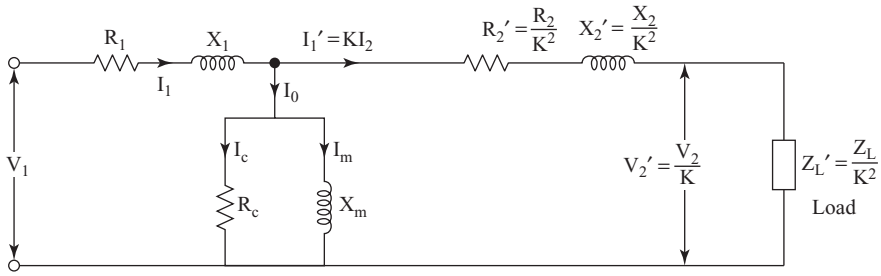


Figure 3.13 *Exact equivalent circuit of a transformer*

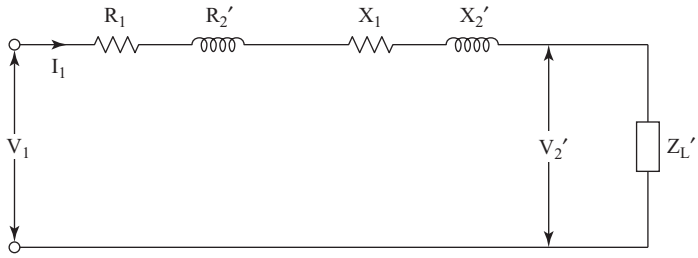


Figure 3.14 *Approximate equivalent circuit of a transformer*

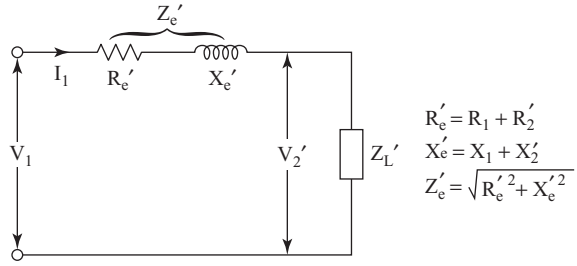


Figure 3.15 *Approximate equivalent circuit of a transformer with secondary parameters referred to the primary side*

3.9 PHASOR DIAGRAM OF A TRANSFORMER

The complete phasor diagram of a transformer at a lagging Pf load has been shown in Fig. 3.16 (a).

Procedure for drawing the phasor diagram is given below.

Draw the flux vector as the reference vector along the x-axis. The voltage induced in the two windings, E_1 ans E_2 will lag flux by 90°. If it is a step-up transformer, E_2 will be bigger in length than E_1. Otherwise, they can also be shown as equal. Show two phasors E_1 and E_2 lagging flux ϕ by 90°. Draw $-E_1$ in opposite direction to E_1, as has been shown in Fig. 3.16. Draw I_2 lagging E_2 by a certain angle. Draw I_1' opposite to I_1. Draw I_o lagging $(-E_1)$ by a large angle. Add I_1' and I_o vectorially to get I_1. Now we have to locate V_1 and V_2. We apply Kirchhoff's law in the primary winding circuit and write

$$V_1 - I_1 R_1 - jI_1 X_1 - E_1 = 0$$

or,
$$V_1 = E_1 + I_1 R_1 + jI_1 X_1 \tag{3.5}$$

Similarly, for the secondary circuit we write $E_2 - I_2 R_2 - jI_2 X_2 - V_2 = 0$

or,
$$V_2 = E_2 - I_2 I_2 - jI_2 X_2. \tag{3.6}$$

Using equation (3.5) develop the phasor diagram step by step.

Add $I_1 R_1$ with E_1 (E_1 drawn as $-E_1$) vectorially. The direction of I_1R_1 is in the direction of I_1. Then add $j\, I_1X_1$. This vector will make 90° with the direction of I_1 in the anticlockwise direction. The resultant vector will be V_1. To get V_2 we have to subtract vectorially I_2R_2 and jI_2X_2 from E_2. The no-load current, I_o is shown as the vector sum of I_m and I_c. I_m is the magnetizing component of the no-load current which produces the flux and is phase with the flux ϕ.

It must be clarified here that E_1 has been taken as $-E_1$ and then the total phasor diagram drawn only for the sake of clarity.

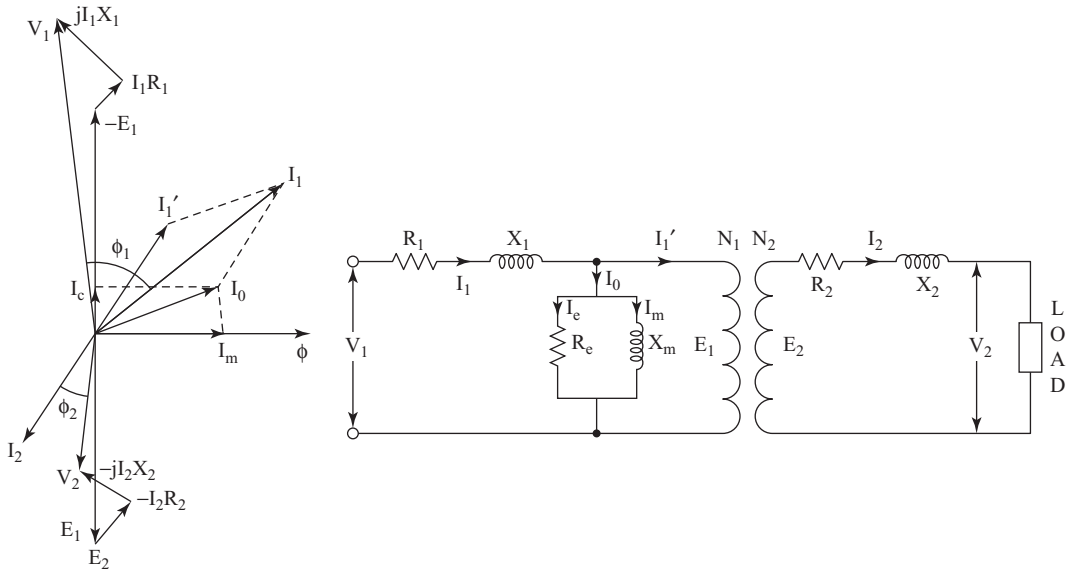


Figure 3.16 *(a) Phasor diagram of a transformer on load; (b) equivalent circuit*

3.10 CONCEPT OF VOLTAGE REGULATION

Voltage regulation of a transformer is defined as the percentage change in terminal voltage from full-load to no-load condition and is expressed as the percentage of the full-load voltage.

$$\text{Percent voltage regulation} = \frac{E_2 - V_2}{V_2} \times 100.$$

A high value of voltage regulation means that there is a large change in the terminal voltage when load is applied to the transformer, which is not desirable.

The expression for voltage regulation in terms of load current, load power factor, and transformer circuit parameters can be found from the simplified equivalent circuit of the transformer. For convenience, we will consider the approximate equivalent circuit of the transformer with primary circuit parameters referred to the secondary side as shown in Fig. 3.17. The phasor diagram relating the various quantities has also been shown. R_e'' and X_e'' are the equivalent resistance and reactance, respectively, of the transformer referred to the secondary side.

The equation relating E_2 and V_2 is

$$E_2 = V_2 + I_2 R_e'' + jI_2 X_e''$$

The phasor diagram shown in Fig. 3.17 (b) has been drawn using the above equation. I_2 has been shown lagging the voltage V_2 by the power factor angle ϕ. We add I_2R_e'' with V_2, I_2R_e'' being taken in the same direction as I_2. Then jI_2X_e'' is added which is at 90° anticlockwise with I_2. The phasor sum of V_2, I_2R_e'', jI_2X_e'' gives E_2. The approximate value of the voltage drop, i.e., the difference between E_2 and V_2 can be calculated. The phasor diagram shown in Fig. 3.17 (c) will be used to develop an expression for voltage regulation as has been done in section 3.16.

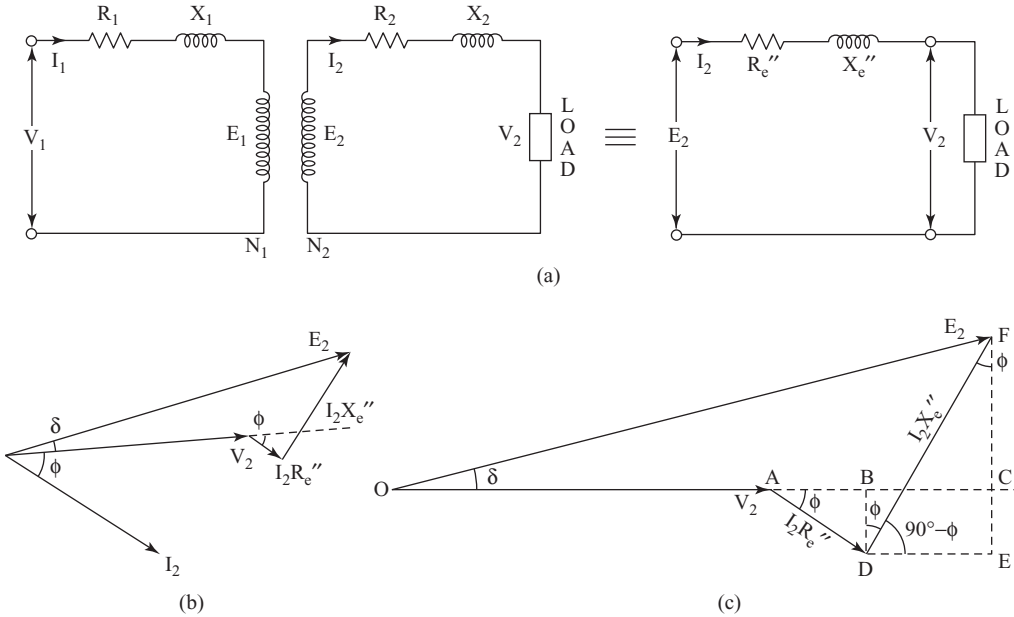


Figure 3.17 *(a) Simplified equivalent circuit of a transformer; (b) phasor diagram; (c) use of the phasor diagram for calculation of the voltage drop*

3.11 CONCEPT OF AN IDEAL TRANSFORMER

The no-load current I_o is about three to five per cent of the rated current, i.e., I_1 or I_2. The voltage drop due to I_o on $(R_1 + jX_1)$ will therefore be small. If we neglect this small effect of I_o, then the parallel branch of R_c and X_m can be shifted towards the left as shown in Fig. 3.18.

From the equivalent circuit, it can be observed that the circuit parameters of an actual transformer has been shown separately from its windings. Thus, the transformer windings are now considered without any resistance and leakage reactance, and having a loss less core. The windings only causes a change in voltage from E_1 to E_2. Such a transformer is often called an ideal transformer. However, an ideal transformer never exists in reality. An ideal transformer will have no loss in it and hence efficiency will be 100 per cent, which is not possible to achieve. Since there will be no voltage drop in the windings due to loading, the regulation will be zero which is again an ideal concept only.

An ideal transformer is one which has no core loss, no winding losses, no resistance of its windings, no winding leakage reactances, and no voltage drop in the windings. The efficiency will be 100 per cent and voltage regulation will be zero. Such ideal conditions are not possible to achieve.

3.12 TRANSFORMER TESTS

The performance of a transformer in terms of its voltage regulation (i.e., percentage change in voltage from full-load to no-load) and efficiency under various loading conditions can be calculated using the approximate equivalent circuit explained earlier. However, the circuit parameters like R_e and X_e, R_c and X_m have to be known. These parameters and the losses in the transformer can be determined by performing two tests, viz the open-circuit test or the no-load test, and the short-circuit test. These tests are explained as follows.

3.12.1 Open-circuit Test or No-load Test

In this test the transformer primary winding is supplied with its rated voltage keeping the secondary winding unconnected to the load, i.e., with no-load on the secondary. Normally, the supply is given to the low-voltage winding. The high-voltage winding is kept open. Three measuring instruments, viz a wattmeter, a voltmeter, and an ammeter are connected to the low voltage side as shown in Fig. 3.19 (a).

The equivalent circuit of the transformer has also been shown under no-load condition in Fig. 3.19 (b). The wattmeter connected on the L.V. side will record the input power, W_o to the transformer. The

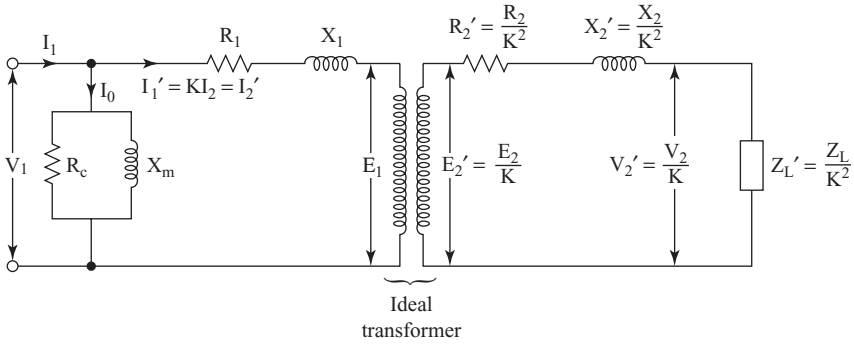


Figure 3.18 *An ideal transformer*

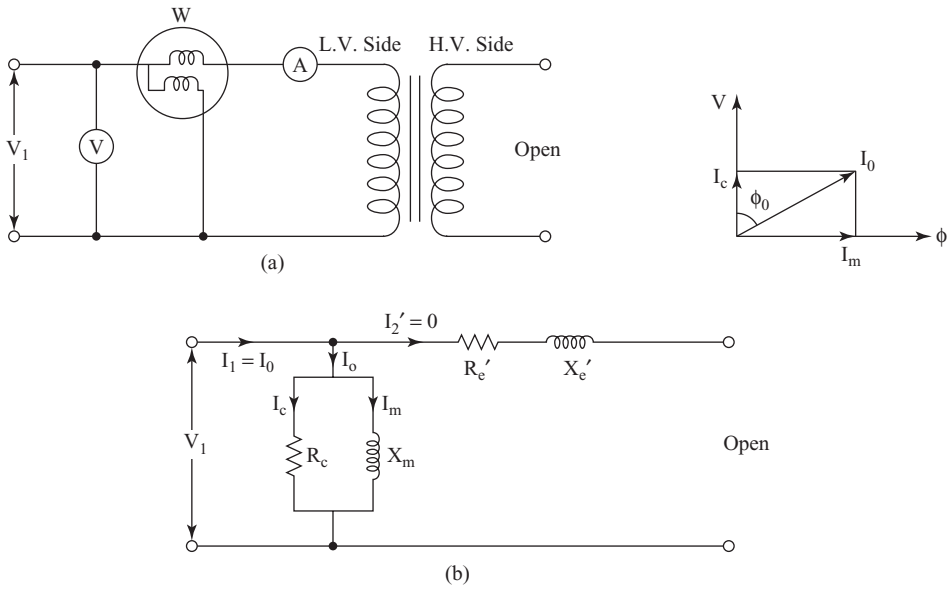


Figure 3.19 *(a) Transformer on no load; (b) equivalent circuit on no load*

supply voltage is measured by the voltmeter and the no-load line current is measured by the ammeter reading, I_o. The input power, W_o is

$$W_o = V_1 I_0 \cos \phi_0 \text{ W}$$

$$\cos \phi_0 = \frac{W_O}{V_1 I_0}$$

$$I_c = I_0 \cos \phi_0$$

$$I_m = I_0 \sin \phi_0$$

From the equivalent circuit on no-load [see Fig. 3.19 (b)]

$$R_c = \frac{V_1}{I_c} \ \Omega \ \text{ and } \ X_m = \frac{V_1}{I_m} \ \Omega$$

Since on no-load the output is zero, the input power is utilized in supplying the no-load losses. At no-load there will be no current in the secondary winding, and hence no copper loss will take place in that winding. The primary winding current on no-load is small. There will be losses in the iron core which will have two components, viz hysteresis loss and eddy current loss. Thus, the wattmeter reading on no-load can be approximately equated to core loss only. From the no-load test data we, will be able to know the core loss of the transformer, no-load current, the no-load power factor, the magnetizing reactance, and the resistance R_c corresponding to core loss. Note that core loss calculated on no-load will be the same as on full-load or at any other load. That is why core loss of a transformer is considered to be a constant loss as it does not depend on the load currents. Core loss depends on the supply voltage and its frequency.

3.12.2 Short-circuit Test

In this test, the secondary winding is short circuited with a wire and a reduced voltage is applied across the primary winding.

One voltmeter, one wattmeter, and an ammeter are connected in the primary circuit for the measurement of the applied voltage under the short-circuit condition, V_{sc}, the power consumed, W_{sc} and the current, I_{sc}, respectively. It may be noted that for convenience, the low-voltage winding is usually short circuited which forms the secondary winding. The instruments are connected in the high-voltage winding circuit where the rated current is comparatively lower than the low-voltage side, as has been shown in Fig. 3.20 (a). This is done by using an auto transformer. Under the short-circuit condition only a very small percentage of rated voltage, say about five per cent, has to be applied to the primary winding to circulate the full-load current through the windings. Current in the primary winding (H.V. winding) will also be lower than that of the low-voltage winding. Thus, by conducting the short-circuit test from the high voltage winding side with the low-voltage winding short-circuited, we can have an accurate measurement.

It may also be noted that in the short-circuit test we create a condition when a full-load rated current will flow through both the primary and secondary windings. Therefore, the copper losses in the two windings will be equal to the amount of copper loss that would otherwise occur when the transformer is actually supplying full-load at the rated voltage. Creating this type of a loading condition of a transformer is called phantom loading or fictitious loading. The actual circuit diagram for the short-circuit test and the equivalent circuit of the transformer under the short-circuit condition have been shown in Fig. 3.20.

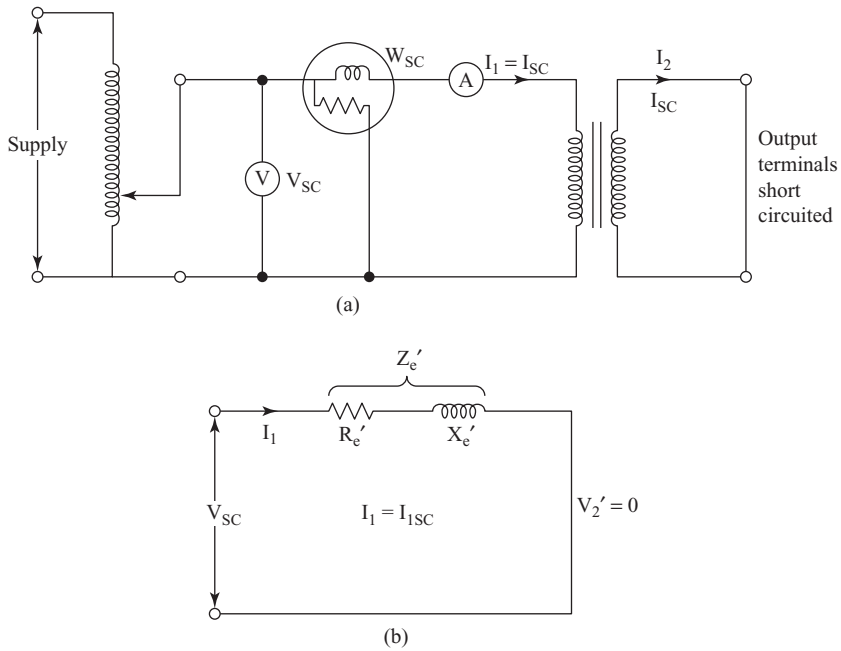


Figure 3.20 *Short-circuit test on a transformer: (a) circuit diagram; (b) equivalent circuit under the short-circuit condition*

From the equivalent circuit shown, it may be observed that the core loss component of the equivalent circuit as was shown on the no-load test has been neglected here. This is because the voltage applied in the short-circuit test is not the full-load rated voltage but a small fraction of it. Since core loss is proportional to the applied voltage, for a small voltage applied under short circuit, the core loss component has been neglected. Thus, we can equate the wattmeter reading to copper losses in the two windings. The short-circuit test readings are taken by adjusting the voltage applied to the primary through a variac, i.e., an auto transformer so that the rated current flows (to be noted from the transformer name plate) through the windings. From the readings of the three instruments, the following calculations are made.

W_{sc} = Copper losses in the two windings having the rated current flowing through them.

$$W_{sc} = I_{1(sc)}^2 R_e'$$

or,
$$R_e' = \frac{W_{sc}}{I_{1\,sc}^2}$$

$$Z_e' = \frac{V_{sc}}{I_{1(sc)}}\,\Omega$$

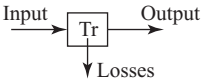
$$Z_e'^2 = R_e'^2 + X_e'^2$$

or,
$$X_e' = \sqrt{Z_e'^2 - R_e'^2}\,.$$

From the data obtained from the no-load test and the short-circuit test, the efficiency and regulation of a transformer can be calculated without actually loading the transformer.

3.13 EFFICIENCY OF A TRANSFORMER

The whole input power to a transformer is not available at the output, some is lost in the iron core as core loss and some is lost in the windings as copper loss. Since the transformer is a static device, there is no friction and windage loss (rotational loss) in the transformer.



$$\text{Input} = \text{Output} + \text{Losses}$$

Efficiency,
$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

$$= \frac{\text{Output power}}{\text{Output power} + W_i + W_{cu}}$$

or,
$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + W_{cu}}$$

where W_i = iron loss or core loss
and W_{cu} = copper loss in primary and secondary windings
V_2 = output voltage
I_2 = load current
$\cos\phi_2$ = load power factor.

The load at which efficiency will be maximum, that is the condition for maximum efficiency, can be determined by differentiating the expression for efficiency with respect to the load current, I_2.

3.14 CONDITION FOR MAXIMUM EFFICIENCY

The expression for η after dividing the numerator and denominator of the right-hand side of the expression for η by I_2 can is written as

$$\eta = \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + (W_i / I_2) + (I_2^2 R_e'' / I_2)}$$

where R_e'' is the equivalent resistance of the transformer windings referred to the secondary side. Efficiency η will be maximum if the denominator is minimum. We can, therefore, minimize the denominator by differentiating it with respect to the load current, I_2 and equating to zero as

$$\frac{d}{dI_2}\left[V_2 \cos\phi_2 + \frac{W_i}{I_2} + I_2 R_e'' \right] = 0$$

or,
$$-\frac{W_i}{I_2^2} + R_e'' = 0.$$

or,
$$I_2^2 R_e'' = W_i$$

When copper loss at a particular load equals the core loss, efficiency will be maximum at that load. The condition for maximum efficiency of a transformer is

$$\text{Core loss} = \text{Copper loss}$$

The value of the load current at maximum efficiency is determined as

$$I_2^2 = \frac{W_i}{R_e''}$$

or,
$$I_2 = \sqrt{\frac{W_i}{R_e''}}.$$

If we want to know at what percentage of full-load, the efficiency will be maximum, (ie if we would like to know if efficiency is maximum at 75% of the full load or at 80% of the full load or at 100% of the full load) we can determine as follows.

Determination of load at which efficiency will be maximum

Let us assume that x is the fraction of the full load at which the efficiency is maximum. The core loss or the iron loss W_i will remain constant at all loads. The copper loss will vary as the square of the load.

This means if at full load copper loss is $= W_{cu}$

at half of full load copper loss $= \frac{1}{4} W_{cu}$

at one-third of full load copper loss $= \frac{1}{9} W_{cu}$.

Thus, at x load, copper loss $= x^2 W_{cu}$

satisfying the condition for maximum efficiency

$$W_i = x^2 W_{cu}$$

or,
$$x = \sqrt{\frac{W_i}{W_{cu}}}$$

where W_{cu} is the full-load copper loss and x is the fraction of the full-load at which efficiency will be maximum.

If we want to know the kVA of the transformer at maximum efficiency, we would determine it as follows:

$$\text{kVA at } \eta_{max} = x \times \text{Full-load kVA}$$

Therefore, kVA at maximum efficiency $= \sqrt{\frac{W_i}{W_{cu}}} \cdot$ Full-load kVA.

Efficiency of a transformer is often expressed in terms of energy output in 24 hours in a day to the energy input. Such a calculated efficiency is known as all-day efficiency which is explained as follows.

3.15 ALL-DAY EFFICIENCY

A transformer when connected to the load has to remain energized all the time ready to supply the load connected to it. Even when all the loads are switched off, i.e., no one is utilizing any electricity, the transformer has to remain on. Thus, irrespective of the load on the transformer, the core loss will occur for all the 24 hours of the day. However, the copper loss will depend on the magnitude of the load current, and will eventually vary from time to time. All-day efficiency is calculated by considering the energy output (power multiplied by time, i.e., energy) in 24 hours to the energy input in 24 hours as

$$\text{All-day Efficiency} = \frac{\text{Output energy in 24 hours.}}{\text{Output energy in 24 hours} + \text{Iron loss in 24 hours} + \text{copper loss in 24 hours}}$$

Both commercial efficiency which is the ratio of the output power to the input power and the all-day efficiency as stated above are calculated for distribution transformers. Distribution transformers are connected to the load all the time.

All-day efficiency of such transformers which are always connected to the load at the output side is somewhat less than their commercial efficiency, which is calculated on the basis of output power and the corresponding input power.

3.16 CALCULATION OF REGULATION OF A TRANSFORMER

When a transformer is not supplying any load, the voltage across the output terminals is the same as that induced in the secondary winding, i.e., E_2. Now, when the transformer is connected to the load, the voltage available across the output terminals, V_2 becomes somewhat less than E_2.

The reduction in the output voltage from no-load to load is due to the voltage drop in the winding resistance and leakage reactance. The students are to refer to the phasor diagram as shown in Fig. 3.17 (c) for determination of voltage regulation which has been redrawn here.

In the phasor diagram shown in Fig. 3.21, we will consider E_2, i.e., length OF as equal to length OC as the angle δ is actually very small. This approximation is made to simplify the determination of an expression for voltage regulation.

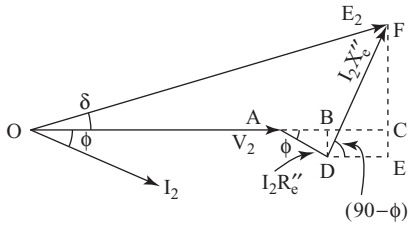


Figure 3.21 *Phasor diagram of a transformer as in Fig. 3.17 (c)*

Thus,

$$E_2 = OF = OC = OA + AB + BC = OA + AB + DE$$
$$= V_2 + I_2 R_e'' \cos\phi + DF \cos(90 - \phi)$$
$$= V_2 + I_2 R_e'' \cos\phi + I_2 X_e'' \sin\phi$$

Percentage regulation $= \dfrac{E_2 - V_2}{V_2} \times 100$

$$= \frac{I_2 R_e'' \cos\phi + I_2 X_e'' \sin\phi}{V_2} \times 100$$

If the power factor is leading, then we will have

Percentage regulation $= \dfrac{I_2 R_e'' \cos\phi - I_2 X_e'' \sin\phi}{V_2} \times 100.$

Regulation can also be calculated by transferring the secondary circuit parameters to the primary side.

3.17 FACTORS AFFECTING LOSSES IN A TRANSFORMER

Since a transformer is a static device, there is no rotational part in it, and hence there is no rotational or frictional losses. Due to current flow through the windings, there will be I^2R loss in both the primary and secondary windings. Thus,

$$\text{Copper loss} = I_1^2 R_1 + I_2^2 R_2$$

Copper loss is proportional to the square of the current. With the secondary circuit resistance referred to the primary side, the total effective resistance, $R_e' = R_1 + \dfrac{R_2}{K^2}$

Similarly, the primary circuit resistance when referred to the secondary side, the total effective resistance of the transformer windings

$$R_e'' = R_2 + K^2 R_1 \quad \text{where } K = \frac{N_2}{N_1}$$

The copper loss $= I_2^2 R_e'' = I_1^2 R_e'$

When the load current is changed, say from full load, I_2 to half load $\frac{I_2}{2}$, the copper loss becomes one-fourth of its value at full load.

The losses that take place in the iron core is called iron loss or core loss. Iron loss consists of two parts, viz Hysteresis loss and eddy-current loss. These are explained in detail as follows.

(a) Hysteresis loss

When alternating voltage is applied to the primary winding of the transformer, the core gets magnetized. The magnetization of the core takes place in alternate directions every half cycle of the supply voltage. Magnetization in alternate directions basically means that the magnetic dipoles of the magnetic material changes their orientation in opposite directions every half cycle. This gives rise to loss of energy which is expressed as

$$W_h = \eta B_m^{1.6} f V \ W$$

where W_h is the hysteresis loss in Watts
B_m is the maximum value of flux density in Wb/m^2
f is the supply frequency
V is the volume of the iron core in m^3
η is the steinmetz constant.

To keep the hysteresis loss low, the material with a lower value of steinmetz constant, such as silicon steel, is chosen as the core material.

(b) Eddy current loss

When the core is subjected to an alternate magnetic field, EMF is induced in the core material also. This EMF causes circulating current in the core, and thereby producing loss-resulting generation of heat. If the core gets heated up, it produces an undesirable effect on the insulation material used in the windings. Eddy current loss is expressed as

$$W_e = K B_m^2 f^2 t^2 \ W$$

where W_e is the eddy current loss in Watts
B_m is the maximum value of flux density
f is the supply frequency
t is the thickness of the core material.

Eddy current loss is minimized by using a thin laminated sheet steel as the core material instead of a solid core. The laminated steel sheets are assembled together and are insulated from each other using insulating varnish. This creates an obstruction to the flow of eddy current, and hence reduces the eddy current loss.

Example 3.1 A transformer has 1000 turns on its primary and 500 turns on the secondary. When a voltage, V of frequency f is connected across the primary winding a maximum flux of 2×10^{-3} Wb is produced in the core which links both the windings. Calculate the value of the EMF induced in the two windings.

Solution:

Let E_1 and E_2 be the EMFs induced in primary and secondary windings, respectively. Here, $N_1 = 1000$ and $N_2 = 500$.

$$E_1 = 4.44 \phi_m f N_1$$
$$= 4.44 \times 2 \times 10^{-3} \times 50 \times 1000.$$
$$= 444 \text{ V}$$

and

$$E_2 = 4.44 \phi_m f N_2$$
$$= 4.44 \times 2 \times 10^{-3} \times 50 \times 500$$
$$= 222 \text{ V}$$

We obsere that,

$$\frac{E_1}{E_2} = \frac{444}{222} = 2$$

and

$$\frac{N_1}{N_2} = \frac{1000}{500} = 2$$

Therefore

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

Example 3.2 A transformer has 900 turns on its primary winding and 300 turns on its secondary. A voltage of 230 V at 50 Hz is connected across its primary winding. The cross-sectional area of the core is 64 cm^2. Calculate the magnitude of the induced EMF in the secondary winding. Also calculate the value of maximum flux density in the core.

Solution:

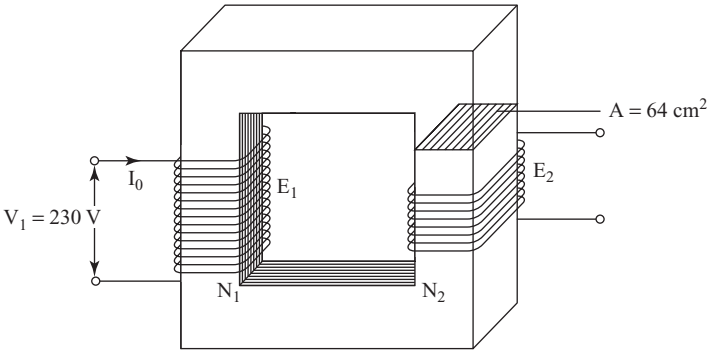


Figure 3.22

Given $V_1 = 230$ V.

The induced EMF in the primary windings is E_1. E_1 is slightly less than V_1 because there will be some voltage drop in the winding.

$V_1 > E_1$, $V_1 - E_1$ = Voltage drop in the primary winding due to current, I_0 flowing through it.

The no-load current, I_0 is very small as compared to the current that would flow when some electrical load is connected across the secondary winding. Here, the transformer is on no-load, i.e., no load has been connected to its secondary winding.

If we neglect the no-load voltage drop in the winding, we can write $V_1 = E_1$.

$$E_1 = 4.44\,\phi_m\,f\,N_1$$

and $$E_2 = 4.44\,\phi_m\,f\,N_2$$

therefore, $$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

or, $$E_2 = E_1\left(\frac{N_2}{N_1}\right) = 230\left(\frac{300}{900}\right) = 76.7\,\text{V}$$

Again, $$E_1 = 4.44\,\phi_m\,f\,N_1$$

$$= 4.44\,B_m\,A\,f\,N_1$$

when B_m is the maximum flux density and A is the cross-sectional area of the core.

Substituting values

$$230 = 4.44\,B_m \cdot \frac{64}{10^4} \cdot 50 \cdot 900$$

or, $$B_m = \frac{230 \cdot 10^4}{4.44 \cdot 64 \cdot 45000} = \frac{2300}{4.44 \cdot 64 \cdot 45}$$

or, $$B_m = 0.18\ \text{Wb/m}^2$$

Example 3.3 A 110 V/220 V transformer is supplied with 110 V, 50 Hz supply to its low-voltage side. It is desired to have maximum value of core flux as 4.2 mWbs. Calculate the required number of turns in its primary winding.

Solution:

$V_1 = 110$ V. Neglecting the winding voltage drop under no-load condition, $V_1 = E_1 = 110$ V.

$$E_1 = 4.44\,\phi_m\,f\,N_1$$

Substituting values, $$110 = 4.44 \times 4.2 \times 10^{-3} \times 50 \times N_1$$

or, $$N_1 = \frac{110 \cdot 10^3}{4.44 \cdot 4.2 \cdot 50} = 119\ \text{turns.}$$

Example 3.4 A 100 kVA, 1100/220 V, 50 Hz transformer has 100 turns on its secondary winding. Calculate the number of turns of the primary winding; the currents that would flow in both the windings when fully loaded, and the maximum value of flux in the core.

Solution:

Given, $$N_2 = 100$$

$$V_1 \approx E_1 \text{ and } V_2 \approx E_2$$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$N_1 = N_2\left(\frac{E_1}{E_2}\right) = 100\left(\frac{1100}{220}\right) = 500\ \text{turns.}$$

Rating of the transformer is = 100 kVA = 100×10^3 VA.

Primary current, $$I_1 = \frac{VA}{V_1} = \frac{100 \cdot 10^3}{1100} = 90.1 \text{ A}.$$

Since volt-ampere rating is the same for the transformer on both the sides,

scondary current, $$I_2 = \frac{VA}{V_2} = \frac{100 \cdot 10^3}{220} = 450.5 \text{ A}$$

For calculating ϕ_m, we will use the EMF equation.

$$E_1 = 4.44\, \phi_m\, f\, N_1$$

Substituting values $$1100 = 4.44\, \phi_m \times 50 \times 500$$

or, $$\phi_m = 9.9 \times 10^{-3} \text{ Wb}$$

Example 3.5 The maximum flux density in the core of a 1100/220 V, 50 Hz, 100 kVA transformer is 3.5 Wb/m^2. Calculate the area of cross section of the core and the number of turns of the primary and secondary windings if the EMF per turn is 5.5 V.

Solution:

$$V_1 = E_1 = 1100 \text{ V}$$

EMF per turn × No. of turns = total induced EMF

or, $$5.5 \times N_1 = E_1$$

or, $$N_1 = \frac{1100}{5.5} = 200 \text{ turns}.$$

$$\frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$N_2 = N_1\left(\frac{E_2}{E_1}\right) = 200\left(\frac{220}{1100}\right) = 50 \text{ turns}.$$

Again, $$E_1 = 4.44\, \phi_m\, f\, N_1$$

or, $$E_1 = 4.44\, B_m\, A\, f\, N_1$$

$$A = \frac{E_1}{4.44\, B_m f\, N_1} = \frac{1100}{4.44 \cdot 3.5 \cdot 50 \cdot 200}$$

$$= 70 \times 10^{-4} \text{ m}^2$$

$$= 70 \text{ cm}^2$$

Example 3.6 The no-load input power to a transformer is 100 W. The no-load current is 3 A when the primary applied voltage is 230 V at 50 Hz. The resistance of the primary winding in 0.5 Ω. Calculate the value of iron loss and no-load power factor.

Solution:

At no-load, the input power is lost as a small amount of I^2R loss in the winding and as core loss.

In an ac circuit, $$\text{power} = VI \cos \phi$$

Let No-load input power be W_0

$$W_0 = V_1 I_0 \cos \phi_0$$

$$\cos \phi_0 = \frac{W_0}{V_1 I_0}$$

$$\cos \phi_0 = \frac{100}{230 \times 3} = 0.15 \text{ lagging.}$$

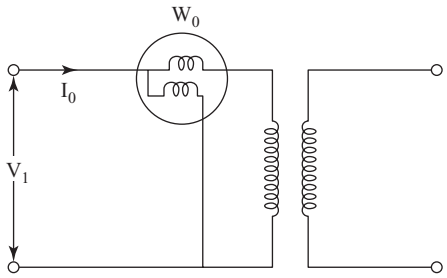


Figure 3.23

$\cos \phi_0$ is the no-load power factor of the transformer. At no-load, primary winding copper loss is equal to

$$I_0^2 R_1 = 3^2 \cdot 0.5 = 4.5 \text{ W}$$

The wattmeter reading W_0 indicates the power loss in the core as also in the winding.

Core loss, or iron loss, $W_i = W_0 - I_0^2 R_1$

$= 100 - 4.5$

$= 95.5$ W

Example 3.7 A 100 kVA, 2400/240 V, 50 Hz transformer has a no-load current of 0.64 A and a core loss of 700 W, when its high-voltage side is energized at rated voltage and frequency. Calculate the components of the no-load current and no-load branch parameters of the equivalent circuit.

Solution:

Neglecting the small amount of copper loss at no-load,

$$V_1 I_0 \cos \phi_0 = \text{Iron loss}$$

$$I_0 \cos \phi_0 = \frac{\text{Iron loss}}{V_1} = \frac{700}{2400} = 0.29 \text{ A.}$$

$$I_c = I_0 \cos \phi_0 = 0.29 \text{ A.}$$

$$\cos \phi_0 = \frac{0.29}{I_0} = \frac{0.29}{0.64} = 0.45$$

$$\sin \phi_0 = 0.89$$

$$I_m = I_0 \sin \phi_0 = 0.64 \times 0.89 = 0.57 \text{ A.}$$

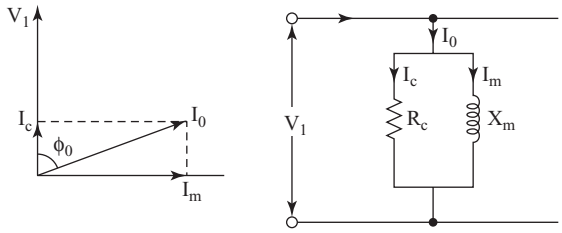


Figure 3.24

$$X_m = \frac{V_1}{I_m} = \frac{2400}{0.57}$$
$$= 4.2 \times 10^3\ \Omega.$$

$$R_c = \frac{V_1}{I_c} = \frac{2400}{0.29} = 8.27 \times 10^3\ \Omega.$$

Example 3.8 A 400/200 V, 50 Hz transformer draws a no-load current of 6 A at 0.2 power factor lagging. The transformer supplies a current of 100 A at 200 V to the load. The load power factor is 0.8 lagging. What is the magnitude of current drawn by the transformer from the supply mains?

Solution:

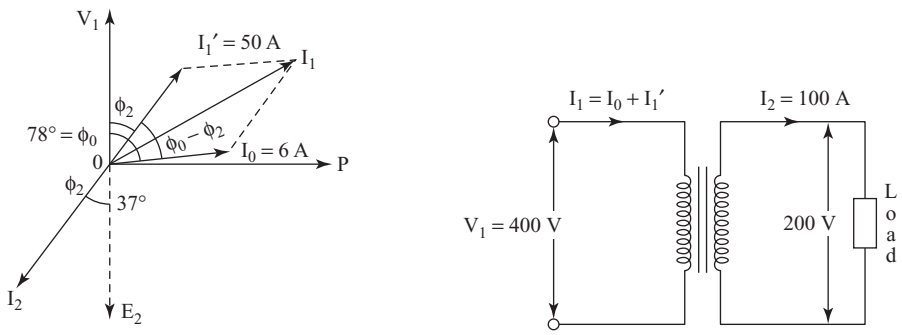


Figure 3.25

The circuit diagram and the phasor diagram showing the currents with reference to supply voltage, V_1 have been shown in Fig. 3.25. I_o is the no-load current making an angle of lag ϕ_0 with V_1 where $\cos \phi_0 = 0.2$ or $\phi_o = 78°$ (lagging). The load power factor, $\cos \phi_2 = 0.8$ or, $\phi_2 = 37°$. I_2 is the load current. I_1' is the additional current drawn by the primary to balance the load current I_2 such that

$$I_1' N_1 = I_2 N_2$$

substituting values

$$I_1' = 100\left(\frac{N_2}{N_1}\right) = 100\left(\frac{V_2}{V_1}\right) = 100\left(\frac{200}{400}\right)$$
$$= 50 \text{ A}.$$

As shown in Fig. 3.25, it is observed that the phasor sum of I_o and I_1' is the primary current when the transformer is loaded. The angle between I_o and I_1' is $\phi_o - \phi_2$, i.e., $(78^o - 37^o) = 41^o$.

Using law of parallelogram,

$$I_1^2 = \left(I_1'\right)^2 + \left(I_o\right)^2 + 2\, I_1'\, I_o \cos\left(\phi_o - \phi_2\right)$$

substituting values

$$I_1^2 = (50)^2 + (6)^2 + 2 \cdot\ 50 \cdot\ 6\cos 41^o.$$

or, $I_1^2 = 2986$

or, $I_1 = 54.6$ A

Example 3.9 A 400/200 V, 50 Hz, 10 kVA transformer has primary and secondary winding resistances of 2.5 Ω and 0.5 Ω and winding leakage reactances of 5 Ω and 1 Ω, respectively. Calculate the equivalent resistance and reactance of the transformer referred to the secondary side. What amount of power will be lost in the windings?

Solution:

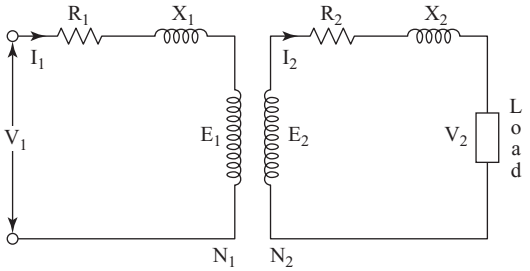


Figure 3.26 (a)

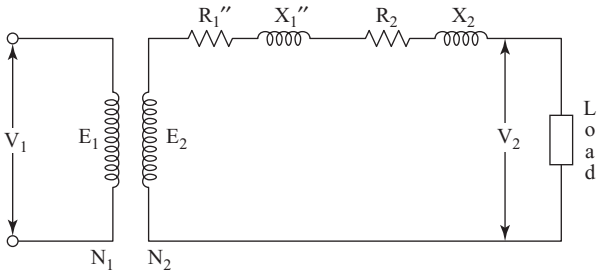


Figure 3.26 (b)

Given, $R_1 = 2.5\ \Omega, X_1 = 5\ \Omega, R_2 = 0.5\ \Omega, X_2 = 1.0\ \Omega.$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{200}{400} = \frac{1}{2} = K. \qquad \text{(assuming } V_1 = E_1 \text{ and } V_2 = E_2\text{)}$$

$$I_1^2\, R_1 = I_2^2\, R_1'' \text{ or, } R_1'' = R_1\left(\frac{I_1}{I_2}\right)^2 = R_1\left(\frac{N_2}{N_1}\right)^2 = K^2 R_1$$

thus, $$R_1'' = \left(\frac{1}{2}\right)^2 R_1 = \frac{2.5}{4} = 0.625\ \Omega$$

Similarly,

$$X_1'' = X_1 K^2 = \frac{5}{4} = 1.25\ \Omega.$$
$$R_e'' = R_1'' + R_2 = 0.625 + 0.5.$$
$$= 1.125\ \Omega.$$
$$X_e'' = X_1'' + X_2 = 1.25 + 1.0$$
$$= 2.25\ \Omega.$$

Given, $$kVA = 10$$
$$VA = 10 \times 1000$$
$$V_2 I_2 = 10 \times 1000$$
$$I_2 = \frac{10 \cdot 1000}{V_2} = \frac{10 \cdot 1000}{200} = 50\ A.$$

This current I_2 is passing through the equivalent resistance, $R_e'' = 1.125\ \Omega$.

Total copper loss = $I_2^2 R_e'' = (50)^2 \times 1.125 = 2.8\ kW.$

Example 3.10 A 25 kVA, 2000/200 V transformer has constant loss, i.e., iron loss of 350 W and full-load copper loss called the variable loss of 400 W. Calculate the efficiency of the transformer at full load and at half load 0.8 power factor lagging.

Solution:

Output in kVA = 25, output in kW = kVA cosϕ = 25 × 0.8 = 20 kW

Efficiency, $$\eta = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{\text{Output} \times 100}{\text{Output} + \text{Losses}}$$

Output in Watts = 20 × 1000 W
Core loss or iron loss = 350 W
Full-load copper loss = 400 W

$$\eta = \frac{\text{Output} \times 100}{\text{Output} + \text{Core loss} + \text{Copper loss}}$$
$$= \frac{200 \cdot 1000 \cdot 100}{20 \cdot 1000 + 350 + 400} = 96.4 \text{ per cent}$$

At half load

Output = $10 \cdot 10^3$

Core loss = 350 W (remains constant at all loads)

Copper loss $= \dfrac{400}{4} = 100$ W (variable loss, varies as square of the load)

$$\eta = \frac{10 \times 1000 \times 100}{10 \times 1000 + 350 + 100} = 95.7 \text{ per cent.}$$

Example 3.11 A 5 kVA, 1000/200 V, 50 Hz single-phase transformer has the following no-load test, i.e., the open-circuit test and the short-circuit test data.

No-load test conducted at the low-voltage side:

$$W_0 = 90 \text{ W}, \qquad I_0 = 1.2 \text{ A}, \qquad V = 200 \text{ V}$$

The short-circuit test conducted at the high-voltage side:

$$W_{sc} = 110 \text{ W}, \qquad I_{sc} = 5 \text{ A}, \qquad V_{sc} = 50 \text{ V}$$

Calculate the efficiency of the transformer at full-load 0.8 p.f. lagging. What will be the equivalent resistance of the transformer windings referred to high voltage side?

Solution:

Power consumed on no-load test can be taken approximately equal to the core loss, and power loss on the short-circuit test when the rated current flows through the windings can be taken as equal to full-load copper loss.

$I_{sc} = 5$ A; and full-load current at the high-voltage side $= \dfrac{\text{VA rating}}{\text{voltage}}$

$$= \frac{5 \cdot 1000}{1000} = 5 \text{ A.}$$

Thus, we see that the short-circuit test was conducted under the full-load condition.

Therefore, $\text{copper loss} = W_{SC} = 110 \text{ W} = \dfrac{110}{1000} \text{kW}$

and $\text{iron loss} = W_0 = 90 \text{ W} = \dfrac{90}{1000} \text{kW}$

Efficiency $\eta = \dfrac{\text{kVA} \cos\phi \times 100}{\text{kVA} \cos\phi + W_{sc} + W_0}$

$$= \frac{5 \times 0.8 \times 100}{5 \times 0.8 + (110/1000) + (90/1000)} = 95.24 \text{ per cent}$$

Full load copper loss $= I_1^2 R_e' = 110$ W or $R_e' = \dfrac{110}{5^2} = 4.4\ \Omega.$

Example 3.12 A 20 kVA, 1000/200 V, 50 Hz has core loss and copper loss as 400 W and 600 W, respectively, under the full-load condition. Calculate the efficiency at full load 0.8 lagging power factor. At what percentage of full load will the efficiency be maximum and what is the value of maximum efficiency?

Solution:

Full-load efficiency at 0.8 lagging p.f., $$\eta = \frac{\text{kW output} \times 100}{\text{kW output} + \text{losses}}$$

$$= \frac{\text{kVA} \cos\phi \times 100}{\text{kVA} \cos\phi + W_c + W_{cu}}$$

Substituting values, $$\eta = \frac{20 \times 0.8 \times 100}{20 \times 0.8 + 0.4 + 0.6}$$

$$= \frac{16 \times 100}{17} = 94 \text{ per cent}$$

Now we have to determine at what load the efficiency will be maximum.

Let x be the fraction of full load at which the efficiency will be maximum. Since copper loss is proportional to the square of the load, and at maximum efficiency core loss equals copper loss

$$x^2 \times W_{cu} = W_c$$

$$x = \sqrt{\frac{W_c}{W_{cu}}}$$

Substituting values $$x = \sqrt{\frac{400}{600}} = 0.82$$

Therefore, efficiency will be maximum when the load is 0.82 of 20 kVA, i.e., $0.82 \times 20 = 16.4$ kVA. At this load, core loss equals copper loss; and maximum efficiency is

$$\eta_m = \frac{16.4 \times 0.8 \times 100}{16.4 \times 0.8 + (400/1000) + (400/1000)} = 95 \text{ per cent}$$

Example 3.13 Efficiency of 400/200 V, 200 kVA transformer is 98.5 per cent at full load at 0.8 lagging power factor. At half load, 0.8 power factor lagging the efficiency is 97.5 per cent. Calculate the values of core loss and full-load copper loss.

Solution:

Let W_c be the core loss and W_{cu} be the full-load copper loss.

$$\eta_{F.L} = 0.985 = \frac{200 \times 0.8}{200 \times 0.8 + W_c + W_{cu}}$$

or, $$0.985 = \frac{160}{160 + W_c + W_{cu}}$$

or, $$\frac{160}{0.985} = 160 + W_c + W_{cu}$$

or, $$W_c + W_{cu} = 2.43 \text{ kW} = 2430 \text{ W} \quad (3.7)$$

At half load, W_c will remain the same but W_{cu} will be one-fourth its full-load value. Efficiency at half load is

$$\eta_{H.L} = \frac{100 \times 0.8}{100 \times 0.8 + W_c + (W_{cu}/4)} = \frac{80}{80 + W_c + (W_{cu}/4)}$$

or, $$0.975 = \frac{320}{320 + 4W_c + W_{cu}}$$

or, $$\frac{320}{0.975} = 320 + 4W_c + W_{cu}$$

or, $$4\,W_c + W_{cu} = 8.2 \text{ kW} = 8200 \text{ W} \qquad (3.8)$$

From (3.7) and (3.8), $W_c = 1923$ W

and $W_{cu} = 507$ W

Example 3.14 The equivalent circuit parameters of a 300 kVA, 2200/200 V, 50 Hz single-phase transformer are: primary winding resistance, $R_1 = 0.1\ \Omega$; secondary winding resistance, $R_2 = 0.01\ \Omega$; primary leakage reactance, $X_1 = 0.4\ \Omega$; secondary leakage reactance, $X_2 = 0.03\ \Omega$; resistance representing core loss, $R_c = 6 \times 10^3\ \Omega$, magnetizing reactance $X_m = 2 \times 10^3\ \Omega$. Calculate the voltage regulation and efficiency of the transformer at full load at 0.8 power factor lagging.

Solution:

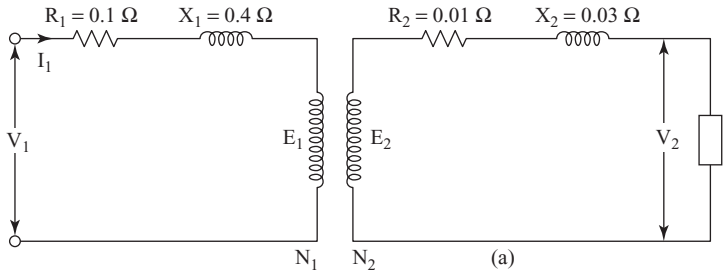


Figure 3.27

We will consider $E_1 = 2200$ V and $E_2 = 220$ V

Transformation ratio, $$K = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{220}{2200} = \frac{1}{10} = 0.1$$

By transferring the secondary quantities to the primary side we will calculate the equivalent resistance and equivalent reactance of the transformer as

$$R_e' = R_1 + \frac{R_2}{K^2} = 0.1 + \frac{0.01}{(0.1)^2} = 1.1\ \Omega$$

$$X_e' = X_1 + \frac{X_2}{K^2} = 0.4 + \frac{0.04}{(0.1)^2} = 1.4\ \Omega$$

$$\text{kVA rating} = 300$$

$$\text{VA rating} = 300 \times 1000$$

$$I_1 = \frac{VA}{E_1} = \frac{300 \cdot 1000}{2200} = 136 \text{ A.}$$

$$\text{p.f} = \cos \phi = 0.8; \sin \phi = 0.6$$

The equivalent circuit with the secondary quantities referred to the primary side and the phasor diagram have been shown in Fig. 3.28.

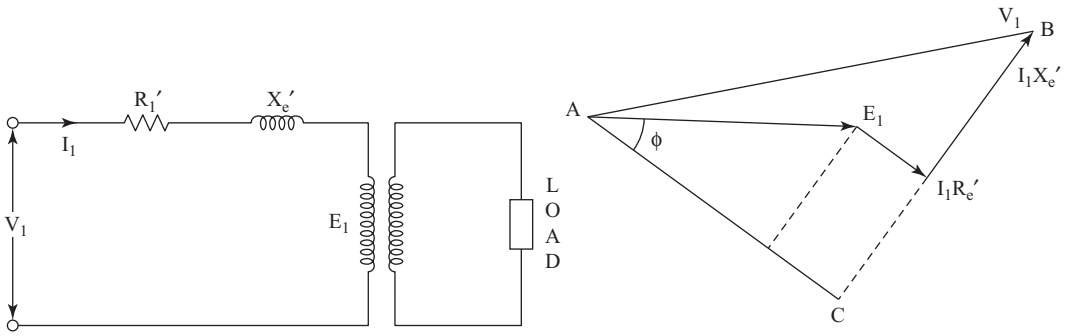


Figure 3.28

$$V_1^2 = AC^2 + CB^2$$

$$= (E_1 \cos\phi + I_1 R_e')^2 + (E_1 \sin\phi + I_1 X_e')^2$$

$$= (2200 \times 0.8 + 136 \times 1.1)^2 + (2200 \times 0.6 + 136 \times 1.4)^2$$

$$= (1909)^2 + (1510.4)^2$$

$$V_1 = 2400 \text{ V.}$$

Voltage regulation $$= \frac{V_1 - E_1}{V_1} \times 100 = \frac{(2400 - 2200)}{2400} \times 100$$

$$= 8.3 \text{ per cent.}$$

To calculate efficiency, we need to calculate the copper loss and core loss.

Full- oad copper loss, $$W_{cu} = I_1^2 R_e' = (136)^2 \times 1.1 = 20.345 \text{ kW.}$$

Core loss $$= V_1 I_c = V_1 \frac{V_1}{R_c} = \frac{V_1^2}{R_c} = \frac{(2400)^2}{6 \times 10^3} = 960 \text{ W}$$

$$= 0.96 \text{ kW.}$$

Efficiency, $$\eta = \frac{\text{Output} \times 100}{\text{Output} + W_{cu} + W_c}$$

$$= \frac{300 \times 0.8 \times 100}{300 \times 0.8 + 20.345 + 0.96}$$

$$= \frac{240 \times 100}{261.3} = 91.84 \text{ per cent.}$$

Example 3.15 A 10 kVA 440/220 V, 50 Hz single-phase transformer gave the following test results when both the following tests were conducted on the high-voltage side:

Open circuit test : 440 V, 1.0 A, 100 W
Short circuit test : 20 V, 22.7 A, 130 W.

Using the test data, calculate the efficiency and voltage regulation at 0.8 power factor lagging.

Solution:

Full-load current on the high-voltage side = $\frac{10 \cdot 1000}{440} = 22.7\text{A}$

This shows that the short-circuit test has been conducted on full load. The wattmeter reading, therefore, represents the full-load copper loss.

$W_{cu} = 130$ W and from oc test data,

$W_c = 100$ W

Full-load efficiency is calculated as

$$\eta = \frac{\text{Output}}{\text{Output} + W_{cu} + W_c} \times 100$$

$$= \frac{10 \cdot 0.8}{10 \cdot 0.8 + 0.13 + 0.1} \cdot 100$$

$$= 97.2 \text{ per cent}$$

Calculation of voltage regulation

Current, $I_1 = \frac{\text{kVA}}{V_1} = \frac{10 \cdot 1000}{440} = 22.7$ A.

From the short-circuit test data, wattmeter reading can be taken as equal to copper losses in the windings.

$$W_{SC} = 130 = I_1^2 R_e' = (22.7)^2 R_e'$$

Therefore, $R_e' = \frac{130}{(22.7)^2} = 0.25\ \Omega.$

$$Z_e' = \frac{V_{sc}}{I_{sc}} = \frac{20}{22.7} = 0.88\ \Omega.$$

$$Z_e^2 = R_e^{2\prime} + X_e^{2\prime}$$

$$X_e' = \sqrt{(0.88)^2 - (0.25)^2} = 0.84\ \Omega.$$

$\cos\phi = 0.8$, $\sin\phi = 0.6$

Regulation $= \frac{(I_1 R_e' \cos\phi + I_1 X_e' \sin\phi)}{V_1} \times 100$

$$= \frac{(22.7 \cdot 0.25 \cdot 0.8 + 22.7 \cdot 0.84 \cdot 0.6)}{440} \cdot 100$$

$$= \frac{(4.54 + 11.44)}{440} \cdot 100$$

= 3.63 per cent

Example 3.16 A 440/220 V single-phase transformer has percentage resistance drop and reactance drop of 1.2 per cent and 6 per cent, respectively. Calculate the voltage regulation of the transformer at 0.8 power factor lagging.

Solution:

Percentage voltage regulation $= \frac{(I_2 R_e'' \cos\phi + I_2 X_e'' \sin\phi)}{E_2} \times 100$

$$= \frac{I_2 R_e''}{E_2} \cos\phi \times 100 + \frac{I_2 X_e''}{E_2} \sin\phi \times 100.$$

Given, $\frac{I_2 R_e''}{E_2} \times 100 = 1.2,\ \frac{I_2 X_e''}{E_2} \times 100 = 6.0,\ \cos\phi = 0.8$

$\sin\phi = 0.6.$

Substituting values regulation $= \frac{I_2 R_e''}{E_2} \cos\phi + \frac{I_2 X_e''}{E_2} \sin\phi$

$= 1.2 \times .0.8 + 6.0 \times 0.6$

= 4.56 per cent

Example 3.17 A 230/115 V, 5 kVA transformer has circuit parameters as $R_1 = 0.2\ \Omega$, $X_1 = 0.8\ \Omega$, $R_2 = 0.1\ \Omega$, $X_2 = 0.2\ \Omega$. Calculate the regulation of the transformer at 0.8 power factor lagging. At what value of power factor will the regulation be zero? Can the value of regulation be negative for any power factor load ?

Solution

The transformation ratio, $K = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{115}{230} = 0.5$

Referring all the circuit parameters to the secondary side,

$$R_e'' = R_2 + R_1''$$
$$= R_2 + K^2 R_1$$
$$= 0.1 + (.5)^2 \times 0.2$$
$$= 0.15\ \Omega$$

$$X_e'' = X_2 + X_1'' = X_2 + K^2 X_1 = 0.2 + (0.25)^2 \times 0.8 = 0.4\ \Omega.$$

$$\cos\phi = 0.8, \sin\phi = 0.6, I_2 = \frac{5 \times 1000}{115} = 43.8\ \text{A}$$

Regulation $= \frac{(I_2 R_e'' \cos\phi + I_2 X_e'' \sin\phi)}{E_2} \times 100$

$$= \frac{(43.8 \cdot 0.15 \cdot 0.8 + 43.8 \cdot 0.4 \cdot 0.6) \cdot 100}{115}$$

$$= 13.7 \text{ per cent}$$

We know regulation can be negative only when the power factor is leading and when the expression for regulation is

$$\text{Regulation} = \frac{(I_2 R_e'' \cos\phi - I_2 X_e'' \sin\phi) \times 100}{E_2}$$

Regulation will be zero when the numerator of the above expression will be zero. That is

$$I_2 R_e'' \cos\phi = I_2 X_e'' \sin\phi$$

or,
$$\tan\phi = \frac{R_e''}{X_e''} = \frac{0.15}{0.4} = 0.375, \quad \text{or} \quad \phi = 20.6^\circ$$

The power factor at which regulation will be zero is cos 20.6° = 0.94 leading

Regulation will be negative if the power factor angle is more than 20.6° leading. To verify, let the angle of lead of load current be higher than 20.6°. If we assume this angle as 37°, the power factor is 0.8 leading. The regulation at 0.8 leading is calculated as

$$\text{Regulation} = \frac{(I_2 R_e'' \cos\phi - I_2 X_e'' \sin\phi)}{E_2} \times 100$$

$$= \frac{(43.8 \times 0.15 \times 0.8 - 43.8 \times 0.4 \times 0.6) \times 100}{115}$$

$$= \frac{(5.256 - 10.512)}{115} \times 100$$

$$= -4.57 \text{ per cent}$$

Example 3.18 Calculate the all-day efficiency of a 25 kVA distribution transformer whose loading pattern is as follows:

15 kW at 0.8 power factor for 6 hours
12 kW at 0.7 power factor for 6 hours
10 kW at 0.9 power factor for 8 hours
Negligible load for 4 hours.

The core loss is 500 W and full-load copper loss is 800 W.

Solution:

Output of the transformer in 24 hours is calculated as

$$15 \text{ kW} \times 6 + 12 \text{ kW} \times 6 + 10 \text{ kW} \times 8 + 0 \times 4$$

$$= 242 \text{ kWh} = \text{output energy}$$

We have to calculate the core loss and copper loss for 24 hours at different loading conditions as Core loss remains constant at all loads. Therefore,

$$\text{Core loss for 24 hours} = 0.5 \times 24 = 12 \text{ kWh}$$

$$= \text{Energy lost in the core}$$

Copper loss varies as the square of the load. The loads on transformer have to be calculated in terms of kVA.

$$15 \text{ kW at } 0.8 \text{ p.f.} = \frac{15 \text{ kW}}{0.8} = 18.75 \text{ kVA}$$

$$12 \text{ kW at } 0.7 \text{ p.f.} = \frac{12 \text{ kW}}{0.7} = 17.14 \text{ kVA}$$

$$10 \text{ kW at } 0.9 \text{ p.f.} = \frac{10 \text{ kw}}{0.9} = 11.11 \text{ kVA}$$

at 25 kVA load copper loss = 0.8 kW.

at 18.75 kVA load copper loss $= 0.8 \times \left(\frac{18.75}{25}\right)^2 = 450 \text{ W} = 0.45 \text{ kW}$

at 17.14 kVA load copper loss $= 0.8 \times \left(\frac{17.14}{25}\right)^2 = 376 \text{ W} = 0.376 \text{ kW}$

at 11.11 kVA load copper loss $= 0.8 \times \left(\frac{11.11}{25}\right)^2 = 157 \text{ W}$

$= 0.157$ kW

All day efficiency

$$= \frac{\text{Energy output in 24 hours}}{\text{Energy output in 24 hours} + \text{Energy loss in core in 24 hours} + \text{Energy loss in copper for 24 hours.}}$$

$$= \frac{242}{242 + 12 + (0.45 \cdot 6 + 0.376 \cdot 6 + 0.157 \cdot 10 + 0 \cdot 4)}$$

$$= \frac{242}{242 + 12 + 6.47} = \frac{242}{260.47} = 0.929$$

Thus $\eta_{\text{all-day}} = 0.929 = 92.9$ per cent

Example 3.19 In a transformer the core loss is 100 W at 40 Hz and 72 W at 30 Hz. Calculate the hysteresis loss and eddy current loss in the transformer core at 50 Hz.

Solution:

We know $W_h = \eta B_m^{1.6} f V$ Watts

and $W_e = K B_m^2 f^2 t^2$ Watts

core loss, $W_i = W_h + W_e$

or, $W_i = a f + b f^2$

Where a and b are constants

or, $\frac{W_i}{f} = a + b f$

Substituting given values,

$$\frac{72}{30} = a + 30\,b \tag{3.9}$$

and

$$\frac{100}{40} = a + 40\,b \tag{3.10}$$

Solving Eqs. (3.9) and (3.10), we get, a = 2.1 and b = 0.01
At 50 Hz, hysteresis loss = af = 2.1 × 50 = 105 W
At 50 Hz, eddy current loss = $bf^2 = 0.01 \times (50)^2 = 25$ W

3.18 REVIEW QUESTIONS

A. Short Answer Type Questions

1. Explain with examples why transformers are required in transmission and distribution of electrical power.
2. Distinguish between a step-up transformer and a step-down transformer.
3. What is the expression for voltage per turn of a transformer?
4. Derive the EMF equation of a transformer.
5. What are the losses in a transformer and how can these be kept low?
6. What is eddy current loss and how can this loss be reduced?
7. Why do we use laminated sheets to build the core of a transformer instead of using a solid core?
8. Distinguish between core-type and shell-type construction of the transformer core.
9. Explain why the frequency of output voltage is the same as input voltage in a transformer.
10. Distinguish between magnetizing reactance and leakage reactance of a transformer.
11. Draw the equivalent circuit of a transformer under the no-load condition.
12. Explain the concept of an ideal transformer.
13. What is meant by voltage regulation of a transformer? Is it desirable to have a high-voltage regulation of a transformer? Justify your answer.
14. How can we calculate the efficiency of a transformer by knowing its losses?
15. Draw the no-load phasor diagram of a transformer. What are the two components of a no-load current ?
16. Draw the full-load phasor diagram of a transformer neglecting the voltage drop in the windings.
17. Derive the condition for maximum efficiency of a transformer.
18. What is all-day efficiency of a transformer? What is its significance?
19. How can you determine the efficiency of a transformer indirectly, i.e., without actually loading the transformer?
20. What is the expression for voltage regulation of a transformer in terms of its equivalent resistance, equivalent reactance, power factor, and the output voltage?
21. How can you determine the efficiency of a transformer of a given rating at any load if the values of full-load losses are known?

22. Explain how the short-circuit test on a transformer is to be conducted. What information do you get from the short-circuit test data?
23. Draw and explain the exact equivalent circuit of a transformer.
24. Explain how in a transformer, the primary current increases as the secondary current, i.e., load current increases.
25. Why is the core of a transformer made of magnetic material?
26. What is the difference between a practical transformer and an ideal transformer?
27. Why do we consider core loss as a constant loss and copper loss as a variable loss?
28. Distinguish between a power transformer and a distribution transformer.
29. Is efficiency of a transformer same at a particular load but at different power factors?
30. Is efficiency of a transformer at a particular load same at 0.8 power factor lagging and 0.8 power factor lagging?
31. What may be the main reason for a constant humming noise in a transformer when it is supplying some load?
32. Why is the efficiency of a transformer higher than any rotating electrical machine of similar ratings?
33. How does regulation of a transformer get affected by load power factor?
34. Write short notes on the following: (i) magnetizing reactance; (ii) leakage reactance; (iii) eddy current loss; (iv) hysteresis loss; (v) all-day efficiency.

B. Numerical Problems

35. A 40 kVA 3200/400 V, single phase, 50 Hz transformer has 112 turns on the secondary winding. Calculate the number of turns on the primary winding. What is the secondary current at full load? What should be the cross-sectional area of the core for a core flux density of 1.2 Wb/m^2?

    [Ans. 896, 100 A, 01362 m^2]

36. A 400 kVA transformer has a full-load core loss of 800 W and copper loss of 2500 W. What will be the values of these losses at $\frac{1}{2}$ load?

    [Ans. 800 W, 625 W]

37. A single-phase transformer is required to step down the voltage from 1100 V to 400 V at 50 Hz. The core has a cross-sectional area of 25 cm^2 and the maximum flux density is 5Wb/m^2. Determine the number of turns of the primary and secondary windings.

    [Ans. 396, 144]

38. A single phase 40 kVA transformer has primary and secondary voltages of 6600 V and 230 V, respectively. The number of turns of the secondary winding is 30. Calculate the number of turns of the primary winding. Also calculate the primary and secondary winding currents.

    [Ans. 860, 6.06 A, 173.9 A]

39. A transformer on no load takes 4.5 A at a power factor of 0.25 lagging when connected to a 230 V, 50 Hz supply. The number of turns of the primary winding is 250. Calculate (a) the magnetizing current, (b) the core loss, and (c) the maximum value of flux in the core.

    [Ans. $I_m = 4.35$, $P_c = 259$ W, $\phi_m = 4.14 \times 10^{-3}$ Wb]

40. A 660 V/ 220 V single-phase transformer takes a no-load current of 2A at a power factor of 0.225 lagging. The transformer supplies a load of 30 A at a power factor of 0.9 lagging. Calculate the current drawn by the primary from the mains and primary power factor. Resistance and reactance of the windings may be neglected.

[Ans. $I_1 = 11.38$ A, $\cos \phi_1 = 0.829$ lag]

41. A 100 kVA transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3 Ω and 0.01 Ω, respectively, and the corresponding leakage reactances are 1.1 Ω and 0.035 Ω, respectively. Calculate the equivalent impedance referred to the primary side.

[Ans. $Z'_e = 2.05$ Ω]

42. A 660 V/220 V single-phase transformer takes a no-load current of 2 A at a power factor of 0.255 lagging. The transformer supplies a load of 30 A at a power factor of 0.9 lagging. Calculate the current drawn by the primary from the mains and primary power factor. Neglect winding resistances and reactances.

[Ans. $I_1 = 11.4$ A, $\cos \phi_1 = 0.83$ lagging]

43. The primary and secondary windings of a 500 kVA transformer have $R_1 = 0.4$ Ω and $R_2 = 0.001$ Ω, respectively. The primary and secondary voltages are 6600 V and 400 V, respectively. The iron loss is 3 kW. Calculate the efficiency on full load at 0.8 power factor lagging.

[Ans. 98.3 per cent]

44. A 5 kVA 200/400 V, 50 Hz single phase transformer gave the following test data:

    (i) L.V. side open-circuit test–220 V, 0.7 A, 60 W

    (ii) H.V. side short-circuit test–22 V, 16 A, 120 W

    Calculate the regulation of the transformer under the full-load condition.

[Ans. 3 per cent]

45. The no-load current of a transformer is 15 A at a power factor of 0.2 lagging when connected to a 460 V, 50 Hz supply. If the primary winding has 550 turns, calculate (i) magnetizing component of the no-load current, (ii) the iron loss, and (iii) maximum value of flux in the core.

[Ans. $I_m = 14.67$ A, 780 W, 2.129 mwb]

46. A single-phase, 100 kVA distribution transformer is loaded as mentioned during 24 hours:

    4 hours : no load

    8 hours : 50 per cent load at power factor = 1

    6 hours : 75 per cent load at power factor = 0.9

    6 hours : full-load at 0.9 power factor

    The full load copper loss and core loss is 5 kW and 2 kW, respectively. Calculate the all-day efficiency of the transformer.

[Ans. 92.5 per cent]

47. A 12 kVA, 200/400 V, 50 Hz single-phase transformer gave the following readings on the open-circuit test and the short-circuit test:

    open-circuit test : 200 V, 1.3 A, 120 W

    short-circuit test conducted on the H.V. side: 22 V, 30 A, 200 W

    Calculate the equivalent circuit parameters as referred to the low voltage side. Also calculate the magnetizing component of the no-load circuit.

[Ans. $R_c = 333$ Ω, $X_m = 174$ Ω, $R'_e = 0.055$ Ω, $X'_e = 0.175$ Ω, $I_m = 1.15$A]

C. Multiple Choice Questions

1. A transformer having number of turns in the primary and secondary winding of 1000 and 500, respectively, is supplied with 230 V at 50 Hz. The induced EMF in the secondary winding will be
   (a) 460 V at 50 Hz (b) 115 V at 25 Hz
   (c) 115 V at 50 Hz (d) 500 V at 50 Hz.
2. The core of the transformers is made of laminated steel sheets so as to
   (a) Reduce hysteresis loss
   (b) Reduce eddy current loss
   (c) Increase output voltage
   (d) Reduce both hysteresis loss and eddy current loss.
3. The EMF induced in the windings of a transformer
   (a) Lags the core flux by 90°
   (b) Leads the core flux by 90°
   (c) Is in phase with the core flux
   (d) Is in opposition to the core flux.
4. To reduce the core losses in a transformer
   (a) The core is made of silicon steel laminations
   (b) The core is fastened very tight so that the core flux do not fly away
   (c) The core is made of solid steel
   (d) The core is made of copper laminations.
5. The no-load current of a 10 kVA, 230 V/115 V transformer wire will be about
   (a) 5 per cent of its rated current
   (b) 20 per cent of its rated current
   (c) 30 per cent of its rated current
   (d) 0.1 per cent of its rated current.
6. The no-load current of a 15 kVA, 230/1100 V single-phase transformer will be about
   (a) 15.33 A (b) 3 A
   (c) 12 A (d) 73.3 A.
7. Efficiency of a transformer is higher tham that of a motor or a generator of similar rating because
   (a) There is no hysteresis and eddy current loss in a motor or a generator
   (b) Designs of transformers are superior than motors and generators
   (c) Transformer is a static device and there is no rotational loss in it
   (d) Transformers are connected to high-voltage transmission lines whereas motors and generators are connected to low-voltage supply lines.
8. Which of the following losses in a transformer vary with load
   (a) Hysteresis loss
   (b) Eddy current loss
   (c) Copper losses in the windings
   (d) Iron loss.
9. Open-circuit test and short-circuit test on a transformer are performed to determine, respectively, the following losses
   (a) Copper loss and core loss
   (b) Core loss and copper loss
   (c) Eddy current loss and hysteresis loss
   (d) Hysteresis loss and eddy current loss.
10. Maximum efficiency of a transformer is obtained at a load at which its
    (a) Core loss becomes the minimum
    (b) Copper loss becomes the minimum
    (c) Copper loss equals core loss
    (d) Core loss becomes negligible.
11. A transformer has 350 primary turns and 1050 secondary turns. The primary winding is connected across a 230 V, 50 Hz supply. The induced EMF in the secondary will be
    (a) 690 V, 50 Hz (b) 690 V, 100 Hz
    (c) 690 V, 150 Hz (d) 115 V, 50 Hz.
12. The rating of a transformer is expressed in
    (a) kW (b) kVA
    (c) kWh (d) $\cos\phi$.
13. The ampere-turns balance equation for a transformer on load can be expressed as
    (a) $I_1N_1 = I_2N_2$ (b) $I_1N_1 >> I_2N_2$
    (c) $I_1N_1 = I_2N_2$ (d) $I_1N_1 < I_2N_2$.
14. Which of the following characteristic assumptions for an ideal transformer are true?
    (a) Coupling coefficient between the windings is unity
    (b) There are absolutely no core and copper losses
    (c) The core is made up material having infinite permeability
    (d) All the characteristics as in (a), (b), and (c).
15. The EMF equation for a transformer is
    (a) $E = 4.44\ \phi\ f\ N^2$ (b) $E = 4.44\ \phi\ f N$
    (c) $E = 4.44\ \phi\ f^2N$ (d) $E = 4.44\ \phi_m\ f N$.
16. Which of the following statements is true for a transformer?
    (a) A transformer is an energy conversion device
    (b) A transformer changes the voltage and frequency from one level to the other

(c) A transformer is a static variable frequency device
(d) A transformer transforms or changes voltage level of energy utilization keeping the frequency constant.

17. Which of the following effects will the secondary load current of a transformer will have on the main flux created by the magnetizing current?
(a) Magnetization (b) Demagnetization
(c) Polarization (d) No effect.

18. On which of the following does the voltage regulation of a transformer depend?
(a) Load power factor
(b) Magnitude of load
(c) Winding resistance and reactance
(d) all factor as in (a), (b), and (c).

19. The no-load current of a transformer is
(a) The algebraic sum of I_m and I_c
(b) Phasor sum of I_m and I_c
(c) Algebraic sum of I_o and I_1^1
(d) Phasor sum of I_o and I_1^1.

20. The no-load current of a transformer as compared to its full-load current can be expressed as
(a) 0 to 2 per cent (b) 2 to 5 per cent
(c) 10 to 20 per cent (d) 20 to 30 per cent.

21. Power factor of a transformer of no load is low due to
(a) Large component of magnitizing current, which lags the voltage by 90°
(b) Large component of loss component of current
(c) Secondary ampere-turns interfering with the primary ampere-turns
(d) The fact that the primary and secondary windings are not firmly coupled.

22. The core loss and copper loss of a transformer on full load are 400 W and 600 W, respectively. Their values at one-third full load will be
(a) 133.3 W and 200 W
(b) 400 W and 66.66 W
(c) 133.3 W and 600 W
(d) 400 W and 200 W.

23. The full-load core loss and copper loss of a transformer are 400 W and 600 W, respectively. At approximately what percentage of full-load will the efficiency be maximum?
(a) 81 per cent (b) 91 per cent
(c) 95 per cent (d) 99 per cent.

24. The full load output of a transformer at unity power factor is 800 W. Its output at half load 0.8 power factor will be
(a) 400 W (b) 320 W
(c) 160 W (d) 640 W.

25. Large capacity transformers are placed in tanks filled with transformer oil. Which of the following are not valid for the above?
(a) Oil provides insulation between the two windings.
(b) Oil cools the transformer.
(c) Oil prevents magnetizing flux from getting reduced with time.
(d) Temperature rise of the winding is kept under control.

26. The full-load copper loss of a transformer is 1200 W. At half load the copper loss will be
(a) 600 W (b) 1200 W
(c) 300 W (d) 900 W.

27. The full-load core loss of a transformer is 1200 W. At half load the core loss will be
(a) 600 W (b) 1200 W
(c) 300 W (d) 900 W.

28. The no-load current of a certain transformer is 2 A. Its magnetizing component may be
(a) 1.8 A (b) 0.2 A
(c) 0.4 A (d) 0.02 A.

29. When the primary and secondary windings of a transformer are perfectly magnetically coupled
(a) The leakage reactance will be high and voltage regulation will be high (i.e., poor)
(b) The leakage reactance will be low and voltage regulation will be low (i.e., good)
(c) The leakage reactance will be low and voltage regulation will be high (i.e., poor)
(d) The leakage reactance will be high and voltage regulation will be low (i.e., good).

Answers to Multiple Choice Questions

1. (c)	2. (b)	3. (a)	4. (a)	5. (a)	6. (b)
7. (c)	8. (c)	9. (b)	10. (c)	11. (a)	12. (b)
13. (c)	14. (d)	15. (d)	16. (d)	17. (b)	18. (d)
19. (b)	20. (b)	21. (a)	22. (b)	23. (a)	24. (b)
25. (c)	26. (c)	27. (b)	28. (a)	29. (b)	

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Three-phase Induction Motors

TOPICS DISCUSSED

- Construction details of three-phase induction motors
- Windings and pole formation
- Production of rotating magnetic field
- Principle of working
- Rotor-induced emf, rotor frequency, rotor current
- Losses in induction motors
- Power flow diagram
- Torque equation
- Starting torque
- Condition for maximum torque
- Torque–slip characteristic
- Variation of torque–slip characteristic with change in rotor–circuit resistance
- Starting of induction motors
- Speed control of induction motors
- Determination of efficiency
- Applications of induction motors

4.1 INTRODUCTION

Three-phase induction motors are used in many industrial applications such as a drive motor. These motors are very rugged, and hence there is virtually no maintenance required. Only three-phase supply is required for the stator. No supply is to be provided to the rotor. The rotor is energized due to electromagnetic induction.

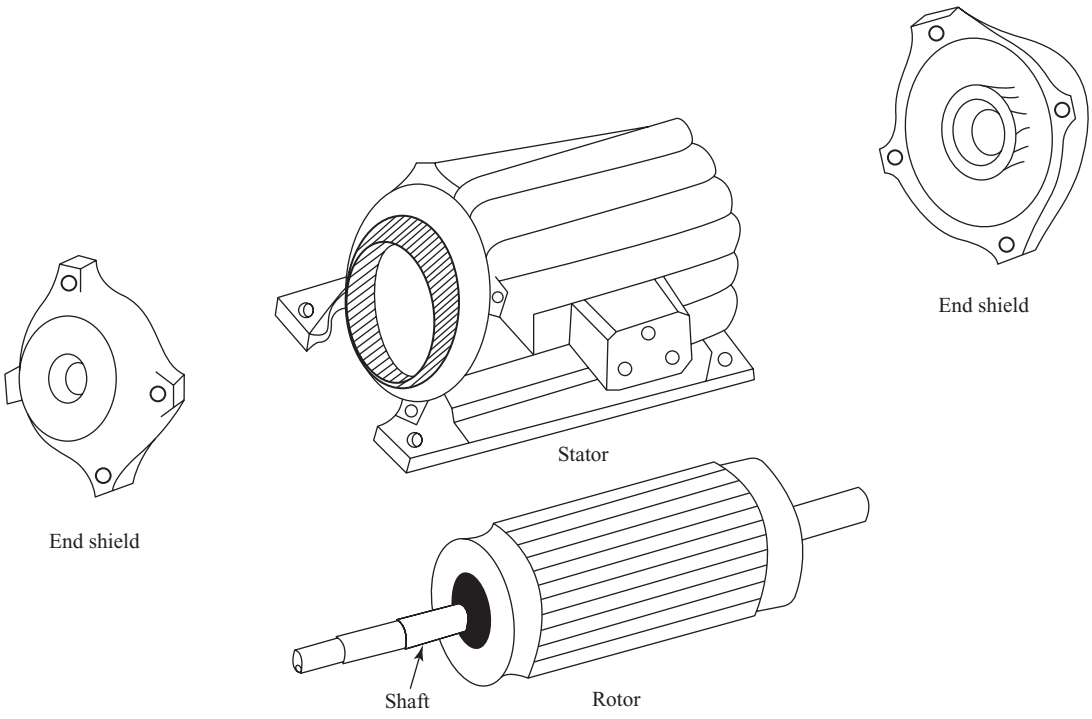
As the name suggests, a three-phase induction motor will have three windings placed in stator slots 120° apart connected either in star or in delta formation. Three-phase supply is provided to these three windings. Due to electromagnetic induction, EMF will be induced into the rotor winding, and if the rotor winding is closed, current will flow through the rotor winding. The interaction between the field produced, due to current flow in the stator windings, when fed from a three-phase supply, and the current-carrying rotor conductors will produce a torque which will rotate the rotor. This is the basic

principle of an induction motor. We will now discuss the constructional details and the principle of working of a three-phase induction motor in detail.

4.2 CONSTRUCTION DETAILS

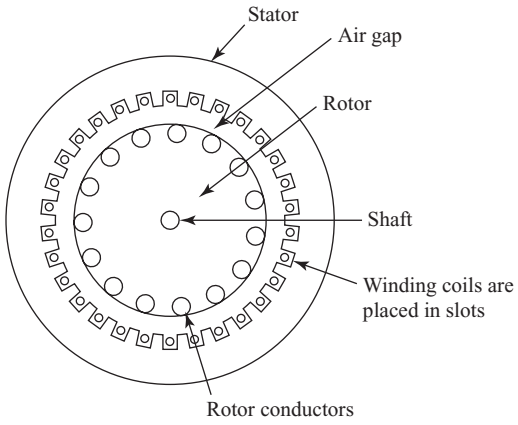
The main parts of any rotating electrical machine, as we already know, are the stator and the rotor. The stator is a hollow cylindrical structure while the rotor is a solid cylindrical body which is placed inside the stator supported at the two ends by two end shields. A small air gap is maintained between the stator and the rotor so that the rotor can rotate freely. The rotor shaft is held at the two ends by two bearings so that the frictional loss is minimum.

Fig. 4.1 (a) shows the stator and the rotor with two end shields from two sides to be brought nearer after placing the rotor inside the stator. When the end shields are fitted to the stator from two sides with the rotor shaft passing through the bearings, the rotor will rest on the bearings and the rotor will remain separated from the stator by a small air gap. The three-phase windings are made on the stator. The windings, made of a number of coils, are placed in slots in the stator. Three-phase winding consists of three identical windings separated from each other by 120° in space. Here, each phase winding has been shown made of three coils only. In actual practice, there will be more coils used per phase. As



(a)

Figure 4.1 *Constructional details of a three-phase induction motor: (a) stator, rotor, and end shields in isometric view; (b) cross-sectional view of stator and rotor (Continued)*



(b)

Figure 4.1 *(Continued)*

shown is Fig. 4.2, R–R′ is one winding, Y–Y′ is the second winding, and B–B′ is the third winding. The axes of the three windings are separated from each other by 120°. The three windings have been shown separated making 120° with each other in Fig. 4.2 (b). The three-phase windings have been shown connected in star formation by joining the end terminals R′, Y′, B′ together. Now three-phase supply can be connected to the three open terminals R, Y, B.

The rotor of an induction motor is of two types, namely, *squirrel-cage type* or *slip-ring type*. In squirrel-cage type, the rotor winding is made of bars inserted in slots made on the rotor surface. The bars are pushed into the slots and are connected from both sides through conducting rings. The connection of the rotor bars with the help of *end rings* has been shown in Fig. 4.3 (a).

Fig. 4.3 (b) shows the three-phase stator windings connected to a three-phase supply with the rotor closed on itself. In Fig. 4.3 (c) is shown the slip-ring-type rotor where the rotor winding is also made in the same way as the stator winding but the open terminals of the windings are connected permanently to three slip rings mounted on the rotor shaft. Extra resistance can be connected in the rotor circuit through brush and slip-ring arrangement. The rotor along with the slip rings mounted on its shaft is free to rotate, while brushes and the extra resistance are stationary.

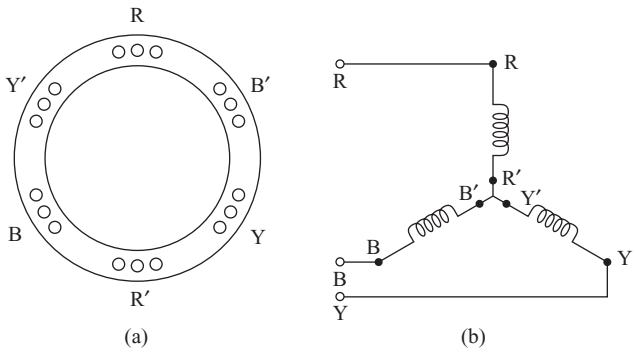


Figure 4.2 *(a) Simple three-phase winding placed on stator slots; (b) stator windings connected in star*

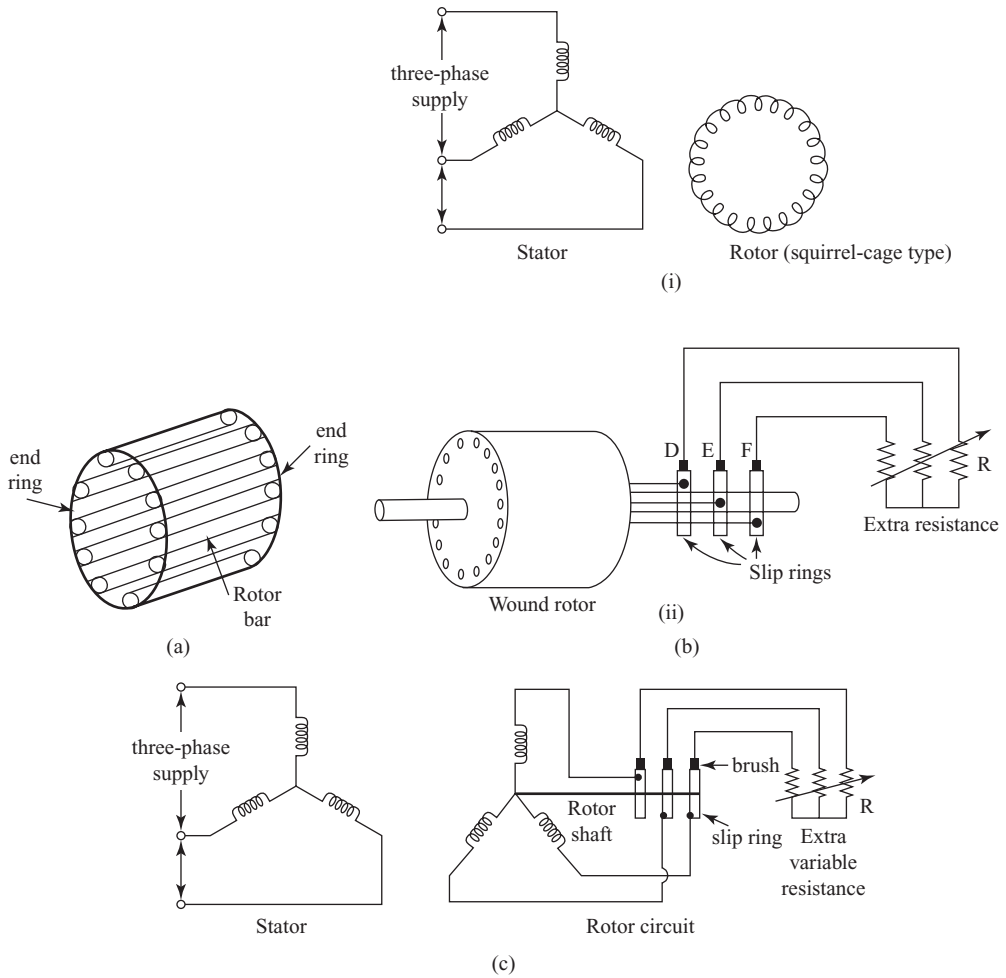


Figure 4.3 *(a) Squirrel-cage-type rotor; (b) stator and rotor circuits (i) squirrel-cage type, (ii) slip-ring type; (c) slip-ring-type induction motor-stator and rotor circuits*

By connecting extra resistance in the rotor circuit during starting, very high starting torque can be developed in slip-ring motors. For squirrel-cage rotor, it is not possible to add any extra resistance in the rotor as the circuit is closed by itself permanently and no terminals are brought out.

4.3 WINDINGS AND POLE FORMATION

Let us now understand how windings can be made for different number of poles. Fig. 4.4 shows a simple two-pole and four-pole stator winding. For simplicity, winding for only one phase has been shown.

In Fig. 4.4 (a) a single coil has been used. The distance between the coil sides is 180° mechanical. This distance is called the span of the coil or simply coil span. When dc supply with fixed polarity is applied, current will flow through the coil and a magnetic field with two poles will be formed as has been shown. If ac supply is given, the polarities will change, the direction of current through the coil will change, and hence the positions of North and South poles will change continuously in every half

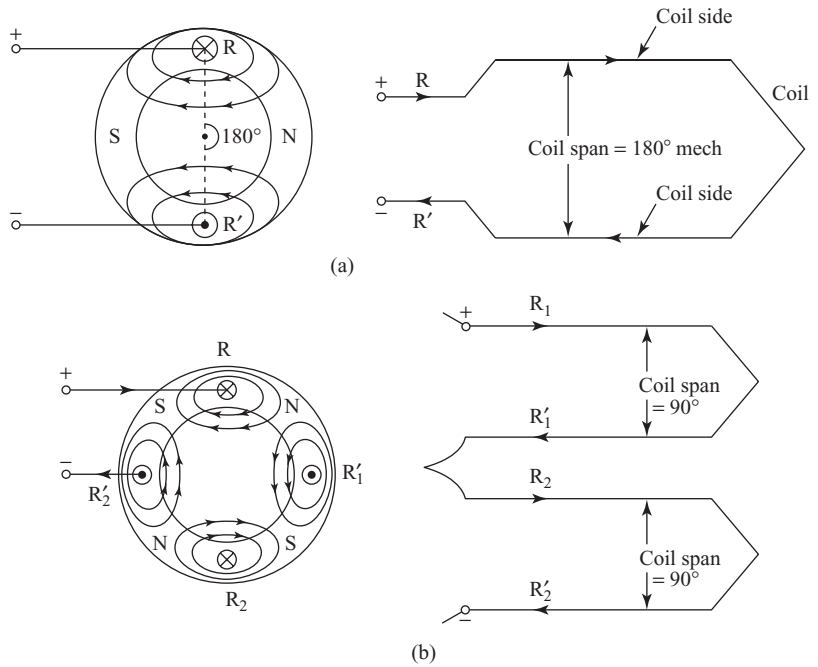


Figure 4.4 *Simplified two-pole and four-pole stator winding*

cycle of power supply. That is to say, when ac supply is applied, a two-pole alternating magnetic field will be produced whose axis will lie along the horizontal direction as shown. Since for a sinusoidal ac supply both magnitude as well as direction of current will change, the magnetic fields produced will have its magnitude as well as direction changing continuously along a fixed axis.

In Fig. 4.4 (b), a simple four-pole stator winding has been shown. Here the coil span has been reduced to 90° mechanical. Two coils connected in series has formed the winding. Four poles are formed with current and flux directions as shown. You can now easily draw a simple six-pole or an eight-pole stator winding with reduced coil spans. In a three-phase induction motor, three separate windings, each wound for two-pole, four-pole, or any even number of poles as required, are made. These windings are connected either in star or in delta and three-phase supply is applied to the windings to produce a resultant magnetic field. The resultant magnetic field is the sum total of the magnetic fields produced by the three winding ampere turns. We will soon see that the resultant magnetic field, when a three-phase supply is applied to a three-phase stator winding, is a rotating magnetic field. A rotating magnetic field is one whose axis goes on rotating continuously when supply is given. That is, the position of North and South poles goes on shifting, as time passes, at a fast speed. The speed N_s depends upon frequency of power supply f, and the number of poles P, for which the winding has been made as

$$N_s = \frac{120f}{P} \tag{4.1}$$

If $f = 50$ Hz, $P = 2$,

$$N_s = \frac{120 \cdot 50}{2} = 3000 \text{ rpm}$$

$$= 50 \text{ rps}$$

4.4 PRODUCTION OF ROTATING MAGNETIC FIELD

Now let us actually see how the field rotates when a three-phase supply is connected to a three-phase stator winding. For the sake of understanding, we will consider only three consecutive instants of time of the three-phase supply voltage, show the direction of current flowing through each of the stator windings, and then draw the resultant magnetic field produced. In a three-phase supply three separate sinusoidal voltages having a displacement of 120° with respect to time is available. We will represent a three-phase supply and assume that these are connected to a two-pole three-phase stator winding as shown in Fig. 4.5 (b).

When three-phase supply is connected to R, Y, and B terminals of the stator which are connected in star, current flowing through the phases will be as follows:

at time t_1 R-phase: zero; y-phase: −ve; B-phase: +ve
at time t_2 R-phase: +ve; y-phase: −ve; B-phase: −ve
at time t_3 R-phase: zero; y-phase: +ve; B-phase: −ve

Positive current in a phase, say R-phase, will be shown as entering through R and leaving through R′, and for negative current direction will be just reverse. Positive current will be represented by a cross and negative current will be represented by a dot. Accordingly, all the current directions have been shown and the resultant field drawn. It is observed that for half cycle of current flow, i.e., in time t_1 to t_3, the magnetic-field axis has rotated by 180°. For one complete cycle of current flow, the resultant magnetic field produced will rotate by 360° i.e., one revolution. As current continues to flow through the three windings, the magnetic-field axis will go on rotating at a speed given by

$$N_s = \frac{120f}{P}$$

If frequency is high, N_s will be high and if number of poles, P is high, N_s will be low.

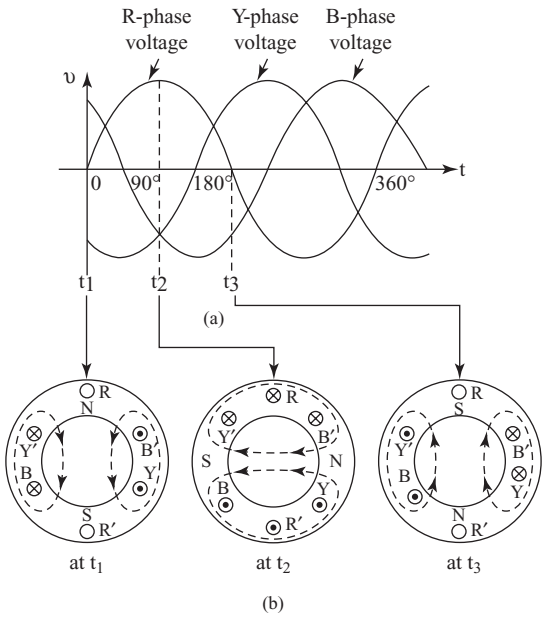


Figure 4.5 *(a) Three-phase supply; (b) two-pole, three-phase stator winding has been drawn three times to show the field produced at time t_1, t_2, and t_3, respectively*

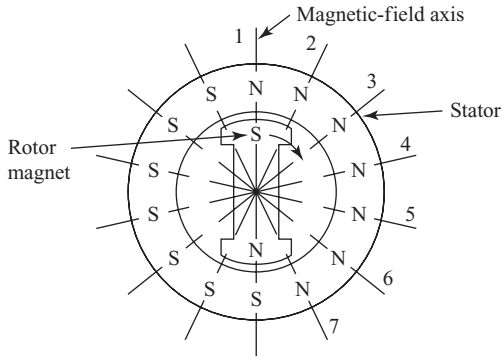


Figure 4.6 *Two magnetic fields always try to align with each other*

Now let us consider an electromagnet, or a permanent magnet, which is free to rotate, placed inside the rotating magnetic field as shown in Fig. 4.6.

The magnetic-field axis rotates in the clockwise direction and shifts its position from 1 to 2, 3, 4 …, and so on. The position of S-pole will be opposite to the position of N-pole, as the poles rotate.

The rotor magnet, in trying to align itself with the rotating magnetic field, will also rotate in the same direction at the same speed. When the rotor rotates at the same speed as the speed of the rotating field, the rotor is said to be in synchronism with the rotating field, and therefore this speed is called synchronous speed. If the rotor is an electromagnet with dc supply given to its windings, the rotor will rotate at synchronous speed when three-phase supply is applied to the stator winding. Three-phase supply to three-phase stator windings produce a magnetic field which rotates at a constant speed. The rotor, when energized or excited by passing current through its windings, becomes an electromagnet. The rotor, magnet, which is free to rotate, aligns itself with the rotating magnetic field. It will continue to rotate at the same speed as the rotating magnetic field. Such a motor will be called a three-phase synchronous motor.

In a three-phase induction motor, however, only three-phase supply is applied across the stator windings. No supply is provided to the rotor winding. The rotor is made a closed winding either directly as in the case of squirrel-cage winding; or, through extra resistance inserted in the rotor circuit as in the case of slip-ring motors. The rotor gets excited due to electromagnetic induction. We will now study the principle of working of a three-phase induction motor.

4.5 PRINCIPLE OF WORKING

In a three-phase induction motor, the stator is wound with a three-phase winding for P number of poles. The poles for which the winding is made could be 2, 4, 6, 8, … etc. The rotor which is placed inside the stator is either squirrel-cage type or slip-ring type. In both cases, current flowing through the three-phase stator winding produces a rotating magnetic field which will be rotating at a speed, N_s where

$$N_S = \frac{120f}{P}$$

For a 50 Hz supply and P = 2, N_s is 3000 rpm. The rotating field will be rotating continuously at a very high speed. The rotating flux will cut the stationary rotor windings at that speed. Due to this cutting of flux, EMF will be induced in the rotor winding. As the rotor circuit has been made to be a closed winding,

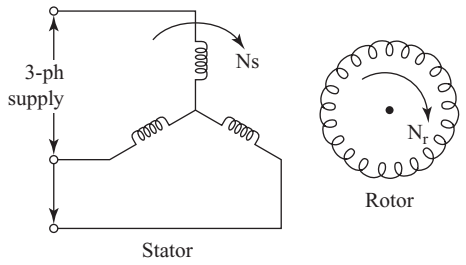


Figure 4.7 *Stator and rotor windings of a three-phase induction motor*

current will flow through the rotor-winding conductors. Thus the rotor circuit gets excited due to electromagnetic induction effect. Because of interaction between the current-carrying rotor conductors and the rotating magnetic field, torque will be developed in the rotor, which will rotate the rotor in the same direction as the rotating magnetic field at a speed N_r.

Emf is induced in the rotor due to cutting of flux. According to Lenz's law, the rotor will oppose the rate of cutting of flux. This is possible only if the rotor starts rotating in the same direction as the rotating magnetic field.

The rotor will attain a speed N_r which is somewhat less than the speed of the rotating magnetic field, N_s. Although the rotor will try to attain a speed of N_s, it will never be able to attain that speed, because if it does, there will be no relative velocity between the rotating field and the speed of the rotor, no EMF induced in the rotor, no current flow in the rotor conductors, no torque developed, and no rotation of the rotor. That is why an induction motor cannot run at synchronous speed, N_s as it is to be excited by electromagnetic induction, which is possible only if there exists a relative velocity between the rotating magnetic field and the rotor.

The difference between the speed of the rotating magnetic field, N_s and the rotor speed N_r is the slip S. Slip is usually expressed as the percentage of N_s, thus,

slip, $$S = \frac{N_s - N_r}{N_s}$$

or, $$SN_s = N_s - N_r \text{ or, } N_r = (1 - S)\, N_s \qquad (4.2)$$

Percentage Slip $$= \frac{N_s - N_r}{N_s} \times 100$$

Slip of a three-phase induction motor is generally 3 to 4 per cent. For example, when a 400 V, 3-ph, 50 Hz supply is connected to a four-pole three-phase induction motor, the speed of the rotating field will be 1500 rpm. The rotor will rotate at a speed less than 1500 rpm, may be, say 1440 rpm. In such a case slip S is

$$S = \frac{(1500 - 1440) \times 100}{1500}$$

$$= 4 \text{ per cent}$$

Changing the direction of rotation

It has been observed that if the sequence of the three-phase supply connected to the three-phase stator windings is changed, the direction of the rotating field produced will change, i.e., the field which was rotating in the clockwise direction will now rotate in the anticlockwise direction. This has been shown in Fig. 4.8. As shown in Fig. 4.8 (b), the connections to R- and Y-phase supply to the stator windings

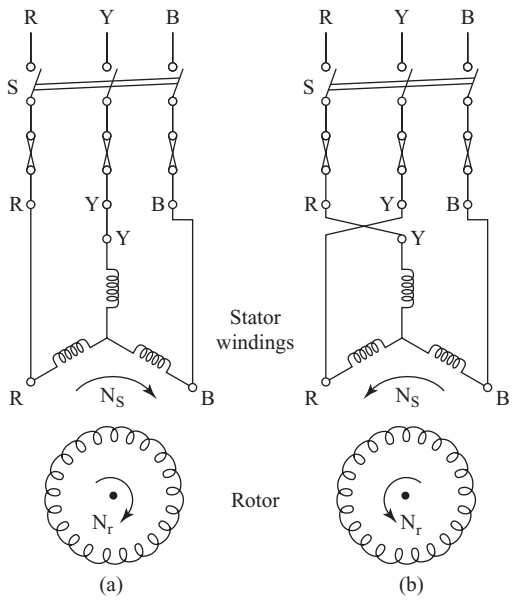


Figure 4.8 *Method of changing the direction of rotation of a three-phase induction motor*

have been interchanged but the connection to phase B remains same as in Fig. 4.8 (a). The direction of rotation of the rotating magnetic field will reverse, and hence the motor will rotate in the opposite direction. Thus, to change the direction of rotation of a three-phase induction motor we have to simply interchange the connections of supply of any two phases.

4.6 ROTOR-INDUCED EMF, ROTOR FREQUENCY AND ROTOR CURRENT

The frequency of supply to the stator is f Hz. The EMF induced in the rotor when the rotor is rotating at a speed N_r is due to the difference of speed of the rotating field and the rotor speed, i.e., $N_s - N_r$.

When the rotor is at standstill, i.e., not yet rotating, the difference between speed of rotation of the stator field and the rotor is $N_s - 0 = N_s$. The EMF induced in the rotor will be maximum and will have a frequency f. When the rotor attains a speed of N_r, the frequency of the induced EMF will get reduced, and let this rotor frequency be called, f_r. Thus, when the relative speed is N_s, the frequency of the induced EMF in the rotor is f. When the relative speed is $(N_s - N_r)$, the frequency is f_r. The relationship between f and f_r is found as follows:

when relative speed is N_s, frequency is f.

When relative speed is $(N_s - N_r)$, the frequency, f_r is $\dfrac{f(N_s - N_r)}{N_s} = \dfrac{N_s - N_r}{N_s} \times f$

$$= Sf$$

therefore, $$f_r = Sf \qquad (4.3)$$

i.e., rotor frequency = Slip × Stator frequency

The rotor-induced EMF is a function of the frequency. If the induced EMF in the rotor under standstill condition is E_{20}, the induced EMF, E_2 in the rotor when it is rotating is, $E_2 = S\,E_{20}$.

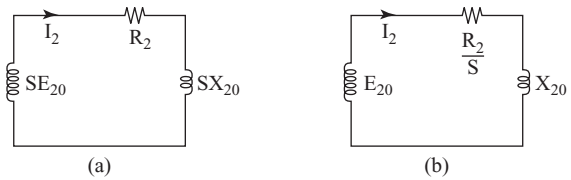


Figure 4.9 *Rotor circuit when the rotor rotates at a slip, S*

The rotor reactance at standstill, $X_{20} = 2\pi fL$.

The rotor reactance X_2 when the rotor is rotating will be corresponding to the rotor frequency f_r. Thus,

$$X_2 = 2\pi f_r L = 2\pi SfL = S2\pi fL = SX_{20}$$

That is, rotor reactance under running condition is equal to slip times the rotor reactance at standstill.

The resistance of the rotor circuit is R_2. R_2 is independent of rotor speed. Resistance does not change with speed. Rotor current is I_2. The rotor circuit when the rotor is rotating is represented as shown in Fig. 4.9 (a).

The EMF induced in the rotor when it is rotating is SE_{20}, the rotor-circuit resistance is R_2, the rotor-circuit reactance is SX_{20}, and the rotor current is I_2.

The current in the rotor circuit is

$$I_2 = \frac{S\,E_{20}}{\sqrt{R_2^2 + S^2X_{20}^2}} \tag{4.4}$$

Dividing both numerator and denominator by S,

$$I_2 = \frac{E_{20}}{\sqrt{(R_2/S)^2 + X_{20}^2}}$$

The equivalent circuit representation of the rotor for the above equation has been shown in Fig. 4.9 (b). This is the condition of the circuit when the rotor is at standstill with E_{20} as the EMF induced, I_2 as current flowing, X_{20} as the rotor-circuit reactance, and R_2/S the rotor-circuit resistance. But we know that the rotor-circuit resistance is R_2 and not R_2/S.

To get R_2 as the resistance we write $R_{2/S}$ as

$$\frac{R_2}{S} = R_2 + \frac{R_2}{S} - R_2 \text{ (By adding and subtracting } R_2\text{)}$$

$$= R_2 + R_2\left(\frac{1}{S} - 1\right)$$

$$= R_2 + R_2\left(\frac{1-S}{S}\right)$$

In $\frac{R_2}{S}$ we see two resistances, viz rotor-circuit resistance R_2 and another resistance $R_2\left(\frac{1-S}{S}\right)$. The question that arises is what is the significance of resistance $R_2\left(\frac{1-S}{S}\right)$. The power lost in this resistance will be equal to $I_2^2 R_2\left(\frac{1-S}{S}\right)$. This power is the electrical equivalent of the mechanical load on the motor shaft.

This power is, therefore, called fictitious (not real but equivalent to) electrical load representing the mechanical power output of the motor. In an induction motor the input power is $\sqrt{3}\ V_L\ I_L \cos\phi$. The output is mechanical whose electrical-equivalent power has been found as above. Not all the input power is converted into mechanical power: some power gets lost in the conversion process.

Example 4.1 A four-pole, three-phase induction when supplied with 400 V, 50 Hz supply rotates at a slip of 4 per cent. What is the speed of the motor?

Solution:

$$N_s = \frac{120f}{p} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$S = \frac{N_s - N_r}{N_s} \quad \text{Substituting values, } 0.04 = \frac{1500 - N_r}{1500}$$

or, $$N_r = 1500 - 60 = 1440 \text{ rpm}$$

4.7 LOSSES IN INDUCTION MOTORS

The input power to an induction motor is taken from the supply mains. For a three-phase induction motor the input power is $3\ V_{Ph}\ I_{ph} \cos\phi$, $\sqrt{3}\ V_L\ I_L \cos\phi$, where V_{ph}, I_{ph} are the phase values and V_L, I_L are the line values. This electrical power input is converted into mechanical power output at the shaft. Power is transferred from the stator to the rotor shaft via the air gap. The whole of the input does not get converted into mechanical output. Some power gets lost in the stator windings as I^2R loss, in the stator core as core loss. Similarly, power is also lost in the rotor as I^2R loss and core loss. When the rotor rotates against the wind (air) friction, power is lost as windage loss and bearing-friction loss and brush-friction loss (in case of slip-ring motors only). Iron loss is the sum of hysteresis loss and eddy curent loss. Hysteresis and eddy current losses depend upon the frequency and flux density. The frequency of the induced EMF and current in the rotor is very small ($f_r = sf$). Therefore, rotor core loss is considered negligible. Stator core loss is constant at all loads since the supply voltage and frequency are normally constant. I^2R losses in the stator and rotor windings are variable, i.e., they vary with change of load (i.e., load current).

Thus, the various losses are

(a) stator copper loss;
(b) rotor copper loss;
(c) iron loss in stator;
(d) iron loss in the rotor (very small);
(e) air-friction loss due to rotation of the rotor;
(f) bearing-friction loss;
(g) brush- and slip-ring-friction loss (in case of slip-ring motors only).

4.8 POWER FLOW DIAGRAM

The flow of power in an induction motor from stator to rotor is depicted in the form of a diagram as in Fig. 4.10. In the figure we have considered rotor core loss as negligible. Power transferred from stator to

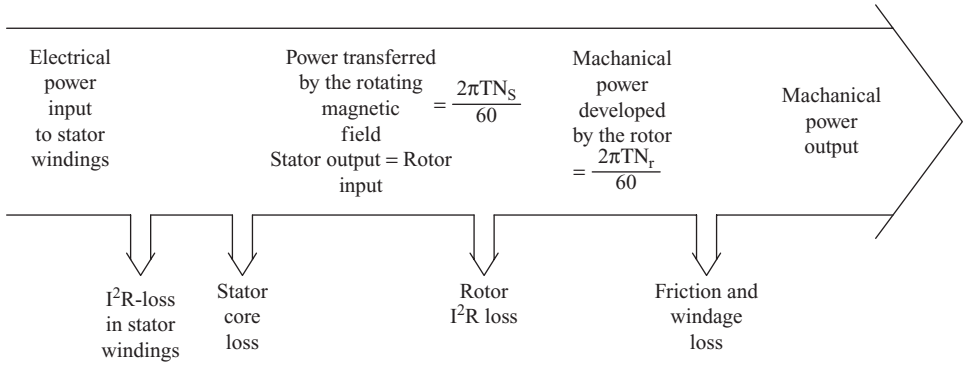


Figure 4.10 *Power flow diagram of an induction motor*

rotor is through the rotating magnetic field. The rotating magnetic field is rotating at synchronous speed, N_s. Therefore, the power transferred $\frac{2\pi TN_s}{60}$ W. The same power is transferred to the rotor through the magnetic field and there is no loss in the air gap in this transfer. Therefore, stator output is taken as equal to rotor input. When the rotor rotates at a speed N_r, the power developed by the rotor is $=\frac{2\pi TN_r}{60}$ W. The difference between rotor input and rotor power developed is expressed as

$$\frac{2\pi TN_s}{60} - \frac{2\pi TN_r}{60} = \text{Rotor } I^2R\text{-loss } (\text{neglecting rotor core loss})$$

Mechanical power output is somewhat less than the power developed. This is because of friction and windage losses.

4.9 TORQUE EQUATION

When three-phase supply is applied to the three-phase stator windings of an induction motor, as shown in Fig. 4.11, a rotating magnetic field is produced which rotates at synchronous speed, N_s where

$$N_s = \frac{120f}{P}$$

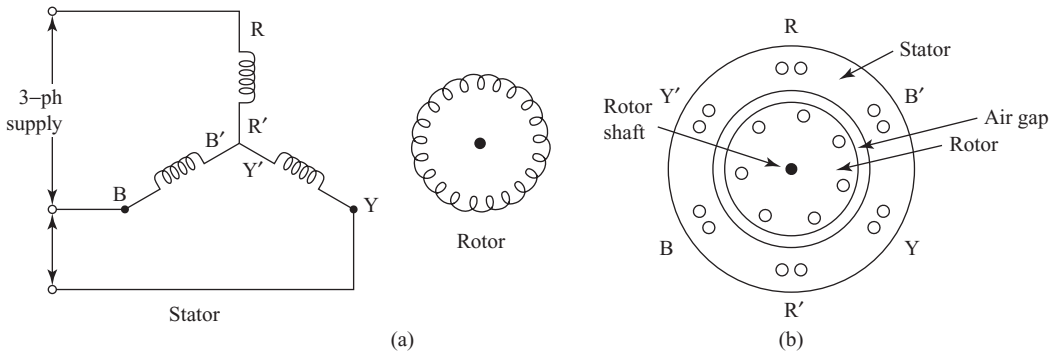


Figure 4.11 *(a) Stator and rotor windings of a three-phase induction motor; (b) simplified corss-sectional view*

Through this rotating magnetic field, power is transferred from stator to the rotor via the air gap. The power transferred from stator to rotor through the magnetic field in equal to $\frac{2\pi TN_s}{60}$. This is the input power to the rotor since there in no power loss in the air gap. Torque will be developed in the rotor which will cause the rotor to rotate at a speed N_r. The power developed by the rotor is $\frac{2\pi TN_r}{60}$.

The difference between the input power and the power developed is the loss in the rotor. In the rotor there will be copper loss and core loss.

Therefore,

$$\frac{2\pi TN_s}{60} - \frac{2\pi TN_r}{60} = \text{Rotor copper loss + Rotor core loss}$$

The core loss (sum of Hysteresis loss and eddy current loss) in the rotor is negligible as the frequency of the induced EMF in the core will have frequency, $f_r = Sf$. f_r is small, and hence core loss can be considered negligible.

Thus,

$$\frac{2\pi TN_s}{60} - \frac{2\pi TN_r}{60} = \text{Rotor copper loss}$$

Dividing both sides by rotor input

$$\frac{(2\pi TN_s/60) - (2\pi TN_r/60)}{(2\pi TN_s/60)} = \frac{\text{Rotor copper loss}}{\text{Rotor input}}$$

$$\frac{N_s - N_r}{N_s} = \frac{\text{Rotor copper loss}}{\text{Rotor input}}$$

$$\therefore \quad \text{Rotor copper loss} = \text{Slip} \times \text{Rotor input} \tag{4.5}$$

The above is an important relationship which can also be used to arrive at the expression for torque as has been shown below.

Putting the value of I_2 from eq. (4.4)

$$\text{Rotor input} \times \text{slip} = I_2^2 R_2$$

$$\frac{2\pi TN_s}{60} \times S = \left[\frac{SE_{20}}{\sqrt{R_2^2 + S^2 X_{20}^2}}\right]^2 R_2$$

$$\frac{2\pi TN_s}{60} \times S = \frac{S^2 E_{20}^2}{R_2^2 + S^2 X_{20}^2} R_2$$

or,

$$T = \frac{60}{2\pi N_s} \frac{S E_{20}^2 R_2}{R_2^2 + S^2 X_{20}^2}$$

If the supply voltage, V is constant, then the induced EMF E_{20} in the rotor at standstill will be constant (like in a transformer)

Therefore,

$$T = K \frac{SR_2}{R_2^2 + S^2 X_{20}^2}, \quad \text{where} \quad K = \frac{60 \times E_{20}^2}{2\pi N_s} \tag{4.6}$$

The value of S varies from 0 to 1. Slip is 0 if the rotor is able to rotate at synchronous speed, N_s. At the moment of start, rotor speed N_r is zero, and hence slip is 1 or 100 per cent. Rotor resistance R_2 is much smaller than the standstill rotor reactance, X_{20}.

Using the expression for torque, we can draw the torque versus slip (or speed) characteristic of the three-phase induction motor.

4.10 STARTING TORQUE

At start, rotor speed $N_r = 0$, therefore, $S = \frac{N_s - N_r}{N_s} = \frac{N_s}{N_s} = 1$

From the torque equation with S = 1

starting torque, $$T_s = \frac{KR_2}{R_2^2 + X_{20}^2} \tag{4.7}$$

The value of starting torque will depend upon the rotor circuit parameters, i.e., R_2 and X_{20}.

The value of X_{20} is generally higher than R_2. Let us assume $R_2 = 1\ \Omega$ and $X_{20} = 8\ \Omega$.

Let us, for the sake of interest, study the value of T_s if R_2 is increased (which can be done by including an extra resistance in the rotor circuit while starting the motor).

With $R_2 = 1$ and $X_{20} = 8$, and using K to be having a value, say 100

$$T_s = \frac{100 \cdot 1}{1 + 8^2} = \frac{100}{65} = 1.54$$

Now let us make $R_2 = 2\ \Omega$, Then T_s is

$$T_s = \frac{100 \cdot 2}{2^2 + 8^2} = \frac{200}{68} = 2.95$$

We further increase R_2 and make $R_2 = 4\ \Omega$

then, $$T_s = \frac{100 \cdot 4}{4^2 + 8^2} = \frac{400}{80} = 5$$

Thus, we see that if rotor-circuit resistance is increased, we can achieve higher starting torque developed by the motor which is a requirement in many applications.

4.11 CONDITION FOR MAXIMUM TORQUE

Condition for maximum torque can be found out by maximizing the expression for torque, i.e., by differentiating T with respect to slip, S and equating to zero. Let us consider the torque equation which is

$$T = K \frac{SR_2}{R_2^2 + S^2 X_{20}^2}$$

or, $$T = K \frac{R_2}{(R_2^2 / S) + S X_{20}^2}$$

To maximize T, we have to minimize the denominator and equate to zero, i.e.,

$$\frac{dT}{dS}\left(\frac{R_2^2}{S} + S X_{20}^2\right) = 0$$

or, $$\frac{dT}{dS}\left(R_2^2 S^{-1} + S X_{20}^2\right) = 0$$

or, $$-R_2^2 S^{-2} + X_{20}^2 = 0$$

or, $$-\frac{R_2^2}{S^2} + X_{20}^2 = 0$$

or, $$R_2^2 = S^2 X_{20}^2$$

or, $$R_2 = S X_{20}, \quad \text{or} \quad S = \frac{R_2}{X_{20}}$$

Thus, the condition for maximum torque is established. Maximum torque in an induction motor will occur at a slip at which,

$$S = \frac{R_2}{X_{20}} \tag{4.8}$$

If we take $R_2 = 1\ \Omega$ and $X_{20} = 8\ \Omega$, maximum torque will occur when the slip is $\frac{1}{8}$, i.e., 0.125.

If $R_2 = 2\ \Omega$ and $X_{20} = 8\ \Omega$, maximum torque will occur when the slip is $\frac{1}{4}$, i.e., 0.25.

When value of R_2 equals the value of X_{20}, i.e., when $R_2 = 8\ \Omega$, and $X_{20} = 8\ \Omega$, maximum torque will occur at S = 1, i.e., at starting.

Example 4.2 A four-pole, three-phase induction motor is supplied with 400 V, 50 Hz supply. The rotor-circuit resistance is 2 Ω and standstill rotor-circuit reactance is 8 Ω. Calculate the speed at which maximum torque will be developed.

Solution:

Given, f = 50 Hz, P = 4. $R_2 = 2\ \Omega$, $X_{20} = 8\ \Omega$

$$N_s = \frac{120f}{P} = \frac{120 \cdot 50}{4} = 1500 \text{ rpm}$$

Condition for maximum torque is given by

$$R_2 = SX_{20}$$

substituting values $$S = \frac{R_2}{X_{20}} = \frac{2}{8} = 0.25$$

at this slip, rotor speed,

$$N_r = (1 - S)N_s$$
$$= (1 - 0.25)1500$$
$$= 1125 \text{ rpm}$$

The value of maximum torque

We will put the condition for maximum torque, i.e., $R_2 = S X_{20}$ in the torque equation as

$$T = K \frac{S R_2}{R_2^2 + S^2 X_{20}^2}$$

putting $R_2 = S X_{20}$, Maximum torque T_m is

$$T_m = \frac{K.S.S X_{20}}{S^2 X_{20}^2 + S^2 X_{20}^2}$$

or, $$T_m = \frac{K S^2 X_{20}}{2 S^2 X_{20}^2}$$

or, $$T_m = \frac{K}{2 X_{20}} \tag{4.9}$$

Thus, we see that the value of maximum torque, T_m is independent of rotor resistance R_2 but the slip at which maximum torque is developed changes with value of R_2.

4.12 TORQUE–SLIP CHARACTERISTIC

The variation of torque with slip can be studied using the torque equation. The shape of the torque–slip characteristic is predicted as follows:

Let us write the torque equation derived earlier. Referring to (4.7), we have

$$T = \frac{K S R_2}{R_2^2 + S^2 X_{20}^2}$$

Let us assume $R_2 = 1\ \Omega$ and $X_{20} = 8\ \Omega$.

The value of S changes from 0 to 1. When the value of S is very small, say 0.01, 0.02, 0.03, 0.04, etc., the value of $S^2 X_{20}^2$ will be small as compared to R_2^2. Thus, for small values of S, we can write

$$T = \frac{K}{R_2} S$$

or,
$$T \propto S$$

and for larger values of S, say 0.2, 0.3, 0.4, etc., R_2^2 is smaller than $S^2 X_{20}^2$, and hence we can write

$$T = \frac{K S R_2}{S^2 X_{20}^2} = \frac{K R_2}{X_{20}^2} \frac{1}{S}$$

or,
$$T \propto \frac{1}{S}$$

Thus, we see that for lower values of slip, torque is directly proportional to slip, and for higher values of slip, torque is inversely proportional to slip. We further calculate the values of torque at S = 0 and S = 1. Also, we calculate the value of maximum torque and the slip at which maximum torque occurs. These values will help us draw the torque–slip characteristic.

At
$$S = 0, T = 0$$

With
$$K = 100,\ T_m = \frac{K}{2 X_{20}} = \frac{100}{2 X 8} = 6.25$$

The value of slip at which T_m occurs is

$$S = \frac{R_2}{X_{20}} = \frac{1}{8} = 0.125$$

and the value of starting torque at S = 1 is

$$T_s = \frac{K R_2}{R_2^2 + X_{20}^2} = \frac{100 \cdot 1}{1^2 + 8^2} = \frac{100}{65} = 1.54$$

We will now draw the torque–slip characteristic using the above values.

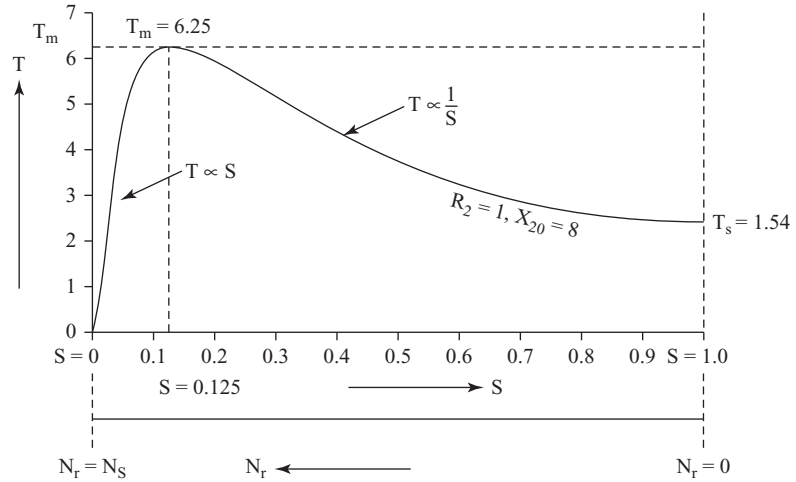


Figure 4.12 *Torque–slip or torque–speed characteristic of an induction motor*

Since just at start, speed is zero, and slip is 1, we can represent S = 1 as $N_r = 0$. Again when S = 0, N_r must be equal to N_s since

$$S = \frac{N_s - N_r}{N_s}$$

Thus, the torque–slip characteristic can also be shown as torque–speed characteristic as has been shown in Fig. 4.12.

4.13 VARIATION OF TORQUE–SLIP CHARACTERISTIC WITH CHANGE IN ROTOR-CIRCUIT RESISTANCE

We will now study the effect of variation of rotor-circuit resistance on the torque-slip characteristics of an induction motor.

We have known that starting torque, maximum torque, and the slip, S at which maximum torque occurs changes with change of rotor-circuit resistance, the reactance remaining unchanged. The basic shape of the torque–slip charactesic however remains the same. We will draw four torque–slip characteristic of a three-phase induction motor for

(i) $R_2 = 1\ \Omega, X_{20} = 8\ \Omega$ (iii) $R_2 = 4\ \Omega, X_{20} = 8\ \Omega$
(ii) $R_2 = 2\ \Omega, X_{20} = 8\ \Omega$ (iv) $R_2 = 8\ \Omega, X_{20} = 8\ \Omega$

Firstly, we calculate T_s, T_m, S at which T_m occurs.

(i)
$$T_{s1} = \frac{KR_2}{R_2^2 + X_{20}^2} = \frac{100 \cdot 1}{1^2 + 8^2} = 1.54 \quad \text{(assuming } K = 100\text{)}$$

$$T_{m1} = \frac{K}{2X_{20}} = \frac{100}{2 \cdot 8} = 6.25$$

$$S_{m1} = \frac{R_2}{X_{20}} = \frac{1}{8} = 0.125$$

(ii)
$$T_{s2} = \frac{K R_2}{R_2^2 + X_{20}^2} = \frac{100 \cdot 2}{2^2 + 8^2} = \frac{200}{68} = 2.95$$

$$T_{m2} = \frac{K}{2 \cdot 20} = \frac{100}{2 \cdot 8} = 6.25$$

$$S_{m2} = \frac{R_2}{X_{20}} = \frac{2}{8} = 0.25$$

(iii)
$$T_{s3} = \frac{K R_2}{R_2^2 + X_{20}^2} = \frac{100 \cdot 6}{4^2 + 8^2} = 5.0$$

$$T_{m3} = 6.25$$

$$S_{m3} = \frac{R_2}{X_{20}} = \frac{4}{8} = 0.5$$

(iv)
$$T_{s4} = \frac{KR_2}{R_2^2 + X_{20}^2} = \frac{100 \cdot 8}{8^2 + 8^2} = 6.25$$

$$T_{m4} = 6.25$$

$$S_{m4} = \frac{R_2}{X_{20}} = \frac{8}{8} = 1.0$$

To draw the torque–slip characteristics, we will take the value of T_{s1}, T_{m1}, and S_{m1} in each case. For example, with $R_2 = 1\ \Omega$, $X_{20} = 8\ \Omega$, $T_{s1} = 1.54$. $S_{m1} = 0.125$ and $T_{m1} = 6.25$ and with $T = 0$ at $S = 0$, we can draw the T–S characteristic with its slope on $T \propto S$ for lower values of slip and $T \propto \frac{1}{S}$ for higher values of slip. We can now draw the four characteristics as shown in Fig. 4.13.

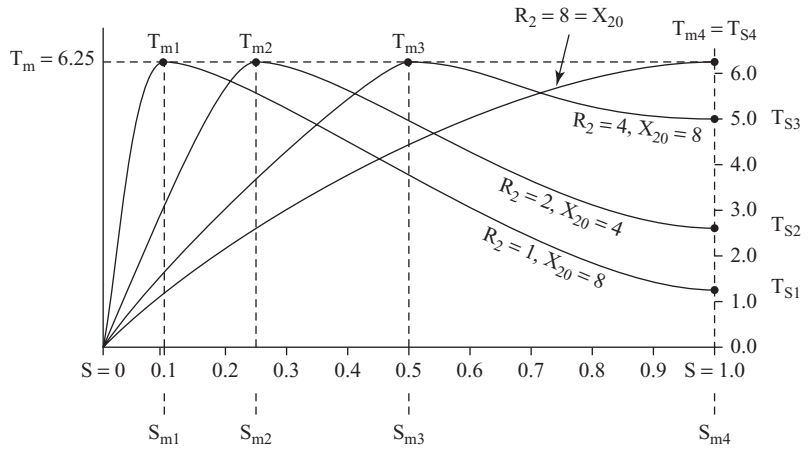


Figure 4.13 *Torque–slip characteristics with increasing rotor-circuit resistance*

By observing the characteristics, we can conclude that

(i) increasing rotor-circuit resistance increases the starting torque;

(ii) maximum torque remains the same for all values of rotor-circuit resistance;

(iii) starting torque becomes equal to maximum torque when rotor-circuit resistance is made equal to rotor-circuit reactance;

(iv) slip at which maximum torque is developed changes with change of rotor-circuit resistance;

(v) increasing rotor-circuit resistance beyond the value of rotor-circuit reactance will reduce the starting torque (this can be verified by calculating T_s at $R_2 = 12\ \Omega$ (say) and $X_{20} = 8\ \Omega$).

4.14 STARTING OF INDUCTION MOTORS

There is similarity between a transformer and an induction motor. An induction motor is like a short circuited transformer. This has been shown in Fig. 4.14. The only difference is that the secondary, i.e., the rotor in an induction motor is cylindrical in shape and is free to rotate while a transformer is totally a static device. The secondary winding, i.e., the rotor winding of an induction motor is closed, i.e., shorted. The similarity between an induction motor and a short-circuited transformer has been shown in Fig. 4.14 (a and b).

With the secondary winding of a transformer short circuited, if full voltage is applied across the primary windings, very high current will flow through the windings. As in the case of transformers, when full voltage applied across stator terminals of an induction motor and the rotor is stationary, very

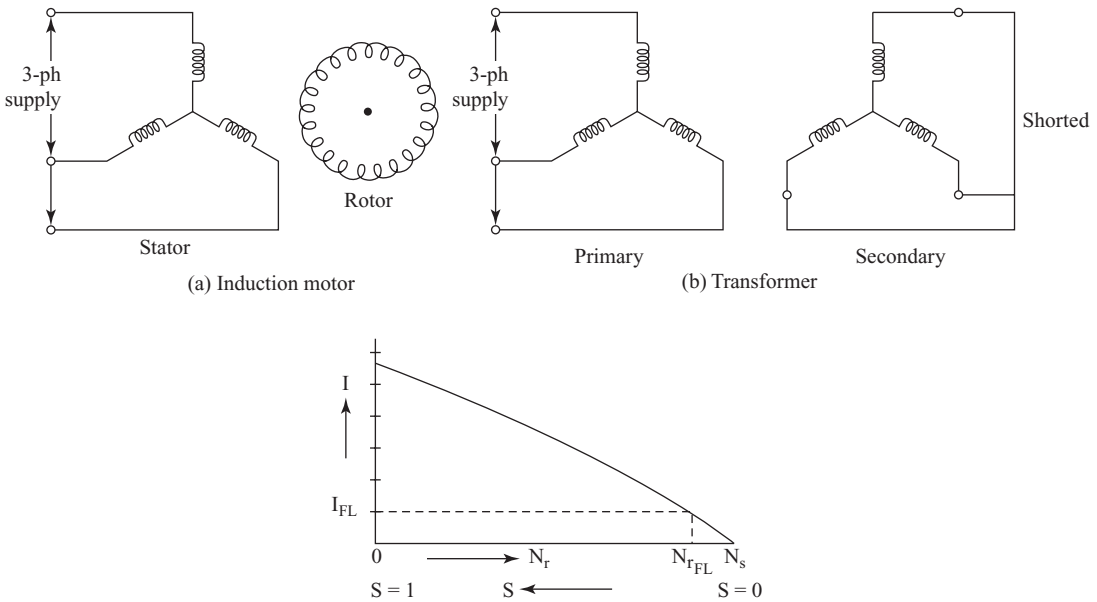


Figure 4.14 *(a) Induction motor; (b) transformer with its secondary windings shorted; (c) high current drawn by an induction motor if started with full voltage*

high current will flow through the rotor and stator windings. If this high current is allowed to flow for a considerable time the motor windings will be burnt out. However, as the motor picks up speed, the EMF induced in it will be SE_{20} so that

$$I_2 = \frac{SE_{20}}{\sqrt{R_2^2 + S^2 X_{20}^2}}$$

As speed increases, i.e., slip decreases, I_2 will go on decreasing; accordingly the current drawn from the supply will also be gradually reducing as has been shown in Fig. 4.14 (c). An induction motor at starting, therefore, will draw very high current if started with full voltage applied across its stator terminals. The starting current may be as high as six-times its full-load current. Thus, it becomes necessary to limit the starting current of an induction motor.

Starting of three-phase induction motors by applying full voltage directly to the stator windings is restricted to small motors upto 5 kW rating. If large motors are started this way, heavy current will be drawn (usually six to eight times the rated current) by the motors. This will not only be harmful to the motors in the long run but will also create heavy voltage drop in the electrical distribution lines, which will disturb the working of other electrical gadgets and machines connected to the line. Higher the rating of the motor, higher will be the disturbance of the line voltage. Starting of motors upto 5 kW rating may be done by applying full voltage. This is called direct-on-line or DOL starting of motors. Reduced voltage starting should be done for larger motors by using a three-phase auto-transformer or by connecting the stator windings of the motor first in star formation and giving the supply and as the motor picks up sufficient speed, connecting the windings in delta formation.

Thus, there are three types of starters used in starting of three-phase induction motors. They are

(i) direct-on-line starters;
(ii) star–delta starters;
(iii) auto-transformer starters.

4.14.1 Direct-on-line Starting

Small induction motors upto the rating of 5 kW are allowed to be started direct-on-line by the electricity boards. For large motors, starters have to be used.

The simplest method of starting a three-phase induction motor is to connect it to a three-phase supply through a switch as shown in Fig. 4.15 (a). However, in case of short-circuit or overload condition due to any wrong connection or excessive loading, the motor will not be protected. So, in a DOL starter provision for overload protection and short-circuit protection must be made. For short-circuit protection, cartridge-type fuses are used. These are connected in series with the lines supplying power to the motor. For overload protection, a thermal overload relay is used. The contact of the relay will open when, due to overload current, the relay will get heated up.

The switching of supply to the motor is done through a contactor and not by a manual switch. Push-button switches are provided for starting and stopping the motor. When the START push button is pressed, the contactor coil gets energized, its main contacts close, and the motor starts running. For stopping the motor the OFF push button is to be pressed due to which the contactor coil will not receive supply and will be de-energized. The contacts of the contractor will open, thereby stopping the motor. In case of over load also the contactor will get de energized due to opening of the contact of the overload relay. The working of the DOL starter is further explained using Fig. 4.15 (b) as follows.

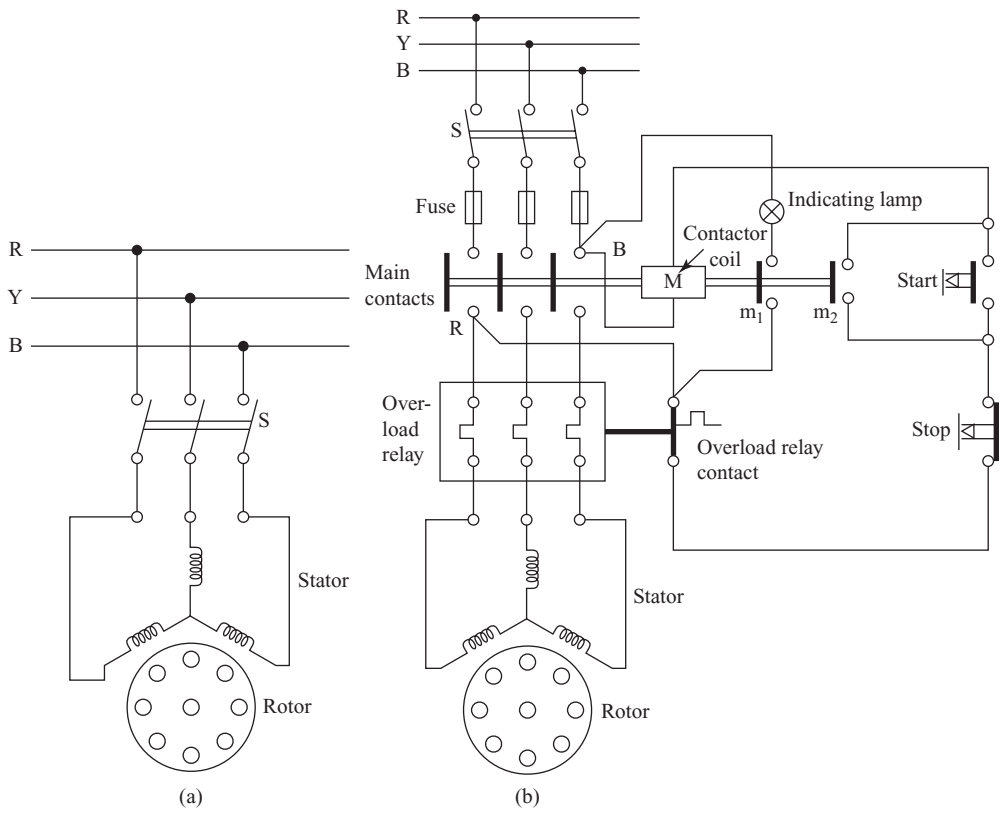


Figure 4.15 *DOL starting of three-phase induction motors: (a) direct connections to supply through a switch but without any overload or short-circuit protection; (b) push-button-operated DOL starter with overload and short-circuit protection*

When the start push button is pressed the contactor coil, M gets energized as the coil gets connected to R-phase and B-phase. The main contacts as well as the auxiliary contacts m_1 and m_2 will close. The motor will get full supply and start rotating. Since the auxiliary contact m_2, is closed, pressure on the start push button can be released and the contactor will continue to remain energized. Indicating lamp will get supply through contact m_1, and will glow, showing that the motor is running. To switch off the motor the stop push button is to be pressed which will de-energize the contactor coil thereby opening its contacts and making power supply to the motor disconnected. In case the motor is overloaded, current drawn will increase beyond the rated capacity, the overload relay contact will open, the contactor coil will get de-energised and, as a consequence, the motor will get disconnected from the supply.

4.14.2 Manual Star–Delta Starter

Fig. 4.16 Shows a manual star–delta starter where the switch S is placed in start position to connect the stator winding terminals A_2, B_2, C_2 in star and connecting the other ends of the windings, i.e., A_1, B_1 and C_1 to the supply. The motor will start running with the stator windings star connected and with full supply voltage applied to them. Once the motor picks up speed, the switch is placed in run position and

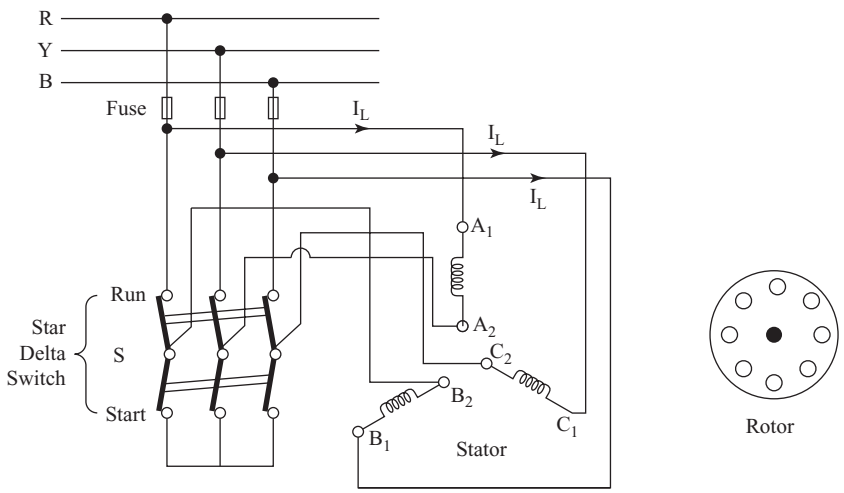


Figure 4.16 *Manual star–delta starter*

the stator windings will get connected in delta and the same supply voltage applied to them. This way the current drawn by the motor from the supply lines during starting is reduced to one-third the value of the current that would have flown if the windings were delta connected during starting period.

This can be understood from the following calculations.

When windings are star connected, the line and phase quantities are represented as shown in Fig. 4.17 (a).

Here,
$$I'_{Ph} = \frac{V}{\sqrt{3}\ Z_{ph}}$$
$$= I'_L$$

When windings are delta connected, the relationships of line and phase quantities are shown as in Fig. 4.17 (b).

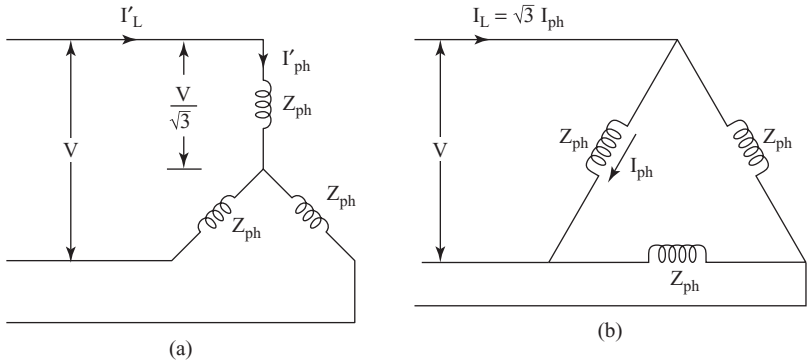


Figure 4.17 *(a) Stator windings star connected during starting; (b) stator windings delta connected in running condition*

Here,
$$I_{Ph} = \frac{V}{Z_{Ph}}$$
$$I_L = \sqrt{3}\, I_{Ph} = \frac{\sqrt{3}V}{Z_{Ph}}$$

If we take the ratio of line current drawn when the windings are star connected to the line current drawn when they are delta connected, we get

$$\frac{I'_L}{I_L} = \frac{I'_{Ph}}{\sqrt{3}\, I_{Ph}} = \frac{V \times Z_{Ph}}{\sqrt{3}Z_{Ph}\sqrt{3}V} = \frac{1}{3}$$

or,
$$I'_L = \frac{1}{3} I_L$$

Thus, current drawn during starting is reduced to one-third if the windings are first star connected and then delta connected. Note that in the manual star–delta starter shown in Fig. 4.16, over load protection device has not been shown.

Star–delta starters are available in automatic form. The windings are first connected in star before full voltage is applied. After the rotor picks up sufficient speed, a time-delay relay (TDR) operates and then the windings get connected in delta. Such starters are called push-button-operated star–delta starters. The operating time of the TDR can be adjusted according to the time taken by the rotor to pick up sufficient speed.

In auto-transformer starters, reduced voltage is applied to the stator windings at starting with the help of a three-phase auto-transformer.

4.15 SPEED CONTROL OF INDUCTION MOTORS

Different types of electric motors are manufactured for use as drives in various kinds of industrial applications. The selection criterion for use are their ruggedness, cost, ease of use, supply requirement, and control of speed. Induction motors have become universally accepted as a first choice because of their satisfying all the above-mentioned requirements. Because of availability of power electronic control devices, speed control of induction motors has become easy now. The basic methods of speed control of induction motors will be discussed. The electronic method of speed control is beyond the scope of this book. The details of use of electronic control are dealt with in a separate subject of power electronics.

We have known that the slip of an induction motor is expressed as

$$S = \frac{N_s - N_r}{N_s}$$

or,
$$SN_s = N_s - N_r$$

or,
$$N_r = N_s(1 - S)$$

or,
$$N_r = \frac{120f}{P}(1 - S)$$

where N_r is the speed of the motor, f is the frequency of supply, and P is the number of poles of the motor.

This equation indicates that the speed N_r of the motor can be changed by any of the following methods:

(a) by changing the supply frequency f;

(b) by changing the number of poles, p for which the stator windings are wound;

(c) by changing the slip;

(d) in addition, speed of slip-ring induction motors can be changed by changing the rotor-circuit resistance.

These methods are described in brief as follows:

(a) *Control of speed by changing supply frequency*

By changing the supply frequency, the speed of the motor can be increased or decreased smoothly. However, the supply frequency available from the electricity supply authority is at 50 Hz which is fixed. Frequency conversion equipment is, therefore, needed for speed control of motors. Variable frequency supply can be obtained from a separate motor generator set, rotary convertors, or solid-state electronic devices. The frequency changing device should change frequency and applied voltage simultaneously as a direct ratio. If frequency is increased, the supply voltage must also be increased and if frequency is decreased, supply voltage must also be decreased proportionally. This will keep the torque developed constant and the operating efficiency high.

(b) *Control of speed by pole changing*

Three techniques, viz (i) use of separate stator windings wound for two different number of poles; (ii) use of consequent pole technique, and (iii) use of pole-amplitude modulation can be used.

Instead of one winding for, say eight poles on the stator we may use two separate windings insulated from each other, say one for eight poles and the other for 10 poles. The synchronous speed corresponding to P = 8 and P = 10 will be 750 rpm and 600 rpm, respectively. The rotor speed corresponding to these synchronous speeds will be somewhat less than these synchronous speeds. Thus, we will get two rotor speeds by having a switching arrangement of power supply to the two stator windings as per our speed requirement. It may be noted that the rotor poles are by induction effect, and hence will be the same as the stator number of poles. If the stator number of poles are increased, the rotor poles will also increase. The number of poles of the stator and rotor must be the same.

In the consequent pole technique, terminals are brought out from the stator winding and by proper switching arrangement the number of poles formed by the stator current is changed. The technique is illustrated in Fig. 4.18. For, simplicity, only two coils have been shown forming the stator winding. Through switching arrangement, supply to the windings are changed as shown in Fig. 4.18 (a) and (b).

The direction of flux around the current-carrying conductors have been shown. The effective number of poles when supply is given at terminal P and taken out from terminal Q is 4. When supply is given at R and taken out from P and Q, the number of poles formed is 2. Thus, we will get two speeds corresponding to P = 4 and P = 2 by changing the power supply points.

Pole-amplitude modulation technique involves reversing the connection of one-half of the windings. It is possible to obtain a different ratio of pole formation, and hence of the rotor speed.

(c) *Speed control by changing the slip*

Slip of the motor, and hence the rotor speed can be changed by introducing, i.e., injecting a voltage of slip frequency directly into the rotor circuit. Depending upon the phase of the injected EMF with respect to the rotor EMF, the slip, and hence the speed will change.

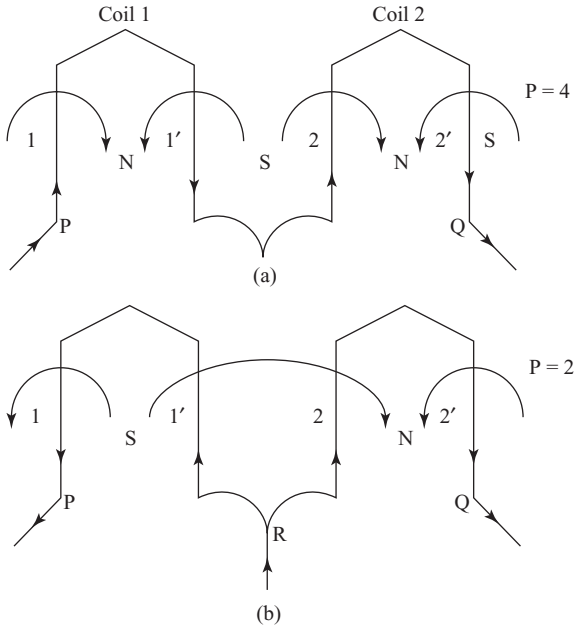


Figure 4.18 *Speed control by using the consequent pole technique: (a) four-pole formation; (b) two-pole formation*

(d) *Speed control by changing rotor-circuit resistance*

In slip-ring induction motors, it is possible to add extra resistance in the rotor circuit. To obtain high starting torque and also to control speed when the motor is running, the extra resistance connected in the rotor circuit is changed. Higher the rotor-circuit resistance, lower will be the speed obtained. However, starting torque will increase with increase of rotor-circuit resistance.

4.16 DETERMINATION OF EFFICIENCY

Efficiency of an induction motor can be determined by loading the motor and measuring the mechanical output and the electrical input. By converting the input and output in either electrical unit or in mechanical unit, the efficiency of the motor can be determined by taking the ratio of output and input. Small induction motors can be tested by this method where it is possible to load the machine in the laboratory.

In testing of large motors, the indirect method is adopted where the losses of the motor are determined from some tests and the efficiency is calculated as

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}}$$

Two types to losses take place in a motor. One is called constant loss and the other is called variable loss. Iron loss and friction and rotational losses are constant at all loads as long as supply voltage and frequency are constant and the speed of rotation does not change much as the load on the motor varies. Copper loss in the stator and rotor varies with load. As the load on the motor changes, the stator and rotor currents vary and the copper losses in the windings also vary.

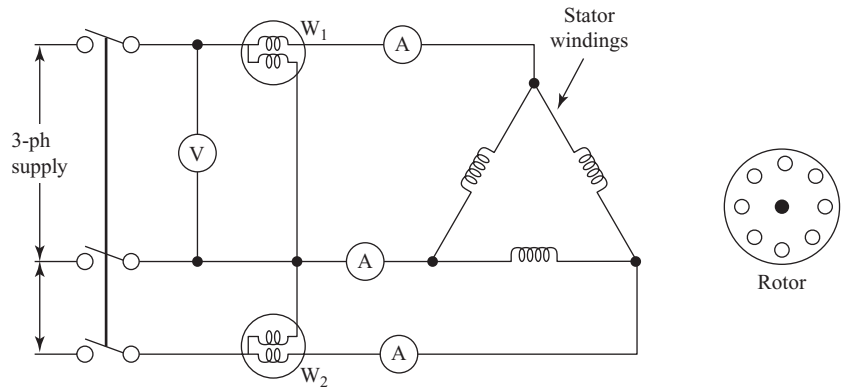
Two tests are performed on the motor to determine the constant losses and variable losses so as to determine the efficiency of the motor. These tests are described as follows.

4.16.1 No-load Test

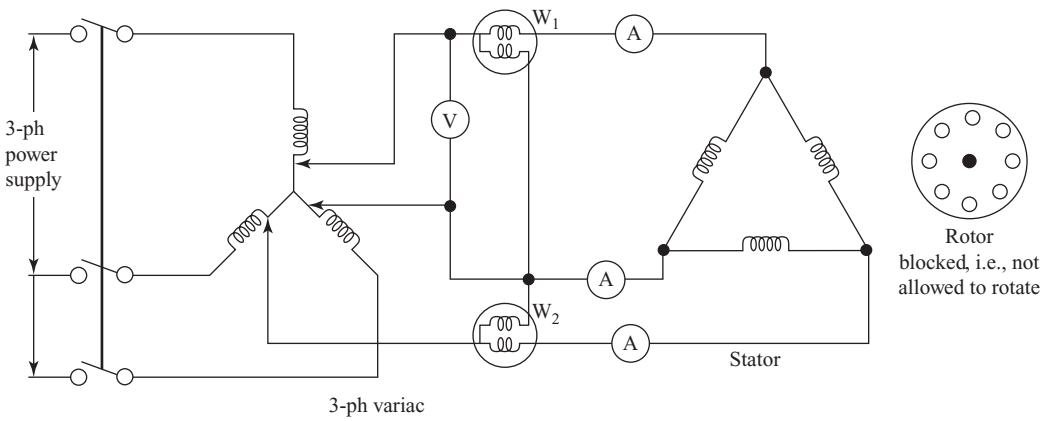
The motor is run on no load with full voltage applied across its stator terminals as shown in Fig. 4.19. Two single-phase wattmeters are connected to measure the three-phase power input. Since with no load connected on the motor shaft, the input power is completely wasted as loss, the sum of the wattmeter reading can be considered equal to the various losses. Therefore,

$$W_1 + W_2 = I^2R \text{ loss in the windings at no load}$$
$$+ \text{ Iron loss in the core}$$
$$+ \text{ Friction and windage loss due to rotation of the rotor}$$

By subtracting the no-load I^2R loss from the input power, the constant losses can be calculated. Note that I^2R loss in the rotor at no load is very small. Thus, we will consider only the I^2R loss in the stator windings.



(a) No-load test on an induction motor



(b) Blocked-rotor test on an induction motor

Figure 4.19 *Circuits for no-load and blocked-rotor tests*

4.16.2 Blocked-rotor Test

The blocked-rotor test is performed by blocking the rotor so that it is not allowed to rotate. Low voltage is applied to the rotor through a three-phase auto-transformer (Variac). The Output voltage of the variac is adjusted so that full-load-rated current flows through the windings. The core loss which is proportional to this low input voltage is small. There is no friction and windage loss as the rotor is not allowed to rotate. The sum of the wattmeter readings can approximately be taken as equal to full-load I^2R loss in the windings.

Thus, we observe that the no-load test and the blocked-rotor test together provide us the account for losses that would take place when the motor is fully loaded with full voltage applied across its terminals. By knowing the losses, we will be able to calculate the efficiency of the motor.

In these two tests, we have created a loading condition that would happen when the motor is actually loaded. That is why this method of finding out the efficiency without actually applying load on the motor shaft is called an indirect method.

4.17 APPLICATIONS OF INDUCTION MOTORS

Around 90 per cent of the electrical motors used in industry and domestic appliances are either three-phase induction motors or single-phase induction motors. This is because induction motors are rugged in construction requiring hardly any maintenance, that they are comparatively cheap, and require supply only to the stator. No supply is required to be given to the rotor. The rotor gets excited by virtue of electromagnetic induction. Further, there is no requirement of brush, slip rings, or commutator. However, slip-ring-type induction motors where extra resistance is added to the rotor circuit are used in applications where high starting torque is required.

Three-phase induction motors are used as drive motors in pumps, lifts, cranes, hoists, lifts, compressors, large capacity exhaust fans, driving lathe machines, crushers, in oil extracting mills, textile and paper mills, etc.

Example 4.3 A 3 hp, three-phase, four-pole, 400 V, 50 Hz induction motor runs at 1440 rpm. What will be the frequency of the rotor-induced EMF?

Solution:

Synchronous speed, $N_s = \frac{120f}{p} = \frac{120 \cdot 50}{4} = 1500\,\text{rpm}$

Rotor speed, $N_r = 1440$ rpm

Slip, $S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1400}{1500} = 0.04$

Stator supply frequency, $f = 50$ Hz

Rotor induced EMF frequency, $f_r = Sf$

$= 0.04 \times 50$

$= 2$ Hz

Example 4.4 The frequency of the rotor-induced EMF of 400 V, three-phase, 50 Hz, six-pole induction motor is 2 Hz. Calculate the speed of the motor.

Solution:

$$N_s = \frac{120f}{P} = \frac{120 \cdot 50}{6} = 1000 \text{ rpm}$$

Rotor frequency, $f_r = Sf$

or, $2 = S \times 50$

or, $S = 0.04$

Again, $S = \frac{N_s - N_r}{N_s}$

Substituting values $0.04 = \frac{1000 - N_r}{1000}$

or, $N_r = 960 \text{ rpm}$

Example 4.5 A slip-ring-type three-phase induction motor rotates at a speed of 1440 rpm when a 400 V, 50 Hz is applied across the stator terminals. What will be the frequency of the rotor-induced EMF?

Solution:

We know the synchronous speed of the rotating magnetic field produced is expressed as

$$N_s = \frac{120f}{P}$$

Here, f = 50 Hz but number of poles of the stator winding has not been mentioned. The number of poles can be 2, 4, 6, 8, etc.

for $P = 2, \quad N_s = \frac{120 \cdot 50}{2} = 3000 \text{ rpm}$

for $P = 4, \quad N_s = \frac{120 \cdot 50}{4} = 1500 \text{ rpm}$

for $P = 6, \quad N_s = \frac{120 \cdot 50}{6} = 1000 \text{ rpm}$

for $P = 8, \quad N_s = \frac{120 \cdot 50}{8} = 750 \text{ rpm}$

We know that an induction motor runs at a speed slightly less than the synchronous speed. Here, N_r = 1440 rpm (given). Synchronous speed corresponding to this rotor speed must, therefore, be 1500 rpm Thus, N_r = 1440 rpm and N_s = 1500 rpm.

Slip, $S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1440}{1500} = 0.04$

Rotor frequency, $f_r = S \cdot f = 0.04 \cdot 50 = 2 \text{ Hz}$

Example 4.6 A six-pole, three-phase synchronous generator driven at 1000 rpm supplies power to an induction motor which runs at a speed of 1440 rpm on full load. Calculate the percentage slip of the motor and the number of poles of the motor.

Solution:

The synchronous generator is rotated at a synchronous speed of 1000 rpm by a prime mover. The synchronous speed N_s is expressed as

$$N_s = \frac{120f}{P} \quad \text{where f is the frequency of the generated EMF}$$

or,
$$f = \frac{N_s \cdot P}{120}$$
$$= \frac{1000 \cdot 6}{120} = 50\,\text{Hz}$$

This synchronous generator now supplies power to the induction motor at a frequency of 50 Hz. The synchronous speed of the rotating magnetic field produced in the motor is

$$N_s = \frac{120 \cdot f}{p} = \frac{120 \cdot 50}{2} = 3000\,\text{rpm (for P = 2)}$$

and
$$N_s = \frac{120 \cdot 50}{4} = 1500\,\text{rpm (for P = 4)}$$

The motor speed is 1440 rpm, which should be slightly less than the synchronous speed. Logically, the number of poles of the motor must be 4.

Percentageslip,
$$S = \frac{N_s - N_r}{N_s} \times 100 = \frac{(1500 - 1440) \times 100}{1500} = 4 \text{ per cent}$$

Example 4.7 A 400 V, 50 Hz, three-phase induction motor is rotating at 960 rpm on full load. Calculate the following for the motor:

Number of poles; full-load slip; frequency of rotor-induced EMF; speed of the rotor magnetic field with respect to the rotor.

Also show that both the stator field and the rotor field are stationary with respect to each other.

Solution:

$$f = 50 \text{ Hz}$$

$$P = 2, 4, 6, \text{ etc.}$$

$$N_s = \frac{120\ f}{p} = \frac{120 \cdot 50}{2} = 3000\,\text{rpm (for P = 2)}$$

$$= \frac{120 \cdot 50}{4} = 1500\,\text{rpm (for P = 4)}$$

Rotor speed is somewhat less than the synchronous speed N_s. Logically, here N_s can only be 1500 rpm, when N_s = 1500 rpm, P = 4.

Full load slip,
$$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1440}{1500} = 0.04$$

Frequency of rotor induced EMF $f_r = S \times f = 0.04 \times 50$

$$= 2 \text{ Hz}$$

Speed of rotor field with respect to rotor, N is

$$N = \frac{120 \cdot f_r}{P} = \frac{120 \cdot 2}{4} = 60 \text{ rpm}$$

The rotor rotates at a speed of 1440 rpm. This means the speed of the rotor with respect to stator, which is stationary, is 1440 rpm.

The speed of the rotor field with respect to the rotor is 60 rpm.

Therefore, the speed of the rotor field with respect to the stator is 1440 + 60 = 1500 rpm. And, the speed of the rotating magnetic field produced by the stator rotates at synchronous speed, N_s with respect to the stator. In this case the speed of rotating field produced by the stator is 1500 rpm. Thus, we see that both the magnetic fields of the stator and rotor are stationary with respect to each other, which is, of course, the essential condition for production of torque.

Example 4.8 A three-phase, four-pole, 50 Hz induction motor rotates at a full-load speed of 1470 rpm. The EMF measured between the slip-ring terminals when the rotor is not rotating is 200 V. The rotor windings are star connected and has resistance and stand-still reactance per phase of 0.1 Ω and 1.0 Ω, respectively. Calculate the rotor current on full load.

Solution:

$$N_r = 1470 \text{ rpm}$$

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1470}{1500} = 0.02$$

Rotor-induced EMF between the slip rings at standstill, $E_{20} = 200$ V.

As the rotor windings are star connected,

$$E_{20} \text{ per phase} = \frac{200}{\sqrt{3}} = 115.4 \text{ V}$$

When the rotor is rotating at a speed of 1470 rpm the rotor-induced EMF per phase, E_2 is

$$E_2 = S\,E_{20} = 0.02 \times 115.4 = 2.3 \text{ V}$$

Rotor current when the rotor is rotating at 1470 rpm,

$$I_2 = \frac{S\,E_{20}}{Z_2} = \frac{S\,E_{20}}{\sqrt{R_2^2 + (SX_{20})^2}}$$

Substituting values

$$I_2 = \frac{2.3}{\sqrt{(0.1)^2 + (0.02 \cdot 1)^2}} = \frac{2.3}{0.102} = 22.5 \text{ A}$$

Example 4.9 A 15 hp, three-phase, four-pole, 50 Hz induction motor has full-load speed of 1455 rpm. The friction and windage loss of the motor at this speed is 600 W. Calculate the rotor copper loss.

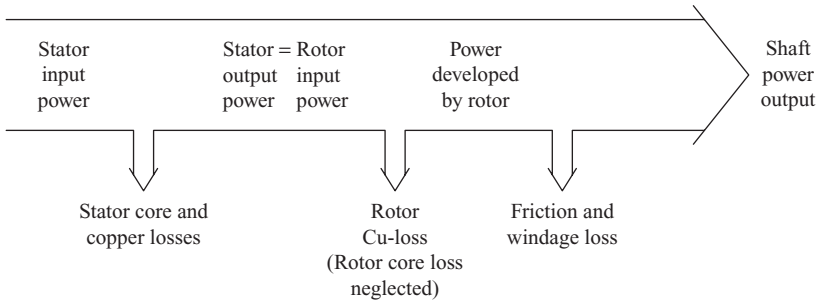
Solution:

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Slip,
$$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1455}{1500} = 0.03$$

Motor output available at the shaft = 15 hp
$= 15 \times 735.5$ W
$= 11032$ W

Now, let us look at the power flow diagram



Power developed by rotor = Shaft power output + Friction and Windage losses
$= 11032 + 600$
$= 11632$ W

We know the relation
Rotor copper loss = S × Rotor input (See eq. 4.3)
(Slip × Rotor input) is lost as rotor copper loss
The remaining power, i.e., (1–S) Rotor input, is developed as the rotor power.
Therefore,
Power developed by rotor = (1–S) Rotor input

$$= \frac{(1-S) \text{ Rotor copper loss}}{S}$$

Thus,
$$\text{Rotor copper loss} = \frac{S}{(1-S)} \times \text{Rotor power developed}$$
$$= \frac{0.3 \times 11632}{(1-0.03)} \text{ W}$$
$$= 360 \text{ W}$$

Example 4.10 A 10 hp, four-pole, 50 Hz, three-phase induction motor has friction and windage loss of 3 per cent of output. Calculate at full load the rotor copper loss, rotor input for a full-load slip of 4 per cent. If at this load stator loss is 6 per cent of the input power, calculate the efficiency.

Solution:

Output power = 10 hp = 10 × 735.5 W
= 7355 W

and slip, S = 0.04

Friction and windage loss = 0.03 × 7355 W
= 220.6 W

Power developed = 7355 – 220.6 = 7134.4 W

Rotor copper loss = S × Rotor input, (4.8)

Power developed = (1–S) Rotor input (4.9)

From Eqs. (4.8) and (4.9),

$$\text{Power developed} = (1-S)\frac{\text{Rotor copper loss}}{S}$$

or,
$$\text{Rotor copper loss} = \frac{S}{(1-S)}\text{Power developed}$$

Substituting values,
$$\text{Rotor copper loss} = \frac{0.04}{0.96}\cdot 7134.4 \text{ W}$$
$$= 297.3 \text{ W}$$

Rotor input – Rotor copper loss = Power developed

Therefore, Rotor input = Power developed + Rotor copper loss
= 7134.4 + 297.3
= 7431.7 W

Let input power be = X W

Out of this, 0.06 X W is wasted as stator losses. Stator output = Rotor input = 0.94 X W. Equating with actual values, 0.94 X = 7431.7 W

or, X = 7906 W

$$\text{Percentage Efficiency} = \frac{\text{Output}}{\text{Input}}\cdot 100 = \frac{7355\cdot 100}{7906} = 93 \text{ per cent}$$

Example 4.11 A four-pole 50 Hz, three-phase induction motor has rotor resistance of 0.5 Ω phase. The maximum torque occurs at a speed or 1470 rpm. Calculate the ratio of starting torque to maximum torque.

Solution:

The synchronous speed,
$$N_s = \frac{120f}{P} = \frac{120\cdot 50}{4} = 1500 \text{ rpm}$$

Rotor speed, N_r = 1200 rpm

Slip,
$$S = \frac{N_s - N_r}{N_s} = \frac{1500-1470}{1500} = 0.02$$

At this slip, torque is maximum. Condition for maximum torque is $R_2 = SX_{20}$.

Thus,
$$X_{20} = \frac{R_2}{S} = \frac{0.5}{0.02} = 25\,\Omega$$

For constant supply voltage, the expression for torque, T is given as

$$T = \frac{K S R_2}{R_2^2 + S^2 X_{20}^2}$$

Value of maximimum torque T_m is

$$T_m = \frac{K}{2X_{20}}$$

At starting, S = 1, Starting torque T_{st} is

$$T_{st} = \frac{K R_2}{R_2^2 + X_{20}^2}$$

Substituting values,

$$\frac{T_{st}}{T_m} = \frac{K R_2}{R_2^2 + X_{20}^2} \cdot \frac{2X_{20}}{K} = \frac{2X_{20} R_2}{R_2^2 + X_{20}^2}$$

$$\frac{T_{st}}{T_m} = \frac{2 \cdot 25 \cdot 0.5}{(.5)^2 + (25)^2} = 0.04$$

That is, the starting torque is only 4 per cent of the maximum torque.

Example 4.12 No-load test and blocked-rotor test were performed on a 10 hp, four-pole, 400 V, 50 Hz, three-phase induction motor to determine its efficiency. The test data are given as follows:

no-load test: V = 400 V, I = 6 A, P = 300 W

blocked-rotor test: V = 40 V, I = 24 A, P = 700 W

Calculate the efficiency of the motor on full load.

Solution:

Losses under blocked-rotor test is considered equal to I^2R losses in the two windings. If R_e is the equivalent resistance of the two windings referred to stator side, then total copper loss

$$3\, I^2\, R_e' = 700 \text{ W}$$

$$R_e' = \frac{700}{3 \times 24 \times 24} = 0.4\ \Omega$$

At no load, I^2 R loss in the windings

$$= 3\, I_0^2\, R_e' = 3 \times 6^2 \times 0.4$$
$$= 43.2 \text{ W}$$

No-load power input = 300 W

Core loss + Frictional and Windage loss = 300 – 43.2

= 256.8 W

Output = 10 hp = 10 × 735.5 = 7355 W

Efficiency,

$$\eta = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{10 \times 735.5 \times 100}{10 \times 735.5 + 700 + 256.8}$$

= 88.5 per cent

Example 4.13 A three-phase, 20-pole slip-ring induction motor runs at 291 rpm when connected to a 50 Hz supply. Calculate slip for full-load torque if the total rotor-circuit resistance is doubled. Assume $R_2 >> SX_2$.

Solution:

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{20} = 300 \text{ rpm}$$

slip,

$$S = \frac{N_s - N_r}{N_s} \times 100 = \frac{(300 - 291)}{300} \times 100 = 3 \text{ per cent}$$

Full-load torque equation is,

$$T = \frac{K S E_{20}^2 R_2}{R_2^2 + S^2 X_{20}^2}$$

$$\text{If } R^2 >> S^2 X_{20}^2$$

$$T = \frac{K S E_{20}^2 R_2}{R_2^2} = \frac{K S E_{20}^2}{R_2}$$

For a given torque, T and rotor-induced EMF at standstill, i.e., E_{20},

$$R_2 \propto S$$

If R_2 is doubled, S will be doubled. The slip at doubled R_2 will be 6 per cent.

4.18 REVIEW QUESTIONS

A. Short Answer Type Questions

1. Explain the principle of working of a three-phase induction motor.
2. Explain why a three-phase induction motor can not run at synchronous speed.
3. What is meant by slip of an induction motor? What is the value of slip at starting and at synchronous speed?
4. How can you change the direction of rotation of a three-phase induction motor? What is meant by phase sequence of power supply?
5. Explain the purpose of making two types of rotor construction for three-phase induction motors.
6. Write an expression for torque developed by an induction motor and write the meaning of each term.
7. Draw the torque-speed characteristic of a three-phase induction motor and explain its shape.
8. Show that stator magnetic field and the field produced by the rotor mmf are stationary with respect to each other.
9. What is the condition for maximum torque developed at starting?
10. Show how a rotating magnetic field is produced when a three-phase supply is connected across a three-phase winding.
11. What is the expression for maximum torque developed in a three-phase induction motor?
12. Draw the power-flow diagram in an induction motor.

13. What are the various losses in an induction motor? State the factors on which they depend.
14. Explain how efficiency of an induction motor can be determined by performing no-load test and blocked-rotor test.
15. Explain why a starter is required to start a large three-phase induction motor.
16. Draw a push-button-operated direct-on-line starter for an induction motor.
17. Draw the connection diagram for a manual star–delta starter.
18. Show the effect of variation of rotor-circuit resistance on the torque-speed characteristic of an induction motor.
19. What is the limit of increasing the rotor-circuit resistance for achieving a high starting torque? Explain your answer.
20. If the applied voltage is reduced to half, what will be the reduction in torque developed?
21. Prove that in an induction motor the rotor copper loss is slip times the rotor input.
22. Show how the maximum torque developed in an induction motor is independent of rotor-circuit resistance.
23. How can you determine experimentally, the full-load copper loss of an induction motor?
24. Establish the relation rotor frequency, $f_r = S \times f$.
25. Mention various applications of three-phase induction motors.
26. What are the advantages and disadvantages of squirrel-cage induction motors over slip-ring induction motors?
27. Explain why the rotor-circuit reactance of an induction motor varies with speed?
28. Establish the relation, $X_2 = S\, X_{20}$ where S is the slip, X_2 is the rotor-circuit reactance under running condition, and X_{20} is the rotor-circuit reactance when the rotor is at rest.
29. Establish the similarity between a transformer and an induction motor.
30. At what slip will torque developed by an induction motor be maximum when rotor resistance equals half the rotor reactance at standstill.
31. In an induction motor, slip is always positive, why?
32. Explain why an induction motor draws heavy current at starting when started on full voltage.

B. Numerical Problems

33. A four-pole, three-phase, 50 Hz induction motor rotates at a speed of 1440 rpm. Calculate its slip in percentage. Also calculate the frequency of the induced EMF in the rotor circuit.

    [Ans. 4 per cent; 2 Hz]

34. A six-pole, three-phase, 400 V, 50 Hz induction motor is running at a speed of 940 rpm. Calculate its slip.

    [Ans. 6 per cent]

35. A four-pole, three-phase, 400 V, 50 Hz induction motor develops an induced EMF in the rotor of 2 Hz. What is the speed of the motor?

    [Ans. 1440 rpm]

36. A four-pole, three-phase induction motor in connected to a 50 Hz supply. Calculate synchronous speed; the rotor speed when slip is 4 per cent, and the rotor frequency when the rotor is running at 1425 rpm.

    [Ans. 1500 rpm, 1440 rpm, 2.5 Hz]

37. The input power to the rotor of a 400 V, 50 Hz, six-pole, three-phase induction motor is 75 kW. The frequency of the rotor induced EMF is 2 Hz. Calculate slip; rotor speed, and power developed by the rotor; rotor $I^2 R$ loss.

[Ans. 0.04, 960 rpm, 72 kW; 1 kW]

38. A 400 V, 50 Hz, six-pole, three-phase induction motor running at 975 rpm draws 40 kW from the mains. The stator loss is 1kW. The friction and windage loss is 2 kW. Calculate (i) slip; (ii) $I^2 R$ loss in the rotor; (iii) the shaft output in hp, and (iv) the efficiency of the motor.

[Ans. 2.5 per cent; 975 W; 49 hp; 90 per cent]

C. Multiple Choice Questions

1. For production of a rotating magnetic field using stationary windings we must connect
   (a) a single-phase supply to a single-phase winding
   (b) a three-phase supply to a three-phase winding
   (c) a single-phase supply to a two-phase winding
   (d) either a single-phase winding or a two-phase winding to a single-phase supply.

2. When a three-phase 50 Hz, 400 V supply is applied across a four-pole, three-phase winding, a rotating magnetic field is produced which is rotating at
   (a) 200 rpm (b) 1600 rpm
   (c) 1500 rpm (d) 3000 rpm.

3. A four-pole, three-phase induction motor is rotating at 1440 rpm when a 400 V, 50 Hz supply is applied to its stator terminals. The slip of the motor expressed in percentage is
   (a) 4 per cent (b) 2 per cent
   (c) 5 per cent (d) 6 per cent.

4. The synchronous speed N_s, frequency f, and number of poles, P are related as
   (a) $N_s = \frac{120P}{f}$ (b) $N_s = \frac{120f}{P}$
   (c) $N_s = \frac{fP}{120}$ (d) $\frac{Pf}{60}$.

5. When an induction motor is yet to start, i.e., at standstill, its slip is
   (a) 0 (b) 1
   (c) infinity (d) near to 0.

6. The speed of a three-phase induction motor when supplied with three-phase, 400 V, 50 Hz supply is 1440 rpm. The number of poles for which the windings are made must be
   (a) 2 (b) 4
   (c) 6 (d) 8.

7. A three-phase induction motor develops a torque of 500 Nm at normal supply voltage. If the supply voltage is reduced to half, what will be the torque developed?
   (a) 250 Nm
   (b) 125 Nm
   (c) will remain constant at 500 Nm
   (d) 62.5 Nm.

8. The slip of an induction motor can be expressed as
   (a) $S = \frac{Nr - Ns}{Nr}$ (b) $S = \frac{Ns - Nr}{Nr}$
   (c) $\frac{Ns - Nr}{Ns}$ (d) $\frac{Nr}{Ns - Nr}$.

9. A four-pole, 50 Hz, 400 V, three-phase induction motor is running at 1440 rpm. The frequency of rotor-induced EMF is
   (a) 2 Hz (b) 4 Hz
   (c) 1 Hz (d) 50 Hz.

10. A four-pole, three-phase induction motor when fed from a 400 V, 50 Hz supply, runs at 1440 rpm. The frequency of EMF induced in the rotor is
    (a) 2 Hz (b) 45 Hz
    (c) 50 Hz (d) 3 Hz.

11. If an induction motor by some means is rotated at synchronous speed, then
    (a) the EMF induced in the rotor will be maximum
    (b) the EMF induced in the rotor will be zero
    (c) the torque developed by the rotor will be maximum
    (d) the frequency of induced EMF will be slip times the supply frequency.

12. Increase in the rotor-circuit resistance of an induction motor will
    (a) increase the starting torque
    (b) decrease the starting torque

(c) increase the maximum torque developed
(d) decrease the maximum torque developed.

13. Torque developed by a three-phase induction motor depends on
(a) the supply voltage
(b) the rotor-circuit resistance
(c) the slip of the rotor
(d) all the above.

14. A large capacity three-phase induction motor is started using a star–delta starter instead of starting direct-on-line. The starting current
(a) is increased three times
(b) remains constant
(c) is reduced to one-third its value
(d) is reduced to half its value.

15. With rotor-circuit resistance, R_2 of 2 Ω, the maximum torque of an induction motor is developed at 10 percent slip. When the maximum torque is developed at 20 percent slip, the value of R_2 will be
(a) 1 Ω (b) 2 Ω
(c) 0.5 Ω (d) 4 Ω.

16. The speed of a three-phase slip-ring-type induction motor can be controlled by
(a) changing the frequency of supply voltage to the motor
(b) changing the rotor-circuit resistance
(c) changing the number of poles
(d) all the methods as in (a), (b) and (c).

17. The four no-load speeds of three-phase induction motors operating on 400 V, 50 Hz supply are 576 rpm, 720 rpm, 1440 rpm, and 2880 rpm. The number of poles of the motor, respectively, are
(a) 8, 6, 4, 2 (b) 10, 8, 4, 2
(c) 12, 8, 4, 2 (d) 16, 8, 4, 2.

18. The power factor of a three-phase induction motor on no load is found to be 0.15 lagging. When the motor is fully loaded its power factor would be around
(a) 0.05 lagging (b) 0.15 lagging
(c) 0.85 lagging (d) 0.85 leading.

19. The three supply terminals R, Y, B when connected, respectively, to the three stator terminals a, b, c of a three-phase induction motor, the motor rotates in the clockwise direction. The motor will rotate in the reverse direction if
(a) R is connected to c, Y is connected to a, and B is connected to b
(b) R is connected to b, Y is connected to a, connection of B to c remains unchanged
(c) the supply voltage is reduced to half its value
(d) the frequency of supply is gradually reduced.

20. A three-phase induction motor has standstill rotor-circuit resistance, X_{20} of 8 Ω and resistance of 2 Ω. The maximum torque will be developed at starting if the circuit parameters are
(a) $R_2 = 2\ \Omega$, $X_{20} = 4\ \Omega$
(b) $R_2 = 4\ \Omega$, $X_{20} = 8\ \Omega$
(c) $R_2 = 8\ \Omega$, $X_{20} = 8\ \Omega$
(d) $R_2 = 16\ \Omega$, $X_{20} = 8\ \Omega$.

Answers to Multiple Choice Questions

1. (b)	2. (c)	3. (a)	4. (b)	5. (b)	6. (b)
7. (b)	8. (c)	9. (a)	10. (d)	11. (b)	12. (a)
13. (d)	14. (c)	15. (d)	16. (d)	17. (b)	18. (c)
19. (b)	20. (c)				

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Three-phase System

TOPICS DISCUSSED

- Advantages of three-phase systems
- Generation of three-phase voltages
- Terms used in three-phase systems and circuits
- Three-phase winding connections
- Active and reactive power
- Comparison between star connection and delta connection
- Measurement of power in three-phase circuits

5.1 INTRODUCTION

Generation, transmission, and distribution of electricity is done by three-phase electrical networks consisting of generators, transformers, and transmission and distribution lines forming the power system. In a three-phase system we have three independent voltages induced in the three windings of the generator. To understand the difference between a single-phase voltage and a three-phase voltage, let us consider how these voltages are generated in ac generators. We have known that EMF is induced in a coil if it cuts lines of force. In Fig. 5.1 we have placed one coil in slots of a hollow cylindrical stator core. A two-pole magnet is rotated at a particular speed by some means. The flux lines will cut the conductors and EMF will be induced in the coil. Since North and South poles' flux will cut the conductors alternately, an alternating single-phase voltage will be induced in the coil.

Here, we have seen that the coil is stationary and the magnetic field is being rotated. However, we could have had the field stationary and the coil rotating by placing the field magnets on the stator and the coil on a cylindrical rotor. What is required is to produce a relative motion between the magnetic field and the conductor. The magnitude of the induced EMF will depend upon the number of coils, the strength of the magnetic field, and the speed of rotation of the magnet. The frequency of the induced EMF will depend upon the number of magnetic poles confronted by the coils per revolution. Normally, the number of magnetic poles and the speed of rotation of the magnetic poles by a drive, usually a turbine,

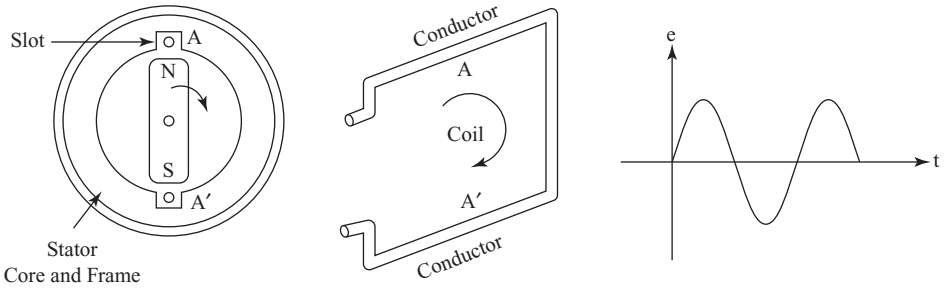

Figure 5.1 *Generation of single-phase voltage*

is kept constant. The number of coils and number of turns used in each coil are kept as per design and are constant once the machine is constructed. That is why the magnitude of the induced EMF and its frequency is constant. Thus, we get a single-phase voltage from the single-phase winding which can be used to supply an electric circuit comprising resistance, inductance, and capacitance elements.

In a three-phase system we will have three-phase voltages induced in the three-phase windings of the generator. In all generating stations three-phase generators are installed.

5.2 ADVANTAGES OF THREE-PHASE SYSTEMS

A three-phase system has a number of advantages over a single-phase system. Some of these are mentioned as follows:

(i) the output of a three-phase machine generating electricity is more than the output of a single-phase machine of the same size;

(ii) the most commonly used three-phase induction motors are self starting. For single-phase motors, as will be explained in a separate chapter, a separate starting winding is required;

(iii) electrical power transmission from the generating station to the places of use is done by transmission lines. It has been seen that three-phase power transmission is more economical than single-phase power transmission;

(iv) the power factor of three-phase systems is better than that of the single-phase systems;

(v) single-phase supply can also be obtained from a three-phase supply;

(vi) the instantaneous power in a single-phase system is fluctuating with time giving rise to noisy performance of single-phase motors. The power output of a symmetrical three-phase system is steady;

(vii) for rectification of ac into dc, the dc output voltage becomes less fluctuating if the number of phases is increased.

Thus, we see that from generation, transmission, distribution, and utilization points of view, three-phase systems are preferred over single-phase systems due to a number of reasons mentioned above.

5.3 GENERATION OF THREE-PHASE VOLTAGES

Due to a number of practical considerations, generators are built to generate poly-phase voltages. Commercial generators are built to generate three-phase voltages. In three-phase generators, three separate windings are made. Windings are made of coils. These windings are placed in stator slots at an angle of 120° apart as shown in Fig. 5.2. RR′ is one phase winding.

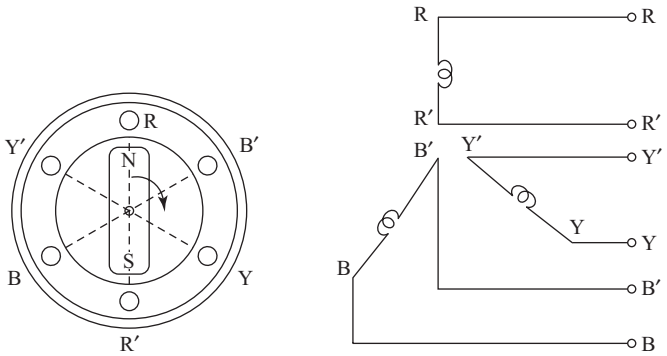

Figure 5.2 *Generation of three-phase voltages*

YY′ is the second phase winding and BB′ is the third phase winding. The three-phase windings are placed at an angular distance of 120°. For simplicity only one coil per phase has been shown. In practice a number of coils connected in series makes one phase winding.

When the magnetic poles are rotated by a prime mover (say a turbine), the magnetic flux of North and South poles will cut the windings in sequence. For clockwise rotation, flux will be cut by coil RR′ first, then by coil YY′, and then by coil BB′. Therefore, EMF will be induced in these coils in sequence. There will be a time phase difference between the EMFs induced in these coils (windings). The time phase difference will be 120°. In terms of time, the phase difference will be the time taken by the magnetic poles to rotate by 120°, i.e., one-third of a revolution. Thus, across the three-phase windings we will get three voltages which are equal in magnitude and frequency but having a time phase difference of 120° between them, as shown in Fig. 5.3.

The equation of voltages are

$$e_R = E_m \sin \omega t \tag{5.1}$$

$$e_Y = E_m \sin (\omega t - 120°) \tag{5.2}$$

$$e_B = E_m \sin (\omega t - 240°) \tag{5.3}$$

Resultant EMF = $E_m \sin \omega t + E_m \sin (\omega t - 120°) + E_m \sin (\omega t - 240°) = 0$

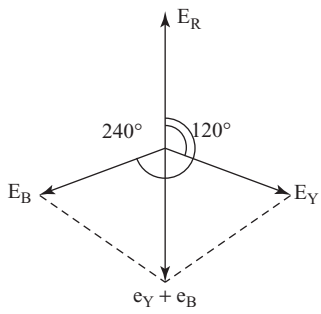

Figure 5.3 *Three-phase voltages displaced in time phase by 120°*

Since the three voltages are equal in magnitude but displaced in time phase by 120°, their phasor sum is zero as shown in Fig. 5.3.

Three-phase supply is required for large-capacity electrical loads. These loads could be three-phase motors used in industrial, commercial, agricultural, and other sectors. For example, the water pump used for irrigation purpose is invariably a three-phase-motor-driven pump requiring a three-phase supply. So, like three-phase supply, we will have three-phase loads. Three-phase supply will be supplying electrical power to three-phase loads. A number of terms are used in connection with three-phase supply and three-phase loads. These are described as follows. Further the three-phase windings can be connected together in the form of star or delta. The voltage between the two-phase windings and current flowing through the phase windings, and the supply line will be different in star and delta connections. These will be studied in detail.

By now we must have realized that by phase we mean a winding. A phase difference between two windings is the physical angular displacement between them. In a three-phase winding, the phase difference between the windings is 120°. Phase sequence is the order in which maximum voltage is induced in the windings. For example, if the magnetic field cuts the conductors of the phase RR′ first and then cuts the conductors of phase YY′, and lastly cuts the conductors of phase BB′, then EMF will be induced in all the phases of equal magnitude but their maximum value will appear in a sequence RYB. Then we call the phase sequence of EMF as RYB. If the magnet system of Fig. 5.2 rotates in the anticlockwise direction, the phase sequence of EMF induced in the three phases will be RBY.

Let us consider an elementary three-phase generator as shown in Fig. 5.4 (a). Three-phase windings are placed in slots in the stator. For simplicity, only one coil per phase has been used. R-R′ is one coil making R-phase winding. Y-Y′ is another coil forming Y-phase winding. B-B′ is one coil forming B-phase winding. These three-phase windings are placed in the stator slots at an angle of 120° in space. For simplicity only one coil per phase has been shown. In actual practice, a number of coils are connected together to form each phase winding. The rotor carries the magnetic poles which produce the magnetic field. Direct current is supplied to the field windings so that the field magnets are excited. When the rotor is rotated by a turbine, the magnetic flux are cut by the stator coils RR′, YY′, and BB′

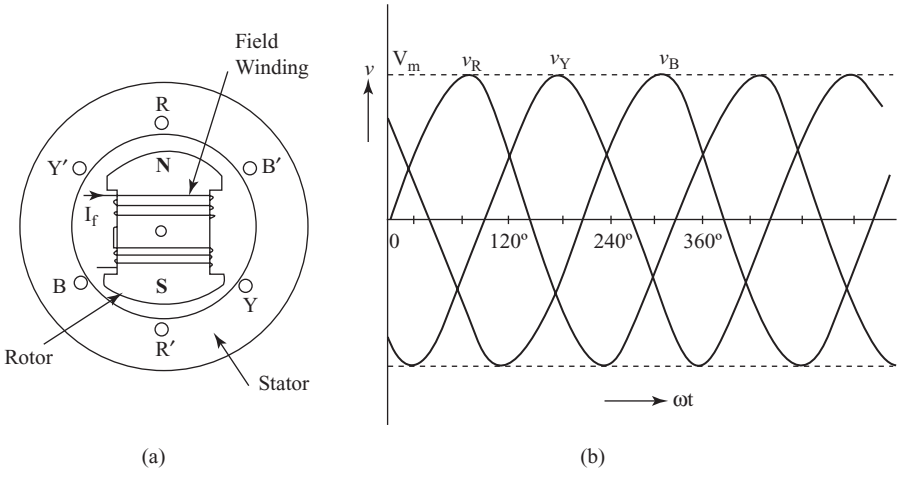

Figure 5.4 *(a) Three-phase two-pole generator; (b) three-phase voltages induced in the windings*

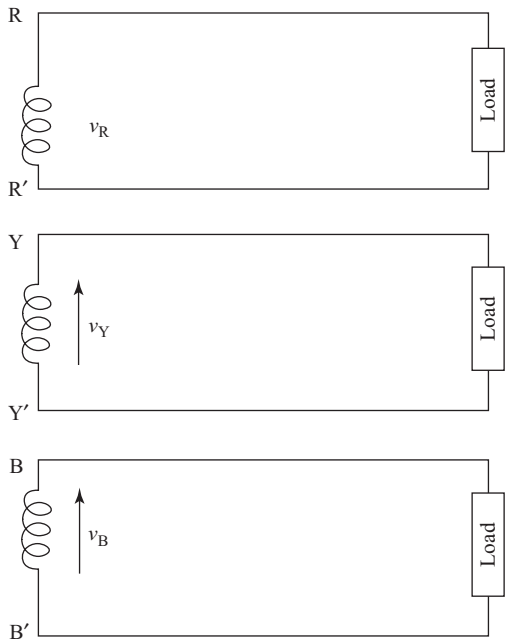

Figure 5.5 *The three-phase windings are connected to the load independently through six wires*

in a sequence. The emfs induced in these coils are sinusoidal in nature because of the nature of flux distribution. The voltage induced in the three-phase windings will be identical in nature but they will be displaced in time-phase by 120° as has been shown. The order in which the phase voltages attain their maximum value or peak value, V_m is called the phase sequence. If the rotor rotates in the clockwise direction, voltages in the phases will be induced in the sequence, V_R, V_Y, V_B. If the rotor poles rotate in the opposite direction the phase sequence of the induced voltage will change from RYB to RBY.

The phase voltages should supply power to electrical loads for which the two end terminals of each phase are to be connected to loads as shown in Fig. 5.5. Six wires are required to be taken out from the generator to the load. Instead of taking out six wires from the generator and connecting them to the loads separately as shown in Fig. 5.5, the three-phase windings are connected either in star or in delta so that only three wires are to be taken out from the generator to the load. The loads are also connected either in star or in delta. In the case of the star connection a fourth wire may be taken out from the neutral point.

5.4 TERMS USED IN THREE-PHASE SYSTEMS AND CIRCUITS

These are some of the terms used while describing a three-phase system. These are as follows:

(i) *Balanced supply*: a set of three sinusoidal voltages (or currents) that are equal in magnitude but has a phase difference of 120º, constitute a balanced three-phase voltage (or current) system.

(ii) *Unbalanced supply*: a three-phase system is said to be unbalanced when either of the three-phase voltages are unequal in magnitude or the phase angle between the three phases is not equal to 120º.

(iii) *Balanced load:* if the load impedances of the three phases are identical in magnitude as well as phase angle, then the load is said to be balanced. It implies that the load has the same value of resistance R and reactance X_L and/or X_C in each phase.

(iv) *Unbalanced load:* if the load impedances of the three phases are neither identical in magnitude nor in phase angle then the load is said to be unbalanced.

(v) *Single phasing:* when one phase of the three-phase supply is not available then the condition is called single phasing.

(vi) *Phase sequence:* the order in which the maximum value of voltages of each phase appear is called the phase sequence. It can be RYB or RBY.

(vii) *Coil:* a coil is made of conducting wire, say copper, having an insulation cover. A coil can be of a single turn or many number of turns. Normally a coil will have a number of turns. A single turn of a coil will have two conductors on its two sides called coil sides

(viii) *Winding:* a number of coils are used to make one winding. Normally the winding coils are connected in series. One winding forms one phase.

(ix) *Symmetrical system:* in a symmetrical three-phase system the magnitude of three-phase voltages is the same but there is a time phase difference of 120^0 between the voltages.

5.5 THREE-PHASE WINDING CONNECTIONS

A three-phase generator will have three-phase windings. These phase windings can be connected in two ways:

1. Star connection
2. Delta connection

5.5.1 Star Connection

The star connection is formed by connecting the starting or finishing ends of all the three windings together. A fourth conductor which is taken out of the star point is called the neutral point. The remaining three ends are brought out for connection to load. These ends are generally referred to as R–Y–B, to which load is to be connected. The star connection is shown in Fig. 5.6 (a). This is a three-phase,

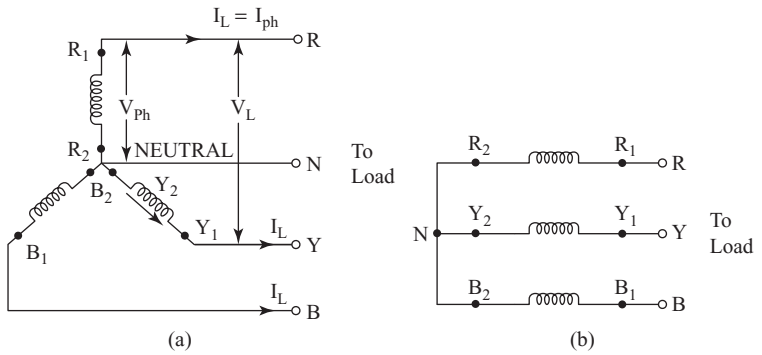

Figure 5.6 *Star connection of phase windings: (a) three-phase four-wire system; (b) three-phase three-wire system*

four-wire star-connected system. If no neutral conductor is taken out from the system it gives rise to a three-phase, three-wire star-connected system.

The current flowing through each line conductor is called line current, I_L. In the star connection the line current is also the phase current. Similarly, voltage across each phase is called phase voltage, V_{Ph}. Voltage across any two line conductors is called line voltage, V_L. When a balanced three-phase load is connected across the supply terminals R,Y,B currents will flow through the circuit. The sum of these currents, i.e., I_R, I_Y, and I_B will be zero. The neutral wire connected between the supply neutral point and the load neutral point will carry no current for a balanced system.

5.5.2 Delta Connection

The delta connection is formed by connecting the end of one winding to the starting end of the other and connections are continued to form a closed loop. In this case, the current flowing through each

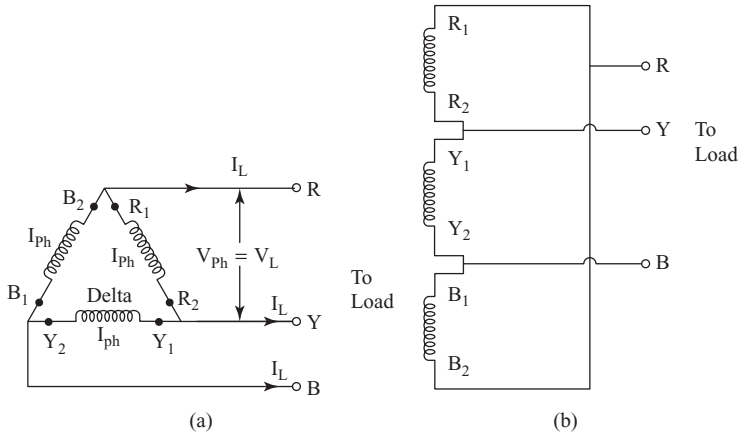

Figure 5.7 *Delta connection of three-phase windings: (a) three-phase three-wire delta connection; (b) connection scheme of windings forming a delta*

line conductor is called line current I_L and the current flowing through each phase winding is called phase current, I_{Ph}. However, we find that the phase voltage is the same as the line voltage in a delta connection. The delta connection of windings has been shown in Fig. 5.7.

5.5.3 Relationship of Line and Phase Voltages and Currents in a Star-connected System

Consider the balanced star-connected system as shown in Fig. 5.8.

Suppose load is inductive and, therefore, current will lag the applied voltage by angle ϕ. Consider a balanced system so that the magnitude of current and voltage of each phase will be the same.

i.e., Phase voltages, $V_R = V_Y = V_B = V_{Ph}$

Line current, $I_R = I_Y = I_B = I_L$

Line voltage, $V_L = V_{RY} = V_{YB} = V_{BR}$

Phase current, $I_{Ph} = I_R = I_Y = I_B$

$I_L = I_{PH}$ for star connection as the same phase current passes through the lines to the load.

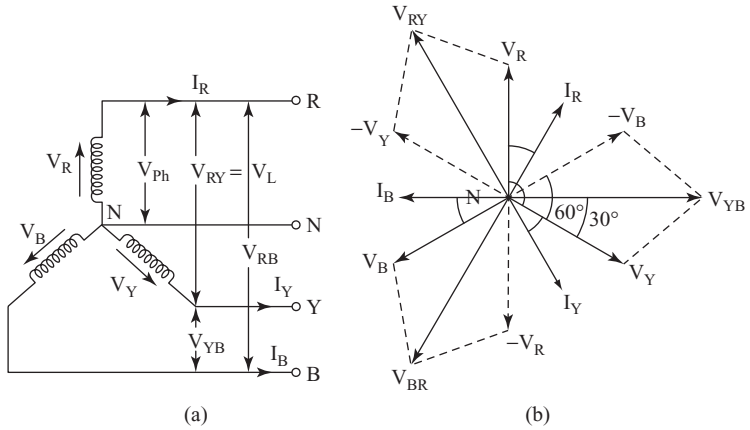

Figure 5.8 *(a) Balanced star-connected system; (b) phasor diagram*

To derive the relation between V_L and V_{Ph}, consider line voltage V_{RY}.

$$V_{RY} = V_{RN} + V_{NY}$$

$$V_{RY} = V_{RN} + (-V_{YN})$$

Similarly $$V_{YB} = V_{YN} + (-V_{BN})$$

and $$V_{BR} = V_{BN} + (-V_{RN})$$

The procedure for drawing the phasor diagram of Fig. 5.8 (b) is as follows.

Draw three phasors V_R, V_Y, and V_B representing the phase voltages. These voltages are of equal magnitude but displaced by 120º. The line voltage phasors, V_{RY}, V_{YB}, V_{BR} are drawn by vectorially adding the phase voltages. For example, to draw line voltage V_{RY} we have to add the phase voltages as

$$V_{RY} = V_{RN} + V_{NY} = V_{RN} + (-V_{YN})$$

The phasor V_{YN} is obtained by reversing V_{NY}. V_{RY} is obtained by vectorially adding V_{RN} and V_{YN} as has been shown in Fig. 5.8 (b). Similarly the other line voltages have been drawn. The phase currents I_R, I_B, and I_Y have been shown lagging the phase voltages by the power factor angle ϕ.

From the phasor diagram shown in Fig. 5.6 (b), the phase angle between phasors V_R and $(-V_Y)$ is 60°.

$\therefore$ $$V_{RY} = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60^\circ}$$

$$V_{RY} = V_L = \sqrt{V_{Ph}^2 + V_{Ph}^2 + 2V_{Ph} V_{Ph} \cdot \frac{1}{2}}$$

$$V_L = \sqrt{3}\, V_{Ph}$$

$$V_L = \sqrt{3}\, V_{Ph} \qquad (5.4)$$

Thus, for the star-connected system
Line Voltage = $\sqrt{3}$ × Phase voltage
Line Current = Phase current

Power: Power output per phase $= V_{Ph} I_{Ph} \cos \phi$

Total power output $= 3 V_{Ph} I_{Ph} \cos \phi$

$$= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos \phi$$

$$P = \sqrt{3}\ V_L\ I_L \cos \phi \tag{5.5}$$

Power = $\sqrt{3}$ × line voltage × line current × power factor

5.5.4 Relationship of Line and Phase Voltages and Currents in a Delta-connected System

Consider the balanced delta-connected system as shown in Fig. 5.9.

In a delta-connected system the voltage aeros the winding, i.e., the phases is the same as that across the line terminals. However, the current through the phases is not the same as through the supply lines.

Therefore, in the case of the delta-connected circuit, phase voltage is equal to the line voltage, but line current is not equal to phase current.

Line voltage $V_L = V_{RY} = V_{YB} = V_{BR}$

Line current $I_L = I_R = I_Y = I_B$

Phase voltage $V_{Ph} = V_{RY} = V_{YB} = V_{BR}$

Phase current $I_{Ph} = I_{RY} = I_{YB} = I_{BR}$

$\therefore V_{Ph} = V_L$ for delta-connected load. (5.6)

In Fig. 5.9 (a) is shown a three-phase delta-connected supply system connected to a three-phase delta-connected load. The line currents are I_R, I_Y, and I_B, respectively. The phase currents are I_{RY}, I_{YB}, and I_{BR}. The phasor diagram in Fig. 5.9 (b) has been developed by first showing the three-phase voltages V_{YB}, V_{BR}, and V_{RY} of equal magnitude but displaced by 120° from each other. Then the phase currents I_{YB},

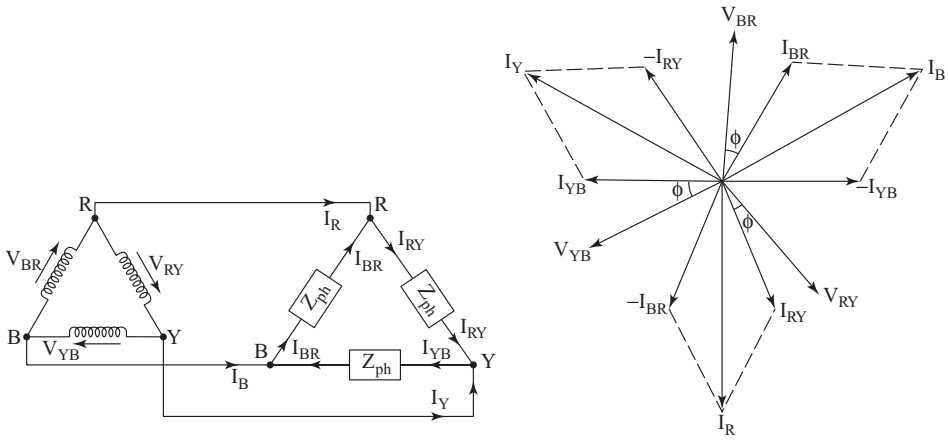

Figure 5.9 *Delta-connected system: (a) a three-phase delta-connected load supplied from a delta-connected supply source; (b) phasor diagram of voltages and currents*

I_{BR}, and I_{RY} have been shown lagging respective phase voltages by power factor angle ϕ. The line currents are drawn by applying KCL at the nodes R, Y, and B and adding the phasors, as has been shown.

To derive the relation between I_L and I_{Ph}, apply KCL at node R, as shown in Fig. 5.9 (b)

$$I_R + I_{BR} = I_{RY}$$

$$\therefore \quad I_R = I_{RY} - I_{BR}$$

Similarly at node Y and B, we can write

$$I_Y = I_{YB} - I_{RY}$$

$$I_B = I_{BR} - I_{YB}$$

Since phase angle between phase currents I_{RY} and $-I_{BR}$ is 60°

$$\therefore \quad I_R = \sqrt{I_{RY}^2 + I_{BR}^2 + 2I_{RY} I_{BR} \cos 60°}$$

$$I_R = I_L = \sqrt{I_{Ph}^2 + I_{Ph}^2 + 2I_{Ph} I_{Ph} \cdot \frac{1}{2}}$$

$$I_L = \sqrt{3} \cdot I_{ph}$$

Thus, for a three-phase delta-connected system,

Line current = $\sqrt{3} \times$ Phase current

Line voltage = Phase voltage

Power: Power output per phase $= V_{Ph} I_{Ph} \cos \phi$

Total power output $= 3V_{Ph} I_{Ph} \cos \phi$

$$= 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos \phi$$

$$\sqrt{3} V_L I_L \cos \phi$$

$$\text{Power} = \sqrt{3} \cdot \text{Line voltage} \cdot \text{Line current} \cdot \text{Power factor}$$

For both star-connected and delta-connected systems, the total power P is

$$P = \sqrt{3}\, V_L\, I_L \cos \phi \tag{5.7}$$

If per phase power is P_h and total power is P_T,

then

$$P_T = 3\, P_h \tag{5.8}$$

5.6 ACTIVE AND REACTIVE POWER

The in-phase component of I_{Ph} along V has been shown in Fig. 5.10 as $I_{Ph} \cos \phi$ and the perpendicular component as $I_{Ph} \sin \phi$. If we multiply all the sides of the triangle ABC by V_{Ph}, the triangle becomes a power triangle where $AB = V_{Ph} I_{Ph} \cos \phi$ is called the *active power*, $BC = V_{Ph} I_{Ph} \sin \phi$ is called the *reactive power*, and $V_{Ph} I_{Ph}$ is called the *apparent power*.

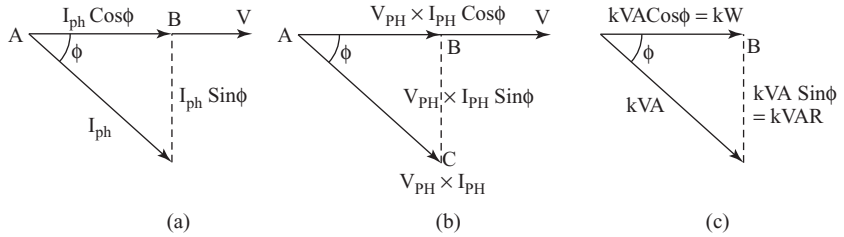

Figure 5.10 *Relationship between active power, reactive power, and apparent power*

Apparent power = kVA (5.9)

Apparent power × cos ϕ = Active power = kW (5.10)

Apparent power × sin ϕ = Reactive power = kVAR (5.11)

Multiplying all the sides of the power triangle by 10^3, i.e., expressing the power in terms of 'kilo', the power triangle is redrawn as has been shown in Fig. 5.8 (c) kVA is kilo Volt Ampere.

$$kVA \cos\phi = kW \text{ (kilo Watt)} \quad (5.12)$$

$$kVA \sin\phi = kVAR \text{ (kilo Var)} \quad (5.13)$$

5.7 COMPARISON BETWEEN STAR CONNECTION AND DELTA CONNECTION

As mentioned earlier, the three windings of a generator can be connected either in star or in delta. Same is the case with transformers. Three-phase electrical loads and the windings of three-phase motors can also be connected in star or in delta.

The relationship between voltages, currents and their phase relationship along with some other related factors have been compared and are presented in a tabular form.

Star Connection	Delta Connection
1. Line current is the same as phase current, i.e., $I_L = I_{Ph}$	1. Line current is $\sqrt{3}$ × the phase current, i.e., $I_L = \sqrt{3}\, I_{Ph}$
2. Line voltage is $\sqrt{3}$ the phase voltage, i.e., $V_L = \sqrt{3}\, V_{Ph}$	2. Line voltage is the same as phase voltage, i.e., $V_L = V_{ph}$
3. Total power = $\sqrt{3}\ V_L\, I_L \cos\phi$	3. Total power = $\sqrt{3}\ V_L\, I_L \cos\phi$
4. Per phase power = $V_{Ph}\, I_{Ph} \cos\phi$	4. Per phase power = $V_{Ph}\, I_{Ph} \cos\phi$
5. Three-phase three-wire and three-phase four-wire systems are possible	5. Three-phase three-wire system is possible
6. Line voltages lead the respective phase voltages by 30°	6. Line currents lag the respective phase currents by 30°

Example 5.1 A 400 V, three-phase, 50 Hz power supply is applied across the three terminals of a delta-connected three-phase load. The resistance and reactance of each phase is 6 Ω and 8 Ω, respectively. Calculate the line current, phase current, active power, reactive power, and apparent power of the circuit.

Solution:

The load is delta connected. Hence

$$V_{Ph} = V_L = 400 \text{ V}$$

$$Z_{Ph} = R + jX = 6 + j8 = \sqrt{6^2 + 8^2} \angle \tan^{-1}\frac{8}{6} = 10\angle 53° \ \Omega$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{400\angle 0°}{10\angle 53°} = 40\angle -53° \text{ A}$$

Power factor, $\cos\phi = \cos 53° = 0.6$ lagging

$$I_L = \sqrt{3}\, I_{Ph} = 1.732 \cdot 40 = 69.28 \text{ A}$$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{6}{10} = 0.6$ lagging

$\sin\phi = 0.8$

Active Power $= \sqrt{3}\, V_L\, I_L \cos\phi = 1.732 \times 400 \times 69.28 \times 0.6$

$= 28798 \text{ W} = 28.798 \text{ kW}$

$\simeq 28.8 \text{ kW}$

Reactive Power $= \sqrt{3}\, V_L\, I_L \sin\phi = 1.732 \times 400 \times 69.28 \times 0.8$

$= 38397 \text{ VAR} = 38.397 \text{ kVAR}$

$\simeq 38.4 \text{ kVAR}$

Apparent Power $= 3\, V_{Ph}\, I_{Ph} = \sqrt{3}\, V_L\, I_L$

$= 1.732 \times 400 \times 69.28$

$= 47997 \text{ VA} = 47.997 \text{ kVA}$

$\simeq 48 \text{ kVA}$

The power triangle is shown in Fig. 5.11.

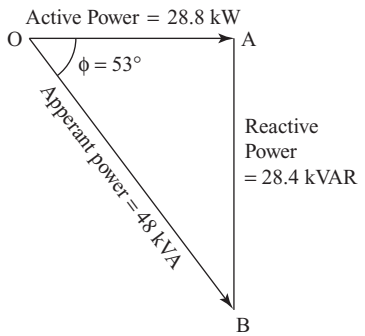

Figure 5.11

Apparent Power in $$\text{kVA} = \sqrt{(\text{Active Power})^2 + (\text{Reactive Power})^2}$$
$$= \sqrt{(28.8)^2 + (38.4)^2}$$
$$= \sqrt{829.44 + 1474.56}$$
$$= \sqrt{2304} = 48$$

Example 5.2 A balanced star-connected load of (8 + j6) Ω per phase is connected to a balanced three-phase, 400 V supply. Find the line current, power factor, power, and total volt-amperes.

Solution:

Phase voltage, $$V_P = \frac{\text{Line Voltage, } V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\,\text{V}$$

Impedence per phase, $$Z_P = \sqrt{R^2 + X_L^2} = \sqrt{8^2 + 6^2} = 10\,\Omega$$

Phase current, $$I_P = \frac{V_P}{Z_P} = \frac{231}{10} = 23.1\ \text{A}$$

Line current $$I_L = I_P = 23.1\ \text{A}$$

Power factor $$\cos\phi = \frac{R}{Z} = \frac{8}{10} = 0.8\,(\text{lagging})$$

Total power, $$P = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$= \sqrt{3} \cdot 400 \cdot 23.1 \cdot 0.8$$

$$= 12{,}800\ \text{W}$$

Total volt amperes $$= \sqrt{3}\, V_L\, I_L$$

$$= \sqrt{3} \cdot 400 \cdot 23.1 = 16{,}000\ \text{VA}$$

Example 5.3 A three-phase four-wire supply system has a line voltage of 400 V. Three non-inductive loads of 16 kW, 8 kW, and 12 kW are connected between R, Y, and B phases and the neutral, respectively. Calculate the current flowing through the neutral wire.

Solution:

Loads connected between the different phases and the neutral are of 16 kW, 8 kW, and 12 kW, respectively.

The current through the neutral wire line is the phasor sum of all the line currents. We will first calculate the line currents and then add them vectorially.

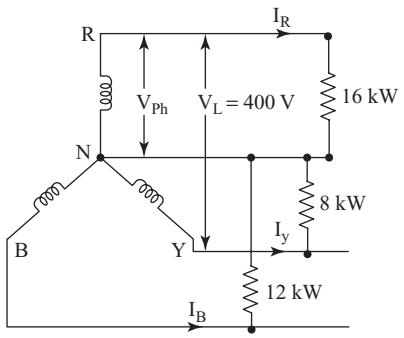

Figure 5.12

For star connection, $$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{VB} = 231\,V$$

The three-phase voltages are equal but have a phase difference of 120° between them.

$$V_R = 231\angle 0°, V_Y = 231\angle -120°, \text{and } V_B = 231\angle -240°$$

$$I_R = \frac{10 \cdot 1000}{V_R} = \frac{16 \cdot 1000}{231\angle 0} = 69.3\angle 0°$$

$$I_Y = \frac{8 \times 1000}{V_Y} = \frac{8 \times 1000}{231\angle -120°} = 34.6\angle 120°$$

$$I_B = \frac{12 \times 1000}{V_B} = \frac{12 \times 1000}{231\angle -240°} = 52\angle 240°$$

Current through the neutral wire, I_N is

$$\begin{aligned} I_N &= I_R + I_Y + I_B \\ &= 69.3\angle 0° + 34.6\angle 120° + 52\angle 240° \\ &= 69.3(\cos 0° + j \sin 0°) + 34.6(\cos 120° + j \sin 120°) \\ &\quad + 52(\cos 240° + j \sin 240°) \\ &= 69.3(1 + j0) + 34.6(-0.5 + j0.866) \\ &\quad + 52(-0.5 + j0.866) \\ I_N &= 69.3 - 17.3 + j\,30 - 26 + j\,45 \\ &= 26 + j\,75 \\ |I_N| &= \sqrt{26^2 + 75^2} = \sqrt{6301} \\ &= 79.4\text{ A} \end{aligned}$$

Example 5.4 A three-phase load has a resistance and reactance of 6 Ω each for all the three phases. The load is connected in star. A 400 V, 50 Hz, three-phase supply is connected across the load. Calculate phase voltage, phase current, power factor, power consumed per phase, and the total power consumed by the load.

Solution:

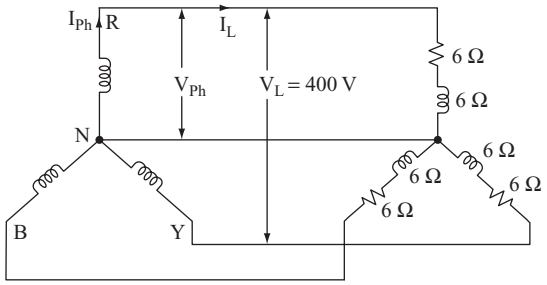

Figure 5.13

$$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$Z_{Ph} = 6 + j\,6\,\Omega = 8.48 \angle 45^\circ \ \Omega$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{231 \angle 0^\circ}{8.48 \angle 45^\circ} = 27.2 \angle -45^\circ \text{ A}$$

$$I_{Ph} = I_L = 27.2 \text{ Aw}$$

Angle between V_{Ph} and I_{Ph} is 45°.

Power factor = cos 45° = 0.7 lagging

Power absorbed by each phase of the load = $V_{Ph}\, I_{Ph} \cos \phi$

$= 231 \times 27.2 \times 0.7$

$= 4398$ W

(Total power consumed = $3 \times 4398 = 13194$ W

Example 5.5 A 400 V, 50 Hz, three-phase supply is provided to a three-phase star-connected load. Each phase of the load absorbs a power of 2000 W. The load power factor is 0.8 lagging. Calculate the total power supplied to the load; the line current.

Solution:

Power consumed by each phase = 2000 W

Power consumed by all the three phases = 3×2000 W

$= 6000$ W

Total power supplied = total power consumed.

$$V_L = 400 \text{ V}, \ V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

Power consumed per phase $= V_{Ph}\, I_{Ph} \cos \phi$

$$V_{Ph}\, I_{Ph} \cos \phi = 2000$$

$$I_L = I_{Ph} = \frac{2000}{231 \cdot \ 0.8} = 10.82 \text{ A}$$

Example 5.6 A 400 V, 50 Hz, three-phase supply is provided to a three-phase delta-connected load. The resistance and inductance of each phase of the load is 8 Ω and 0.04 H, respectively. Calculate the phase current and the line current drawn by the load. Also calculate the total power consumed.

Solution:

The impedance of load per phase,

$$Z_{Ph} = R + j\,\omega L = 8 + j\,2\pi \times 50 \times 0.04$$
$$= 8 + j\,12.56\ \Omega.$$

Since the load is delta connected, $V_{Ph} = V_L = 400$ V

$$I_{ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{400}{8 + j\,12.56} = \frac{400\angle 0}{14.89\angle \tan^{-1}(12.56/8)} = 26.86\angle -58^\circ;$$
$$I_L = \sqrt{3}\ I_{ph} = 1.73 \times 26.86 = 36\ \text{A}$$

Total power consumed $= 3\ V_{Ph}\ I_{Ph} \cos\phi$

$$= 3 \times 400 \times 26.86 \cos\phi\ \text{W}$$
$$= 3 \times 400 \times 26.86 \times 0.52 = 1736\ \text{W}$$

Example 5.7 A three-phase star-connected load consumes a total of 12 kW at a power factor of 0.8 lagging when connected to a 400 V, three-phase, 50 Hz power supply. Calculate the resistance and inductance of load per phase.

Solution:

Total power consumed = 12 kW

Per phase power consumed = 4 kW

So, $$V_{Ph}\ I_{Ph} \cos\phi = 4000\ \text{W}$$

$$I_{ph} = \frac{4000}{V_{ph} \cos\phi}$$

or, $$I_{Ph} = \frac{4000}{(400/\sqrt{3}) \cdot\ 0.8} = 21.6\ \text{A}$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}}$$

$$Z_{Ph} = \frac{V_{Ph}}{I_{Ph}} = \frac{231}{21.6} = 10.7\ \Omega.\ \text{Power factor},\ \cos\phi = 0.8,\ \sin\phi = 0.6$$

$$R = Z_{Ph} \cos\phi = 10.7 \times 0.8 = 8.56\ \Omega$$

$$X = Z_{Ph} \sin\phi = 10.7 \times 0.6 = 6.42\ \Omega$$

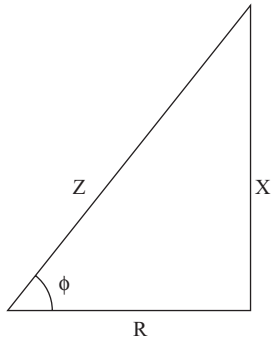

Figure 5.14

$$X = 6.42$$

$$2\pi fL = 6.42$$

$$L = \frac{6.42}{2 \times 3.14 \times 50} = 20.4 \times 10^{-3} \text{ H}$$

$$= 20.4 \text{ mH}$$

Example 5.8 A balanced three-phase star-connected load of 8 + j6 Ω per phase is supplied by a 400 V, 50 Hz supply. Calculate the line current, power factor, active, and reactive power.

Solution:

$$V_L = 400 \text{ V, For star connection, } V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}}$$

$$Z_{Ph} = 8 + j6 = \sqrt{8^2 + 6^2} \angle \tan^{-1} \frac{6}{8} = 10 \angle 37°$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{231}{10 \angle 37°} = 23.1 \angle -37°$$

For star connection,

$$I_{Ph} = I_L = 23.1 \text{ A}$$

Angle of lag of I_{Ph} with V_{Ph} is 37°

Power factor $= \cos \phi = \cos 37° = 0.8$ lagging

Active power $= \sqrt{3}\, V_L\, I_L \cos \phi$

$= 1.732 \times 400 \times 23.1 \times 0.8$

$= 12802 \text{ W} = 12.802 \text{ kW}$

Reactive power $= \sqrt{3}\, V_L\, I_L \sin \phi$

$= 1.732 \times 400 \times 23.1 \times 0.6$

$= 9602 \text{ VAR} = 9.602 \text{ kVAR}$

Example 5.9 A delta-connected three-phase motor load is supplied from a 400 V, thee-phase, 50 Hz supply system. The line current drawn is 21 A. The input power is 11 kW. What will be the line current and power factor when the motor windings are delta connected?

Solution:

Line voltage, $V_L = 400$ V, $V_{Ph} = V_L$ for delta connection

Line current, $$I_L = \sqrt{3}\, I_{Ph} = 21\text{A}$$

Impedance of each winding, $$Z_{Ph} = \frac{V_{Ph}}{I_{Ph}} = \frac{400}{I_L/\sqrt{3}} = \frac{400 \times \sqrt{3}}{21} = 33\ \Omega$$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3} \times 400 \times 21 \times \cos\phi$$

or, $$\cos\phi = \frac{P}{\sqrt{3} \times 400 \times 21} = \frac{11 \times 1000}{\sqrt{3} \times 400 \times 21} = 0.756$$

When the motor windings are star connected

$$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{1.732} = 231\ \text{V}$$

Z_{Ph} will remain the same as the same windings are connected in star

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{231}{33} = 7\,\text{A}$$

In star connection, the line current is the same as phase current, hence, $I_L = I_{Ph} = 7$ A.

Power factor, depends on the circuit parameters

$$\cos\phi = \frac{R}{Z}$$

Since both R and Z remain unchanged, the power factor will remain the same at 0.756. Students may note that line current in a star connection is one-third of the line current in a delta connection.

Example 5.10 A balanced star-connected load of 4 + j6 Ω per phase is connected across a 400 V, three-phase, 50 Hz supply. Calculate line current, phase current, line voltage, phase voltage, power factor, total power, and reactive power.

Solution:

$$Z/_{Ph} = 4 + \text{j}6 = 7.21\angle 56°$$

For star connection, $$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\ \text{V}$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{231\angle 0}{7.21\angle 56°} = 32\angle -56°$$

Power factor, $\cos \phi = \cos 56° = 0.56$ lagging

$I_{Ph} = 32$ A

$I_{Ph} = I_L = 32$ A

$V_L = 400$ V

$V_{Ph} = 231$ V

Total Power $= \sqrt{3} V_L I_L \cos\phi$

$= 3 V_{Ph} I_{Ph} \cos \phi$

$= 3 \times 231 \times 32 \times 0.56$

$= 12418$ W

$= 12.418$ kW

Total reactive power $= \sqrt{3} V_L I_L \sin\phi$

$= 3 V_{Ph} I_{Ph} \sin \phi$

$= 3 \times 231 \times 32 \times 0.83$

$= 18406$ VAR

$= 18.406$ kVAR

5.8 MEASUREMENT OF POWER IN THREE-PHASE CIRCUITS

We have known that in dc circuits power is measured as the product of voltage and current, i.e., power, P = VI. DC power can be measured using a voltmeter and an ammeter. In ac circuits power, P = VI cos ϕ. In three-phase ac circuits, total power is three times the power per phase. Wattmeter is an instrument used for measurement of power in ac circuits. Wattmeters are available as single-phase wattmeters and three-phase wattmeters. Single-phase wattmeters can be used to measure three-phase power. In case of star-connected balanced load with neutral connection, only one single-phase wattmeter can be used to measure the three-phase power. The three-phase power is three times the single-phase power. For unbalanced three-phase loads, i.e., if the currents in the three phases are not the same, two wattmeters are to be used to measure the three-phase power. These methods are described in the following sections.

5.8.1 One-wattmeter Method

In this method, only one single-phase wattmeter can be used to measure the total three-phase power. In this method, the current coil (CC) of the wattmeter is connected in series with any phase and the pressure coil (PC) is connected between that phase and the neutral as shown in Fig. 5.15. One-wattmeter method has a demerit that even a slight degree of unbalance in the load produces a large error in the measurement. In this method one wattmeter will measure only the power of one phase. Hence, total power is taken as three times the wattmeter reading.

$\therefore$ Total Power $= 3 \times V_{ph} I_{ph} \cos\phi$

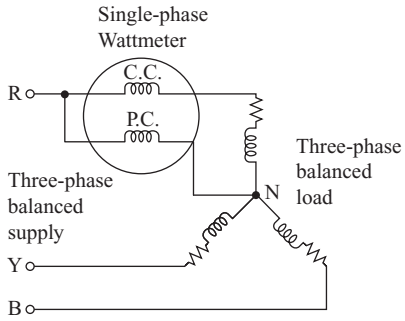

Figure 5.15 *One-wattmeter method of measuring power of a star-connected balanced three-phase load*

5.8.2 Two-wattmeter Method

This method requires only two wattmeters to measure three-phase power for balanced as well as unbalanced loads. In this method two wattmeters are connected in two phases and their pressure coils are connected to the remaining third phase as has been shown in Fig. 5.16.

This method of measurement is useful for balanced and unbalanced loads.

Let us consider the measurement of three-phase power of a star-connected load using two single-phase wattmeters as has been shown in Fig. 5.17(a). We will calculate the power measured by the two wattmeters separately. Let W_1 and W_2 respectively be the two wattmeter readings. Current flowing through the current coil of wattmeter W_1 is I_R. The voltage appearing across its pressure coil is V_{RB}. The wattmeter reading will be equal to, $W_1 = V_{RB}\, I_R$. cos of angle between V_{RB} and I_R. Similarly, the wattmeter reading W_2 will be equal to, $W_2 = V_{YB}\, I_B$ cos of angle between V_{YB} and I_B. We will now draw the phasor diagram, and calculate W_1 and W_2.

From the phasor diagram as shown in Fig. 5.17 (b),

$$W_1 = V_{RB}\, I_L \cos(30-\phi) = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30-\phi) = V_L I_L \cos(30-\phi) \tag{5.14}$$

And

$$W_2 = V_{YB}\, I_Y \cos(30+\phi) = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30+\phi) = V_L I_L \cos(30+\phi) \tag{5.15}$$

We know that the total power in a three-phase circuit is $3V_{ph}\, I_{ph} \cos\phi$ or equal to $\sqrt{3}\, V_L\, I_L \cos\phi$

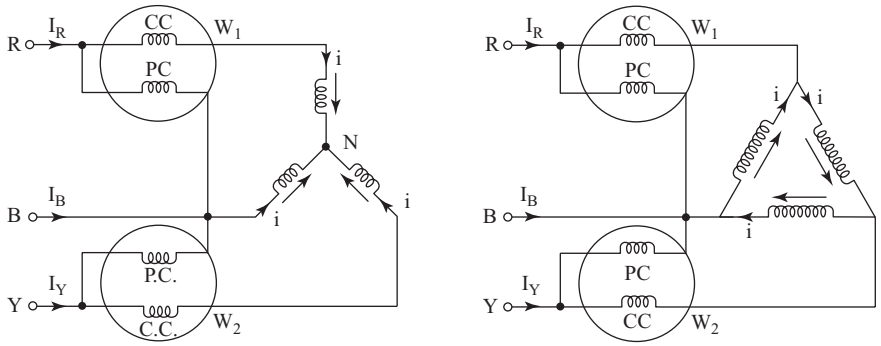

Figure 5.16 *Two-wattmeter method of measuring power for star- and delta-connected load*

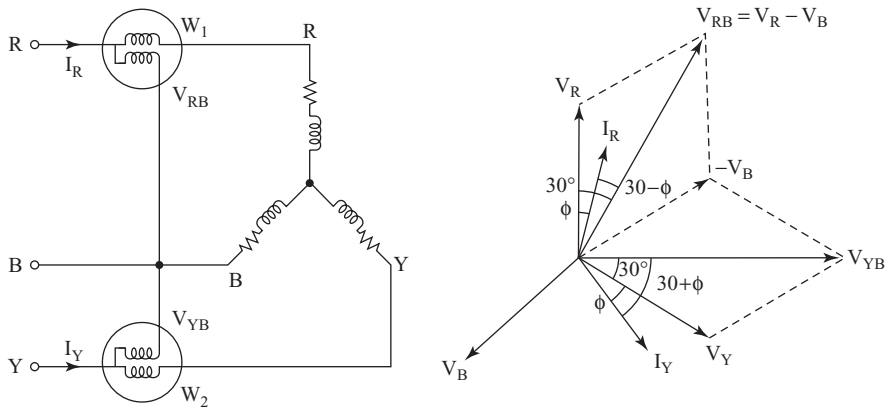

Figure 5.17 *(a) Measurement of three-phase power using two single-phase wattmetters; (b) phasor diagram*

Let us add the two wattmeter readings, i.e., W_1 and W_2.

$$W_1 + W_2 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30 - \phi) + \sqrt{3}\, V_{ph}\, I_{ph} \cos(30 + \phi)$$

$$= \sqrt{3}\, V_{ph}\, I_{ph} [\cos(30 - \phi) + \cos(30 + \phi)]$$

$$= \sqrt{3}\, V_{ph}\, I_{ph}\, 2 \cos\phi \cos 30°$$

$$= \sqrt{3}\, V_{ph}\, I_{ph}\, 2 \cos\phi \frac{\sqrt{3}}{2}$$

$$= 3 V_{ph}\, I_{ph} \cos\phi$$

or,
$$W_1 + W_2 = \sqrt{3}\, V_L\, I_L \cos\phi \tag{5.16}$$

Thus, it is proved that the sum of the two wattmeter readings is equal to the three-phase power.

Now, let us see what we get when the two wattmeter readings are subtracted from each other

$$W_1 - W_2 = \sqrt{3}\, V_{ph}\, I_{ph} [\cos(30° - \varphi) - \cos(30° + \varphi)$$

$$= \sqrt{3}\, V_{ph}\, I_{ph}\, 2 \sin\varphi \sin 30°$$

or,
$$\sqrt{3}(W_1 - W_2) = 3 V_{ph}\, I_{ph} \sin\varphi$$

or,
$$\sqrt{3}(W_1 - W_2) = \sqrt{3}\, V_L\, I_L \sin\varphi \tag{5.17}$$

Dividing eq. (5.17) by eq. (5.16)

$$\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3}\, V_L\, I_L \sin\phi}{\sqrt{3}\, V_L\, I_L \cos\phi} = \tan\phi$$

or,
$$\varphi = \tan^{-1} \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

Power factor, $$\cos\varphi = \cos\tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \quad (5.18)$$

Thus, from the two wattmeter readings, we can calculate the total active and reactive powers and the power factor of the circuit.

Effect of change of power factor on wattmeter readings

We will now study the effect of change of load power factor on the wattmeter readings. Let us rewrite the wattmeter readings as

$$W_1 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30 - \phi)$$

$$W_2 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30 + \phi)$$

We will consider a power factor of unity, 0.5, less than 0.5, and 0 and study the effect on the wattmeter readings.

(i) At unity power factor i.e., when $\cos\phi = 1$ i.e., $\phi = 0$

$$W_1 = \sqrt{3}\, V_{ph}\, I_{ph} \cos 30°$$

$$W_2 = \sqrt{3}\, V_{ph}\, I_{ph} \cos 30°$$

Thus at Power factor = 1, both the wattmeter readings will be positive and of equal value.

(ii) At 0.5 power factor, i.e., $\cos\phi = 0.5$ i.e., $\phi = 60°$.

$$W_1 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(-30°)$$

$$= \sqrt{3}\, V_{ph}\, I_{ph} \cos 30°$$

$$W_2 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30° + 60°) = 0$$

Thus, at power factor equal to 0.5, one of the wattmeters will give zero reading.

(iii) When the power factor is less than 0.5, i.e., when $\phi > 60°$. Let us observe the wattmeter readings.

$$W_1 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30 - \phi)$$

$$W_2 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30 + \phi)$$

When $\phi > 60$, W_1 will give positive readings but W_2 will give a negative reading.

Thus, for power factor less than 0.5, i.e., for $\phi > 60°$, one of the wattmeters will give a negative reading.

(iv) When load is purely inductive or capacitive, so power factor will be zero, i.e., $\phi = 90°$

$$W_1 = V_L\, I_L \cos(30° - 90°) = V_L\, I_L \cos 60°$$

$$W_2 = V_L\, I_L \cos(30° + 90°) = -V_L\, I_L \sin 30°$$

Both the wattmeters show equal but opposite readings. Hence, the total power consumed will be zero.

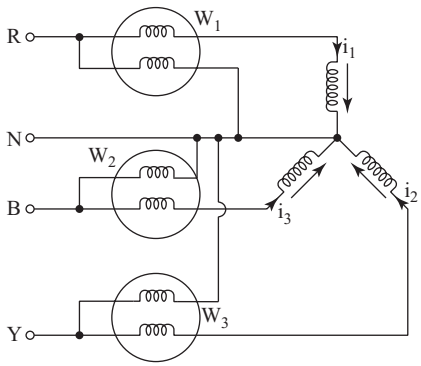

Figure 5.18 *Measurement of three-phase balanced or unbalanced power using three single-phase wattmeters*

5.8.3 Three-wattmeter Method

In this method, three wattmeters are used to measure three-phase power. Three wattmeters are connected in each phase as has been shown in Fig. 5.18, and their pressure coils are connected between each phase and the neutral. This method is valid for the measurement of three-phase power for balanced and unbalanced loads. The main drawback of this method is the requirement of three wattmeters.

Example 5.11 In the two-wattmeter method of power measurement for a three-phase load, the readings of the wattmeter are 1000 W and 550 W. What is the power factor of the load?

Solution:

$$W_1 = 1000 \text{ W}$$
$$W_2 = 550 \text{ W}$$

Power factor of load,
$$\cos\phi = \cos\tan^{-1}\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$
$$= \cos\tan^{-1}\sqrt{3}\,\frac{1000 - 550}{1000 + 550}$$
$$= \cos 26.695°$$
$$= 0.893 \text{ lagging}$$

Example 5.12 In the measurement of three-phase power by the two-wattmeter method, for a certain load, one of the wattmeters reads 20 kW and the other 5 kW after the current coil connection of one of the wattmeters has been reversed. Calculate the power and power factor of the load

Solution:

$$W_1 = 20 \text{ kW}$$
$$W_2 = -5 \text{ kW}$$
$$P = W_1 + W_2 = 20 - 5 = 15 \text{ kW}$$

Power factor of the load $= \cos\tan^{-1}\dfrac{W_1 - W_2}{W_1 + W_2}\sqrt{3}$

$= \cos\tan^{-1}\dfrac{20-(-5)}{20+(-5)}\sqrt{3}$

$= 0.3273$ lagging

Example 5.13 Draw the connection diagram for measurement of power in a three-phase star-connected load using the two-wattmeter method. In one such a measurement the load connected was 30 kW at 0.7 pf lagging. Find the reading of each wattmeter.

Solution:

The connection diagram for measurement of power in a three-phase Y-connected load using the two-wattmeter method has been shown in Fig. 5.16. We know that the reading of the two wattmeters will be $\sqrt{3}\ V_{ph}\ I_{ph} \cos(30 - \phi)$ and $\sqrt{3}\ V_{ph}\ I_{ph} \cos(30 + \phi)$, respectively. For star connection, $\sqrt{3}\ V_{ph} = V_L$ and $I_{ph} = I_L$.

The total load, $P = 30$ kW

Power factor, $\cos\phi = 0.7$ lagging

Phase angle, $\phi = \cos^{-1}(0.7) = 45.57°$ lagging

So, $V_L\ I_L = \dfrac{P \text{ in kW} \times 1000}{\sqrt{3}\cos\phi}$

$= \dfrac{30 \times 1000}{\sqrt{3} \times 0.7} = 24743.6$ VA

Reading of wattmeter, $W_1 = V_L\ I_L \cos(30 - \phi)$

$= 24743.6 \cos(30 - 45.57°)$

$= 23.835$ kW

Reading of wattmeter, $W_2 = V_L\ I_L \cos(30 + \phi)$

$= 24743.6 \cos(30 + 45.57°)$

$= 6.165$ kW

To check, total power, $P = W_1 + W_2 = 23.835 + 6.165$

$= 30$ kW

Example 5.14 A three-phase balanced load connected across a 3ϕ, 400 V ac supply draws a line current of 10 A. Two wattmeters are used to measure input power. The ratio of two wattmeter readings is 2:1. Find the readings of the two wattmeters.

Solution:

Let the ratio of wattmeter readings be X, i.e., $\dfrac{W_2}{W_1} = X$

and $$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

Now we will divide both numerator and dinominator by W_1. Then tan ϕ will be

$$= \sqrt{3}\left(\frac{1 - W_2 / W_1}{1 + W_2 / W_1}\right) = \sqrt{3}\left(\frac{1 - X}{1 - X}\right)$$

and power factor, $$\cos\phi = \frac{1}{\sec\phi} = \frac{1}{\sqrt{1 + \tan^2\phi}}$$

$$= \frac{1}{\sqrt{1 + 3[(1 - X)/(1 + X)]^2}}$$

Substituting $$\frac{W_2}{W_1} = \frac{1}{2} = 0.5$$

$$\cos\phi = \frac{1}{\sqrt{1 + 3(1 - 0.5 / 1 + 0.5)}} = 0.866$$

or, $$\phi = \cos^{-1}(0.866)$$

$$= 30°$$

Wattmeter reading, $W_1 = V_L I_L \cos(30° - 30°) = 400 \times 10 \times \cos 0° = 4000$ W

Wattmeter reading, $W_2 = V_L I_L \cos(30° + 30°) = 400 \times 10 \times \cos 60° = 1000$ W

Example 5.15 Three equal impedances, each consisting of R and L in series are connected in star and are supplied from a 400 V, 50 Hz, three-phase, three-wire balanced supply system. The power input to the load is measured by the two-wattmeter method and the two wattmeters read 3 kW and 1 kW, respectively. Determine the values of R and L connected in each phase.

Solution:

Reading of wattmeter 1, $W_1 = 3$ kW

Reading of wattmeter 2, $W_2 = 1$ kW

Total power $P = W_1 + W_2 = 3 + 1 = 4$ kW

Power factor of the circuit, $$\cos\phi = \cos\tan^{-1}\frac{w_1 - w_2}{w_1 + w_2}\sqrt{3}$$

$$= \cos\tan^{-1}\frac{3-1}{3+1}\sqrt{3}$$

$$= \cos 40.89$$

$$= 0.7559 \text{ lagging}$$

Line current, $$I_L = \frac{P}{\sqrt{3}\, V_L \cos\phi}$$

$$= \frac{4 \times 1000}{\sqrt{3} \times 400 \times 0.7559} = 7.64 \text{ A} = \text{Phase current, } I_p$$

Impedance of the circuit per phase $$Z = \frac{V_P}{I_P} = \frac{400\sqrt{3}}{7.64}$$

$$= 30.237\ \Omega$$

$\therefore$ $$R = Z \cos \phi = 30.237 \times 0.7559 = 22.856\ \Omega$$

Reactance per phase, $$X_L = \sqrt{Z^2 - R^2}$$

$$= \sqrt{(30.237)^2 - (22.856)^2}$$

$$= 19.796\ \Omega$$

Inductance per phase, $$L = \frac{X_L}{2\pi f}$$

$$= \frac{19.796}{2\pi \times 50}$$

$$= 0.063\ \text{H}$$

$$= 63\ \text{mH}$$

Example 5.16 The power input to a three-phase motor is measured by two single-phase wattmeters. The total input power has been measured as equal to 15 kW and the power factor calculated as 0.5. What have been the readings of the two wattmeters?

Solution:

Total power = $W_1 + W_2 = 15$ kW

We have to calculate W_1 and W_2

When we measure three-phase power by the two-wattmeter method, the readings of the two wattmeters are

$$W_1 = V_L I_L \cos (30 - \phi)$$

and $$W_2 = V_L I_L \cos (30 + \phi)$$

$$\cos \phi = 0.5,\ \phi = 60°$$

$$\sqrt{3}\, V_L I_L \cos \phi = 15\ \text{kW}$$

$$V_L I_L = \frac{15}{\sqrt{3} \times 0.5} = 17.3\ \text{kVA}$$

$$W_1 = V_L I_L \cos (30 - \phi)$$

$$= 17.3 \cos (30 - 60°)$$

$$= 17.3 \times 0.866 = 15\ \text{kW}$$

$$W_2 = V_L I_L \cos (30 + \phi)$$

$$= 17.3 \cos 90°$$

$$= 17.3 \times 0$$

$$= 0$$

Total power $$= W_1 + W_2 = 15 + 0 = 15\ \text{kW}$$

Thus, at a load power factor of 0.5, one of the wattmeters has given zero reading. This has been explained earlier under the effect of change of power factor on wattmeter readings.

5.9 REVIEW QUESTIONS

A. Short Answer Type Questions

1. What is the difference between a single-phase winding and a three-phase winding ?
2. Draw wave shapes of a three-phase supply.
3. What is the difference between a balanced load and an unbalanced load ?
4. Show that the phasor sum of the three-phase balanced voltages is zero.
5. What do you mean by phase sequence of three-phase voltages?
6. Distinguish between star connection and delta connection of three-phase windings.
7. Derive the relationship between line current, line voltage, phase current, and phase voltage in case of star and delta connection of three-phase windings.
8. Prove that the power in a three-phase circuit is equal to $\sqrt{3}\, V_L\, I_L \cos\phi$.
9. Distinguish between active power and reactive power in a three-phase system.
10. What is the significance of low power factor of any load on the system?
11. Draw the circuit diagram for measurement of three-phase power with two single-phase wattmeters.
12. At what value of load power factor the reading of one of the wattmeters, in the two-wattmeter method of measurement of three-phase power, will be zero?
13. What are the advantages of the three-phase system over the single-phase system?
14. Write the relationship between phase voltage and current in a delta-connected load.
15. Draw the connection diagram for three-phase resistive–inductive balanced load connected across a three-phase supply. Also draw the phasor diagram showing voltages and currents.

B. Numerical Problems

16. Three coils having same resistance and inductance are connected in star. A three-phase 400 V supply is connected across the three coils. The power consumed by each coil is 800 W and the load power factor is 0.8 lagging. What is the total power consumed by the coils. If now the coils are connected in delta across the supply what would be the total power consumed? Also calculate the line current when delta connected.

[Ans. 2400 W, 7200 W, 13 A]

17. A balanced three-phase star-connected load supplied from a 400 V, 50 Hz, three-phase supply system. The current drawn by each phase is $20 \angle -60°$ A. Calculate the line current, phase voltage, and total power consumed.

[Ans. 20 A, 230.94 V, 6928 W]

18. A delta-connected load has a resistance of 15 Ω and inductance of 0.03 H per phase. The supply voltage is 400 V, 50 Hz. Calculate line current, phase current, phase voltage, and total power consumed.

[Ans. 39.1 A, 22.5 A, 400 V, 22.94 kW]

19. Three identical coils of resistance 20 Ω and inductance 500 mH are connected first in star and then in delta across a 400 V, 50 Hz power supply. Calculate phase current, line current, phase voltage, and power consumed per phase.

[Ans. 1.46 A, 1.46 A, 230.94 V, 42.6 W; 2.53 A, 4.38 A, 400 V, 127.8 W]

20. Calculate the phase current and line current of a delta-connected load drawing 75 kW at 0.8 power factor from a 440 V, three-phase supply.

[Ans. 71 A, 122.97 A]

21. The power consumed by a three-phase balanced load has been measured by two single-phase wattmeters. The readings of the two wattmeters are 8.2 kW and 7.5 kW. Calculate the total power consumed and the load power factor.

[Ans. 15.7 kW, 0.997 lagging]

22. In the measurement of three-phase power by two single-phase wattmeters, it has been observed that the ratio of the two wattmeter readings is 3:1. What is the power factor of the load?

[Ans. 0.75]

23. Three identical coils are connected in star across a three-phase 415 V, 50 Hz supply. The total power drawn is 3 kW at a power factor of 0.3. Calculate the resistance and inductance of each coil.

[Ans. 5.16 Ω, 52.3 mH]

24. Two single-phase wattmeters are used to measure three-phase power. The readings of the two wattmeters are 2000 W and 400 W, respectively. Calculate the power factor of the circuit. What would be the power factor if the reading of the second wattmeter is negative?

[Ans. 0.65, 0.36]

25. Three identical coils each having a resistance of 10 Ω and inductive reactance of 10 Ω are first connected in star and then connected in delta across a 400 V, 50 Hz power supply. Calculate in each case the line current and the readings of two wattmeters connected for the measurement of power.

[Ans. 16.33 A, 49 A, 6309 W and 1690 W, 18931 W and 5072 W]

C. Multiple Choice Questions

1. Three-phase system is used
   (a) For transmission of electrical power
   (b) For generation of electrical power
   (c) For distribution of electrical power
   (d) For generation, transmission, and distribution of electrical power.
2. In a three-phase system the phase sequence indicates.
   (a) The amplitude of voltages
   (b) The order in which the voltages obtain their maximum values
   (c) The phase difference between the three voltages
   (d) The frequency in which the phase voltages are changing.
3. In a star-connected system the relationship between the phase and line quantities are
   (a) $V_{ph} = V_L$ (b) $V_{ph} = \sqrt{3}\, V_L$
   (c) $3I_{ph} = I_L$ (d) $I_{ph} = I_L$.
4. In a delta-connected system the relationship between the phase and line quanties are
   (a) $\sqrt{3}\, V_{ph} = V_L$ (b) $V_{ph} = \sqrt{3}\, V_L$
   (c) $I_{ph} = I_L$ (d) $I_L = \sqrt{3}\, I_{ph}$.
5. Line currents drawn by a three-phase star-connected bnalanced load is 12 A when connected to a balanced three-phase four-wire system. The neutral current will be
   (a) 36 A (b) 4 A
   (c) 0 A (d) 3 A.
6. Power in a balanced three-phase system circuit is
   (a) $3 V_p I_p \cos\phi$ (b) $3 V_L I_L \cos\phi$
   (c) $\sqrt{3}\, V_p I_p \cos\phi$ (d) $V_p I_p \cos\phi$.
7. Reactive power of a three-phase circuit is
   (a) $\sqrt{3}\, V_p I_p \sin\phi$ (b) $\sqrt{3}\, V_L I_L \sin\phi$
   (c) $\sqrt{3}\, V_L I_L \cos\phi$ (d) $\sqrt{3}\, V_L I_L$.
8. power in a single-phase ac circuit can be expressed as
   (a) $\sqrt{3}\, V_p I_p \cos\phi$ (b) $3 V_p I_p \cos\phi$
   (c) $V_p I_p \cos\phi$ (d) $V_p I_p \cos\phi$.
9. One single-phase wattmeter can be used to measure power in a three-phase circuit when
   (a) The load is balanced
   (b) The load is delta connected and is balanced
   (c) The load is balanced, star connected, and the neutral wire is available
   (d) The load is balanced and star connected.

10. A balanced three-phase sinusoidal power supply means
    (a) Three sinusoidal voltages of the same frequency and maximum value displaced in 120° time phase
    (b) Three sinusoidal voltages of any frequency but having the same maximum value
    (c) Three sinusoidal voltages of some frequency and maximum value with no time phase displacement between them
    (d) Three sinusoidal voltages of any value but having a time phase displacement of 120° between them.
11. An unbalanced three-phase supply system will have
    (a) Three unequal voltages
    (b) Three voltages having unequal time phase displacement between them
    (c) Three voltages of unequal magnitude and angular displacement among them
    (d) All of the above.
12. In the two-wattmeter method of measuring three-phase power, the reading of the two wattmeters will be equal when the power factor of the circuit is
    (a) 0 (b) 1
    (c) 0.5 (d) 0.866.
13. In the two-wattmeter method of measuring three-phase power, the reading of one of the wattmeters can be negative when the power factor angle is
    (a) More than 60° (b) Less than 60°
    (c) More than 30° (d) Less than 30°.
14. Four equal resistance of 100 Ω each connected in delta is supplied from 400 V three-phase star-connected supply, the line current drawn will be
    (a) 12 A (b) 4 A
    (c) 6.928 A (d) 13.856 A.

Answers to Multiple Choice Questions

1. (d)	2. (b)	3. (d)	4. (d)	5. (e)	6. (a)
7. (b)	8. (c)	9. (c)	10. (a)	11. (d)	12. (b)
13. (a)	14. (c)				

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6 Electric Power Systems

TOPICS DISCUSSED

- Generation of electricity
- Sources of energy for electricity generation
- Thermal power generation from fossil fuel
- Hydroelectric power-generating stations
- Nuclear power-generating stations
- Non-conventional or alternative generating stations
- Transmission and distribution of electricity
- Domestic wiring
- Circuit protective devices and safety precautions
- Efficient use of electricity

6.1 INTRODUCTION

We have known that energy is available in nature in different forms like in fossil fuel such as coal, gas, and oil; high-speed wind; falling water; sun rays, etc. All these forms of energy can be converted into electrical energy which can be transmitted to various places for use. Electrical energy is clean and easily controllable.

Electricity in large quantity is produced in power houses through a turbine–generator set. The turbines are rotated by pressurized steam or the potential energy of water. The generator is rotated by the turbine. The generator produces electricity which is sent for use to various places through transmission lines and distribution networks. An electrical power system consists of generation, transmission, distribution, and utilization of electricity.

The first electric supply system was set up in 1882 in New York City, USA. The system involved electricity generation through a dc generator driven by a steam engine. The electricity generated was distributed for local use by underground cables for lighting purpose only. Gradually, power systems started growing in size. Large size ac generators were installed in power houses to generate electricity in the Mega Watt range and at higher voltage. By using transformers, the generated electricity was stepped up to high voltages such as 132 kV, 220 kV, 480 kV and so on. The magnitude of stepping up

of voltage depended upon the distance to which the generated electricity had to be transmitted for use. Thus, high-voltage and extra-high-voltage transmission systems came into existence. At the receiving end of the transmission line, the voltage level is brought down for distribution to different distribution substations where the voltage level is further brought down for supplying to consumers. The distribution system is constituted by overhead lines and underground cables.

A number of control devices are used at various stages of the power system for safe, efficient, reliable, and economic use of electricity. The control devices include different types of relays, switchgear, switch, fuse, etc.

Transformers are used at various places in the power system for stepping-up or stepping-down the system voltage. The voltage level of the power system has been designated as follows.

Extra high voltage, i.e., EHV: above 220 kV and upto 800 kV.
High voltage, i.e., HV: above 66 kV and upto 220 kV.
Medium voltage, i.e., MV: above 1 kV but less than 66 kV.
Low voltage is LV: 1 kV or less than 1 kV.

The power system can broadly be divided into three sub-systems, i.e., Generation, Transmission, and Distribution excluding the Utilization part. These are discussed in some detail as follows.

6.2 GENERATION OF ELECTRICITY

Electricity for commercial use is generated in power houses by converting primary sources of energy to electricity. The sources of energy are either *non-renewable* or *renewable*. For example coal, oil, gas, etc. are non-renewable sources of energy. They get exhausted once used. On the other hand, solar energy, energy of river water, and wind are renewable sources of energy. They do not get exhausted. The various sources of energy are mentioned in the following section.

6.3 SOURCES OF ENERGY FOR ELECTRICITY GENERATION

(a) *Coal*: Coal is burnt in a furnace to generate steam. Steam at high pressure and temperature is supplied to a turbo-generator which produces electricity. Use of coal leaves a huge quantity of ash and produces fuel gases. They pollute the environment, and therefore need to be handled carefully.

(b) *Fuel oil*: Fuel oil is a product of oil refinery after the crude oil has been processed. Fuel oil produces no ash. The design of its furnace is simpler as compared to a coal-fired one.

(c) *Gas*: An important energy source, it contain's into methane (CH_4). Liquefied petroleum gas (LPG) is one of the petroleum products produced in oil refineries.

(d) *Solar*: Electricity using the sun's rays can be produced through the photovoltaic system or through solar thermal technologies. Solar photovoltaic system converts the sun's energy directly using solar cells (semiconductor device). Solar thermal technologies convert radiant energy of the sun into thermal energy and then into electricity.

(e) *Wind*: Wind energy system converts the kinetic energy of wind into electricity by using wind turbines coupled with a generator. The wind velocity of a particular location depends on the height and the nature of the terrain. Wind results from temperature gradients between the Equator and the Poles, and between land and sea.

(f) *Water*: The potential energy stored in water can be utilized as the water is allowed to fall from a higher to a lower level. This can be achieved by constructing a dam on a river at a suitable place

where the gradient is high. Another energy source is from the kinetic energy and the potential energy of ocean waves. A tidal energy of water is extracted by creating a dam at a suitable place, using a turbo-generator and a sluice gate in the dam to allow the tidal flow to enter or leave the tidal basin.

(g) *Biomass*: These are agricultural and forestry wastes, municipal wastes, the waste from the crushing of sugar canes (bagasse) etc. Biomass is burnt to produce steam which will feed a turbo-generator to produce electricity. Alternatively, biomass may be processed to produce gaseous output to run a gas turbine–generator set.

(h) *Geothermal*: Steam and hot water produced inside the earth can be used to generate electricity.

The different technologies used to produce electricity from various energy sources are discussed in brief in the following section.

6.4 THERMAL POWER GENERATION FROM FOSSIL FUEL

In a thermal-power-generating station the heat energy from coal, oil or gas is utilized to produce steam and run steam turbines. The steam turbines are coupled with generators to produce electrical power output. A coal-based thermal plant is described in brief in the following section.

6.4.1 Coal-fired Thermal Power Stations

Coal is first powdered, i.e., pulverized and burnt in a furnace and the heat energy is used to boil water in the boiler to produce steam. The steam from the boiler is taken to the turbine. The turbine works as a prime mover (drive) to the generator. Thus, the mechanical energy of the turbine is converted into electricity. The generated electricity is transmitted to various places through transmission lines after stepping up the voltage with the help of a transformer. Here, the chemical energy in coal is first converted to heat energy, then to mechanical energy, and finally to electrical energy. The conversion efficiency in a thermal-power-generating station is low due to heat and other types of losses at various stages of conversion. Fig. 6.1 shows a schematic representation of a coal-fired power station. As can be seen from the figure, the steam after passing through the turbine goes to a condenser where it is condensed into water. This water is again fed to the boiler for producing steam. The heat from the used steam is taken out and dissipated in the cooling tower.

In coal-fired stations, the coal is brought from the coal storage area through the coal conveyer to the coal hopper placed near the boiler. The coal is then pulverized, i.e., made into powdered form so that complete combustion of coal takes place in the furnace. The pulverizer may be a rotating drum grinder or other type of grinders.

Sufficient air is required in the furnace for combustion. Air is drawn from the atmosphere by the use of a forced draft fan (FD Fan). The FD fan takes air from the atmosphere and the air is preheated before injecting through air nozzles on the furnace inside wall. The induced draft fan (ID Fan) assists the FD fan by drawing out combustible gases from the furnace and maintaining a certain negative pressure in the furnace. Maintaining a certain negative pressure inside the furnace avoids any backfiring through any opening. An *electrostatic precipitator* is used before the ID fan (not shown in the figure) to collect the fine dust particles so that these are not thrown into the atmosphere along with the flue gases and pollute the atmosphere. This is a must requirement provided by the environmental protection law, and the power stations have to abide by this requirement of installing dust collectors from the flue gases.

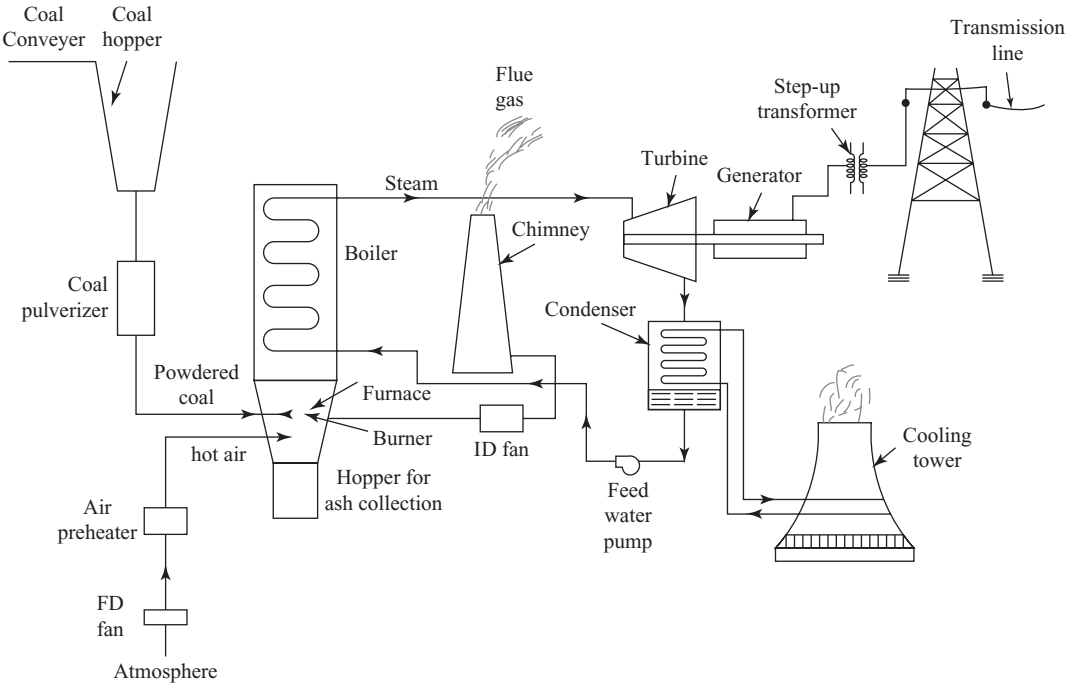


Figure 6.1 *Schematic diagram of a coal-fired thermal power plant*

Steam is taken out continuously from the boiler to the turbine. At the other side of the turbine, used steam is condensed to water and fed back to the boiler through a feed water pump. Thus, there is continuous withdrawal of steam and continuous return of condensed water to the boiler. However, there is some loss of water in the form of steam due to leakage, etc. This has to be made up by adding make-up water to the boiler water system. Boiler water has to be treated to remove calcium, magnesium, and other salts otherwise these may form undesirable deposits in the water flow system. A water treatment plant is, therefore, required in a thermal power plant.

To increase the efficiency of the conversion of heat into mechanical energy, steam is generated at very high temperature and pressure. Further, large capacity turbines and generators of the range of hundreds of megawatts are installed to reduce the capital cost per kilowatt of energy produced.

It may be noticed that in a thermal power station, steam and hot water are produced along with electricity. The waste heat in the form of hot air and steam can be used in certain process industries like paper mills, textile mills, food-processing industry, chemical industries, etc. When the otherwise waste heat is used, the overall plant efficiency is increased.

The system where electricity, steam, and hot water are simultaneously made available, is called *cogeneration*. The efficiency of a thermal power plant is around 40 to 50 per cent. Cogeneration increases the overall efficiency.

6.4.2 Gas-fired Thermal Power Stations

Natural gas is used as the source of energy to generate electricity. A gas turbine is the prime mover which will drive the generator. The hot gases exhausted from the turbines can be passed through a *heat*

exchanger to produce steam. This steam can be used to run a steam turbine which would drive another generator. This way, the total thermal efficiency of the plant can be increased. This method is called the *combined-cycle method of generation of electricity*. If it is decided not to use the hot gases exhausted from the gas turbine for further generation of electricity, the hot gases can be used to produce steam for use in various industrial processes. Gas-fired stations are more environment friendly as compared to coal-fired power stations, as the flue gases emitted contain almost zero sulfur dioxide. As can be imagined, the installation cost of gas-fired stations would be less than the coal-fired ones. However, the operational cost of gas-fired stations will be high due to the higher cost of fuel, i.e., the gas used. In the state of Rajasthan in India, Kota is the place where all types of power stations like nuclear, thermal (coal fired and gas fired), and hydro are in operation.

6.4.3 Oil- and Diesel-oil-fired Thermal Power Stations

In oil refineries, the oil left behind after processing the crude oil can be used in an oil-fired steam power station installed near the refinery. The crude oil pumped out of an oil well can also be used directly to generate steam to run the steam turbine which will rotate the generator to produce electricity.

Diesel oil can be used to run an IC engine which will drive the generator. Due to the high cost of diesel, diesel-oil-fired stations are mostly used for standby power supply.

6.5 HYDROELECTRIC POWER-GENERATING STATIONS

Energy from falling water is used to run water turbines. Water turbines are coupled with electrical generators to produce electricity. Hydroelectric power generation depends upon the height of the falling water and the quantity of the falling water. The first use of water power to produce electricity was a water wheel on the Fox River in Wisconsin, USA in the year 1882. Thereafter, many hydroelectric power plants were constructed including one in Niagara falls in USA. These days hydroelectric power plants are constructed as mega projects with the generating capacity of over 1000 megawatts MW. The initial investment on a hydroelectric power plant is huge because of the money required for the construction of a dam. Most often such plants are constructed in hilly areas where there is sufficient gradient available of the river water. When a dam is constructed to obstruct the flow of water of a river, the upstream areas get flooded with water and creates a water storage area. For example, the dam constructed on the river Satluj at Bhakra–Nangal has created a large water storage area in the form of a lake, called Govind Sagar Lake. This water is used in a controlled way to produce electricity on a continuous basis using water turbine–generator sets installed in the power house at the lower area of the dam. Since a large area gets flooded in the upstream area when a dam is constructed on a river, the people living in such areas are to be shifted permanently from there. Further, there is an environmental impact on the construction of dams in hilly areas due to which the builders often find it difficult to construct new hydroelectric plants in certain areas. However, looking at the large potential of hydroelectric power and its negligibly small running cost, new projects are coming up in many countries. In India, the installed capacity of hydroelectric power generation was over 25,000 MW by 2001. In a hydroelectric power plant, water at a higher elevation flows downward through large pipes called penstocks. This falling water rotates turbines which drive the generators thus converting the mechanical energy into electrical energy.

The advantages of hydroelectric power is that it produces no pollution and is continually renewable. Water is continuously available because of rain and melting of snow, and never gets exhausted. Just to get an idea of how much electricity gets generated from falling water, someone calculated that four

litres of water falling from a height of 30 meters each second could produce 1 kW of electricity which could run ten 60 W light bulbs and five ceiling fans.

Hydroelectric power plants are of three types, namely high- and medium-head-storage type, run-of-river type, and pumped-storage type.

Storage-type hydroelectric installations use a dam to block and store water in a reservoir. The stored water in the reservoir is released to run the turbines to generate electricity. The flow of water is controlled as per the requirement of output.

In the run-of-river type, use is made of the natural flow of water of a river to run a turbine. This type of installation may not require a dam to be constructed. A low-head diversion structure to direct the water flow to the penstock may be used.

Hydroelectric power plant with pumped storage facility use specially designed turbines. These turbines will drive the generator in a conventional way when water from the reservoir is allowed to flow through the turbines via the penstock, and thereby generate electricity during the day when there is huge demand on electricity from the supply system. However, during night when there is less demand for electricity, some of the turbines can be used as pumps to lift water from the outlet area back to the reservoir for future use. Fig. 6.2 shows the schematic diagram for a high-head-reservoir-type hydroelectric power plant. The generated power, P in a hydroelectric plan can be expressed as $P = 9.81\ \rho\ Q\ h\ \eta \times 10^{-6}$ M, where, ρ is the specific weight of water in 1000 Kg/m^3, Q is the quantity of discharge of water through the turbine in m^3/second, h is the head of water in metres, and η is the combined efficiency of the turbine–generator set.

Hydroelectric power generation provides a number of advantages. Some of these are mentioned below.

(i) Hydroelectric plants do not contribute to air pollution, acid rain, ozone depletion or any toxic wastes.

(ii) Many hydroelectric plants provide flood control and water supply for drinking and irrigation.

(iii) The big reservoir in the form of a lake creates recreational facilities like swiming, boating, water-skiing, camping, picknicking, sightseeing, fishing, etc.

(iv) Operation cost of hydroelectric plants is low as there is no fuel cost.

(v) Maintenance cost is low.

(vi) Life of a hydroelectric plant is long.

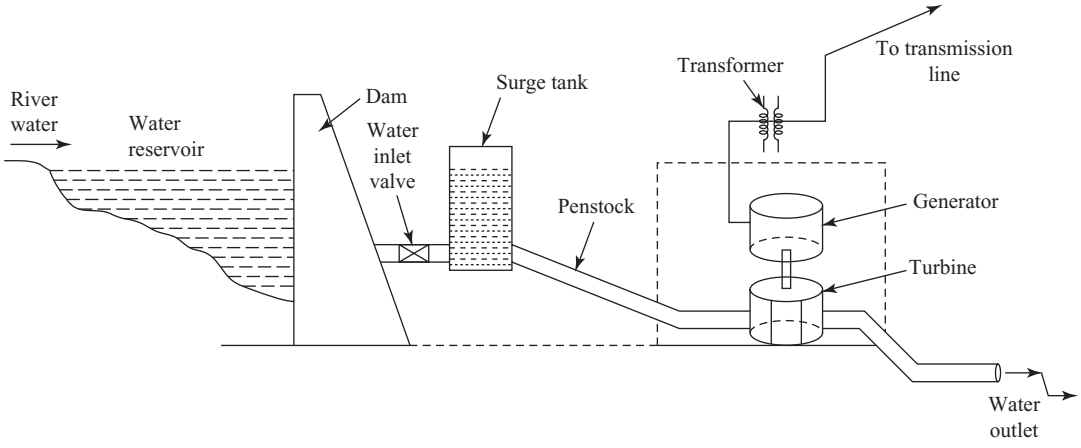


Figure 6.2 *Schematic representation of a hydroelectric power plant*

Table 6.1 *A Few Hydroelectric Plants in India*

Location	Units	Total capacity (MW)
• Bhakra, Himachal Pradesh	5×108; 5×157	1325
• Dehar, Himachal Pradesh	6×165	990
• Salal, Jammu & Kashmir	6×115	690
• Sardar Soravar, Gujrat	6×200; 5×140	1450
• Nagarjun Sagar, Andhra Pradesh	1×110; 7×100.8; 5×30	965
• Idukki, Kerala	6×130	780

However, there are certain demerits of hydroelectric plants like heavy investment required for the construction of dam, etc.; time taken to instal a plant is high; opposition from environmentalists and the people living on the land to be used as a reservoir; cannot be constructed anywhere, i.e., near the load centre, etc.

The first hydroelectric power plant was installed in USA in the year 1882 followed by Sweeden and Japan. In India, the first hydroelectric power plant was installed in 1897 in Darjeeling area in West Bengal.

A few hydroelectric power plants in India are mentioned in Table 6.1.

6.6 NUCLEAR POWER-GENERATING STATIONS

We have known that in a thermal power plant, the heat required to produce steam is obtained by burning coal, oil, or natural gas.

In a nuclear power plant the required heat is produced by a process called fission of uranium atoms. Fission or splitting of uranium atoms takes place in a nuclear reactor. Uranium is therefore called nuclear fuel. Nuclear fuel consists of two types of uranium, namely U=238 and U=235. U=235 splits easily, i.e., its nucleus, which is composed of protons and neutrons, break up and release heat as well as neutrons. This splitting of atoms is a chain reaction. Fission in U=235 is started by bombarding it with neutrons. In the fission reaction, neutrons are released and heat is generated. When neutrons hit other uranium atoms, these atoms also split releasing neutrons as well as heat. These neutrons strike other atoms of uranium. Thus, one fission (splitting of uranium atom) leads to more fission, and hence creates a chain reaction. This makes the fission self-sustaining once the process is started.

This chain fission reaction once started will become an uncontrolled one unless some control mechanism is used. In a nuclear reactor, fission reaction of uranium is controlled using control rods which are inserted or withdrawn to slow or accelerate the fission reaction.

In a nuclear power plant, the nuclear reactor replaces the steam boiler used in a thermal power plant using coal or oil as a fuel. Many other components are similar to those used in a coal or oil (fossil fuel) power plant.

Nuclear fuel enrichment. U=235 is more desirable for fuel than U=238 because it is easier to split U=235 atoms than U=238 atoms. In the uranium ore, however, the percentage of U=235 is less (low enrichment). It is therefore necessary to enrich the fuel to be used by U=235 in relation to the number of U=238 atoms. This is called the enrichment process. After enrichment, the uranium is fabricated into pellets and stacked into long metal tubes. The filled rod is called a fuel rod. Schematic diagram of a nuclear power plant has been shown in Fig. 6.3.

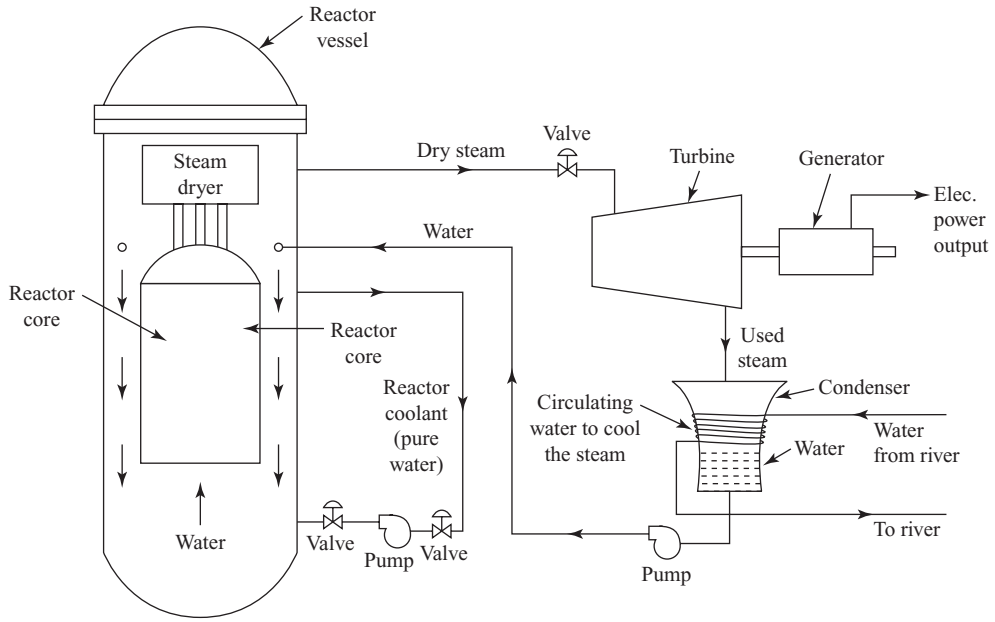


Figure 6.3 *Schematic diagram of a nuclear power plant*

At the nuclear power plant the reactor fuel assemblies consisting of fuel rods, spacer grids, the upper-and lower-end fillings, etc., are inserted vertically into the reactor vessel which is a large steel tank filled with water. The fuel is placed in a precise grid pattern and is called the reactor core. The boiling water reactor operates the same way as a boiler in a fossil fuel power plant.

Pure water called reactor coolant moves upwards through the reactor core absorbing the heat generated because of nuclear fission. Due to heat this water becomes steam but contains some water particles. This steam and water mixer moves up and enters the steam dryer and moisture separator. Dry steam, also called saturated steam leaves the reactor vessel through a steam line (insulated pipes) to the steam turbine. The turbine rotates due to the pressure of the steam. Since the generator is coupled with the turbine, the generator also rotates and produces electrical power which is stepped up and transmitted through transmission lines to various places for use. The exhausted steam is collected into the condenser where it is cooled into water. The condensed water is reused for which it is pumped out of the condenser back to the reactor vessel using a number of pumps. The coolant flow through the core can be varied to change the reactor power as and when required. If a nuclear power plant is not situated near a river, the excess heat of the circulated water is removed by using a cooling tower for transferring some of the heat to the air.

As compared to a coal-fired thermal power plant, the quantity of fuel required in a nuclear power plant is much less. The transportation cost of fuel is therefore less as compared to the transportation of coal or oil. Therefore, a nuclear power station can be built near the place where power is to be utilized, i.e., near the load centre. In a nuclear power plant, radioactive fuel waste has to be removed with outmost care and precaution.

Some of the nuclear power plants in India with their installed capacity are shown in Table 6.2.

Table 6.2 *Some Nuclear Power Plants in Operation in India*

Location	Units	Total capacity (MW)
• Kaiga, Karnataka	220 × 3	660
• Kakrapur, Gujarat	220 × 2	440
• Kalpakkam, Tamil Nadu	220 × 2	440
• Narora, Uttar Pradesh	220 × 2	440
• Rawatbhata (near Kota), Rajasthan	100 × 1 200 × 1 220 × 2	740
• Tarapur, Maharastra	160 × 2 540 × 2	1400

In total, 4120 MW nuclear power is being generated. Over 3000 MW generating capacity projects are under construction.

6.7 NON-CONVENTIONAL OR ALTERNATIVE GENERATING STATIONS

Generation of electricity from energy sources like solar energy, wind energy, ocean wave energy, geothermal energy, etc. are called non-conventional sources of energy.

As these are renewable sources of energy, efforts are being made to generate more and more electricity from these resources. These are being described in brief in the following sections.

6.7.1 Solar Electricity Generation

Sun's heat energy is the readily available energy that can be used for heating water, cooking food, pumping water, and for generation of electricity. The energy received from the sun on earth is so much that someone calculated it as equivalent to about 15,000 times the world's annual energy requirement. In India solar energy is available for 300 to 330 days a year, and hence should be utilized to the full through two different ways, i.e., for heating purpose and for generation of electricity. For generation of electricity *solar photovoltaic cells* are used. Solar photovoltaic uses sun's heat to produce electricity for lighting, running pump motors, and running of electrical appliances. Solar cell, or photovoltaic cell converts sunlight directly into electricity.

Photovoltaic (PV) is the technical term for Solar electric. PV cells are made of silicon. Silicon releases electrons when exposed to light. The amount of electrons released from silicon depends upon the intensity of light falling on it. One PV cell or Solar cell produces about 1.5 W of electricity when exposed to bright sun. Individual solar cells are connected together to form a panel or module capable of producing upto about 100 W of electric power. A number of such panels are then connected in series and parallel to produce a considerable amount of electricity. Common applications of PV systems are found in street lighting and indoor lighting, particularly in far flung and hilly areas where conventional electric supply may be expensive because of its long route.

Commercial solar cells producing electricity have an efficiency of about 15 per cent only. Low-efficiency system means larger arrays would be required for electricity generation, and hence would cost more. However, research is being conducted to improve the efficiency of solar cells so as to make the system economically viable. Government of India provides subsidy to manufacturers and users of solar appliances to encourage their use.

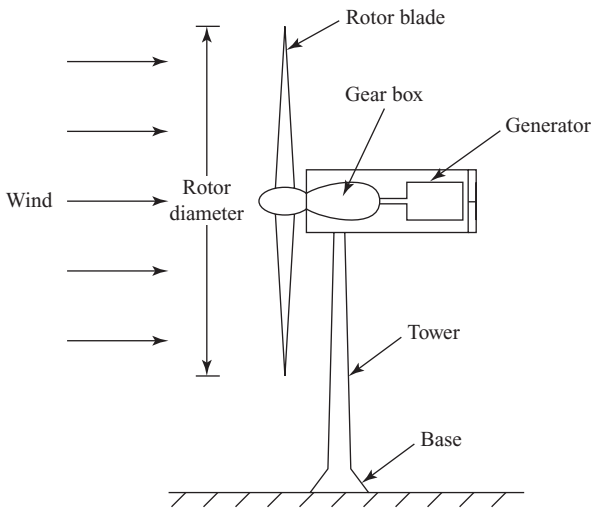


Figure 6.4 *Wind turbine–generator*

6.7.2 Wind Energy to Produce Electricity

When solar radiation enters the earth's atmosphere, due to the curvature of the earth, different regions of the atmosphere are heated differently. The equator region gets heated the most and the polar region the least. Since air tends to flow from warmer region to cooler region, air flow takes place. The energy of this air flow is utilized in the wind mills and wind turbines to produce usable power.

Wind energy is converted into electrical energy through wind turbines coupled with a generator. Wind electric generator converts the kinetic energy in wind to electrical energy by using a rotor, a gear box, and a generator is shown schematically in Fig. 6.4.

Due to flow of wind at a considerable speed, the rotor blades rotate which converts wind energy into rotational shaft energy which in turn rotates the generator through the gearbox arrangement. Wind energy generators of rating from nearly 200 kW to 1000 kW have been installed in different parts in India. Wind speed is the most important factor influencing the amount of energy a wind turbine will produce. The rotor is placed on top of towers to take advantage of strong winds available high above the ground.

India is one of the most promising countries for wind power development with an estimated potential of 20,000 MW. Financial and technical assistance is provided by government of India for wind power development. As on March 2009, the installed capacity of wind power in India was 10,254 MW. Tamil Nadu is the state with most wind-power-generating capacity of 4301.63 MW.

6.7.3 Electricity from Biomass

In most bio power plants, steam is produced by burning of bio-waste material. Bio energy feedstocks are burnt directly in a boiler to produce steam. The saturated steam drives the steam turbine which rotates the generator coupled with it. In certain industries the steam produced in the boiler is also used in manufacturing processes or to heat the buildings. These are called combined heat and power facilities. For example, in a paper industry, the waste wood is used to produce both electricity and steam.

The decay of biomass produce methane gas which can be used to produce electricity. By burning methane, steam can be produced in a boiler and this steam will run a turbine coupled with a generator.

6.7.4 Mini/Micro Hydel Power Generation

In hilly areas there is tremendous potential of generation of electricity where there are hydel (water) resources in the form of falling water. The Himalayan belt in India is gifted with such hydel resources. Mini and micro hydel set (turbine–generator set) can be installed on such falling water resources to generate electricity for local use. These small turbine–generator sets are compact and require no maintenance. Instead of using diesel generator sets, small turbine–generator sets can be used in hilly remote areas for the benefit of small cluster of villages to supply electrical power to farms, schools, houses, etc. The mini hydel plants are rated for 10 kW to 1000 kW. Upto a rating of 100 kW, the plants are called micro-generating plants.

6.7.5 Electricity from Tidal Energy

Sea tides are created due to gravitational effects of the sun and the moon, and the centrifugal force of the earth's rotation. Sufficient head of water can be captured by constructing barrages along the sea shore. The incoming and outgoing tidal waves of varying heights can be blocked by the barrages. A tidal wave height of atleast 7 m is required for economical generation of electricity using water turbines. Tidal power station will fill the reservoirs by opening the sluice gates behind the embarkment along the seashore and use the water height to run turbines. Tidal power stations are constructed across an estuary or a bay. Bay of Bengal in India provides the opportunity to set up such power-generating stations.

6.7.6 Electricity from Ocean Energy

Almost two-third of earth's surface is ocean. The surface water gets heated up due to sun's energy. The sun heats up the surface of the water a lot, and hence the water surface of the ocean works as a solar collector. The warm surface water of the ocean can be used to generate electricity. The warm water of the ocean can be used to vaporize a working liquid of low boiling point such as amonia. The vapour will expand and can run a gas turbine which will drive a generator.

6.7.7 Electricity from Geothermal Energy

Heat energy is trapped in rocks inside the core of the earth. Water absorbs the heat from these rocks and gets converted into steam. This steam when available on earth's surface at specific places can be used to run steam turbines, and thereby generate electricity. Not much has been achieved in tapping geothermal energy so far. In India, in Ladakh area, feasibility study for construction of a 1 MW power station has been made. Geothermal means heat from the earth's interior.

Geothermal energy is generated in the earth's core about 6400 km below the surface of the earth. Temperatures higher than the temperature of the sun is produced in the core of the earth by slow decay of radioactive particles.

The centre of the earth is of solid iron core and its outer is made of very hot melted rock called magma. The magma is surrounded by rock and magma which is also called mantle. The crust is the outermost layer of the earth. The crust forms the land and ocean floors. See Fig. 6.5.

The crust on land area is about 25 to 55 km thick and on oceans 5 to 8 km thick.

Inside the earth the rocks and water absorb heat from the magma. When magma comes closer to the surface it heats up the water trapped in porous rocks or the water running along fractured rocky areas. Such naturally formed hydrothermal resource is called a geothermal reservoir.

Electricity generation in power plants requires steam at very high temperature.

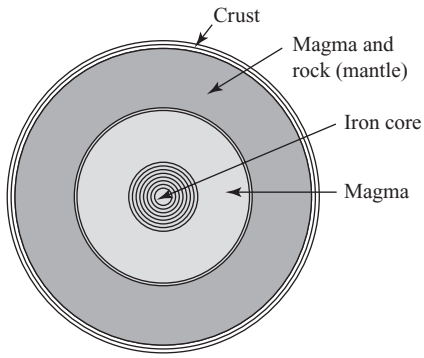


Figure 6.5 *The interior of the earth showing source of geothermal energy*

Geothermal power plants are built where geothermal reservoirs are available within a depth of a few km from the earth's surface. This requires drilling into the earth to extract hot water and dry steam. California in USA produces the most electricity from geothermal energy. Other uses of geothermal energy is to use hot water from the reservoir or naturally available hot springs for heating buildings, bathing, etc. In India, although electricity generation from geothermal energy is at an initial stage, use is made of hot springs for bathing as many believe that taking bath in mineral-rich natural hot water will have a healing effect.

6.8 TRANSMISSION AND DISTRIBUTION OF ELECTRICITY

Electricity generated in power stations are brought to the consumer premises through transmission and distribution systems. Power transmission is from the substation near the power plant to the substation near the populated area or industrial area. In substations, the voltage level of the power generated is either stepped up or stepped down using step-up or step-down transformers. Power is transmitted at high voltage using transmission lines. Fig. 6.6 shows the schematic diagram of a power system.

The distribution system includes overhead lines and underground cables drawn from the transformer substation and taken to the consumer premises, i.e., to industries, commercial establishments, and residential areas.

Power transmission lines at a national level or even at an international level are connected through additional paths and lines so that power can be routed from one power plant to any place where there

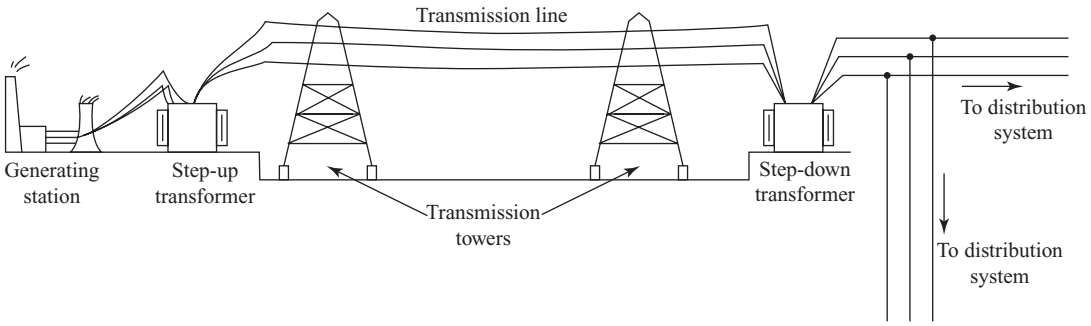


Figure 6.6 *Shows the schematic diagram of power transmission at high voltage*

is need. Surplus power from one station can be supplied to deficit areas. Such a network allowing transmission of power nationwide is called a *national grid.*

The power transmitted is proportional to the square of the voltage $\left(\because P = VI\cos\phi = V\frac{V}{Z}\cos\phi = \frac{V^2}{Z}\cos\phi \right)$. Thus, it is desirable that transmission voltage be made as high as possible to be able to transmit maximum power through a line. However, the height of the lines above the ground has to be increased to keep them away from the ground level. Thus, there is practical limitation to the level of high-voltage transmission. In India, the highest level of transmission voltage at present is 400 kV. In some countries the highest level is 765 kV.

6.8.1 AC Versus DC Transmission

As we have known, P = VI cos Φ. If the voltage level of transmission is increased, the current level goes down. The cross-sectional area of transmission line conductors get reduced, and hence the cost gets reduced. For economical bulk power transmission, the generated voltage level is stepped up. It has been roughly calculated that most economical transmission voltage is 1 kV per 1.6 km or 1 kV per mile. Thus, the most economical transmission voltage for a distance of around 600 km, works out around 400 kV. In India, the transmission voltage varies from 66 kV to 400 kV. For transmission of power to distances more than 600 km, it has been calculated that dc transmission becomes more economical than ac transmission. Electricity is generated by using an alternator, i.e., an ac generator at 11 kV or more. For ac transmission, the three-phase lines from the generator are connected to the primary side of a step-up transformer. The secondary terminals are connected to the transmission lines.

For dc transmission, the alternating voltage is first converted into dc using an ac to dc converter, and is then connected to the transmission lines. At the receiving end of the transmission lines dc is converted to ac and distributed to the consumers through distribution lines, step-down transformers and underground cables. It may be mentioned here that transmission of power is done in two stages. In the first stage, the voltage level may be 220 kV and in the second stage the voltage level is brought down to 132 kV or less. The level of transmission may be brought down even to 33 kV. The first stage of transmission is called primary transmission while the second stage of transmission, at voltages ranging from 132 kV to 33 kV, is called secondary transmission.

6.8.2 Distribution System

The distribution voltage is either 11 kV or 415/230 V. At the distribution substation, power is received from the secondary transmission lines at a voltage of, say 33 kV. The voltage is further stepped down to 11 kV by the distribution transformer installed at the distribution substation. Heavy industries are supplied with 11 kV or even higher voltages, who in turn step down the voltage using their own transformers. For other consumers electricity is supplied at 415/230 V. Three-phase supply is provided at 415 V and single-phase supply is made at 230 V.

Thus, the part of power system which provides electricity to the customers is called the distribution system as shown in Fig. 6.7.

6.8.3 Overhead Versus Underground Distribution Systems

The distribution system can be installed through overhead lines or through underground cables. Overhead lines are generally mounted on concrete or steel poles. The distribution transformer (11 kV/400 V) is mounted on poles. Supply is taken out from the secondary of the transformer through cables. In an underground system cables are laid below the ground level along the street by the side walk. The

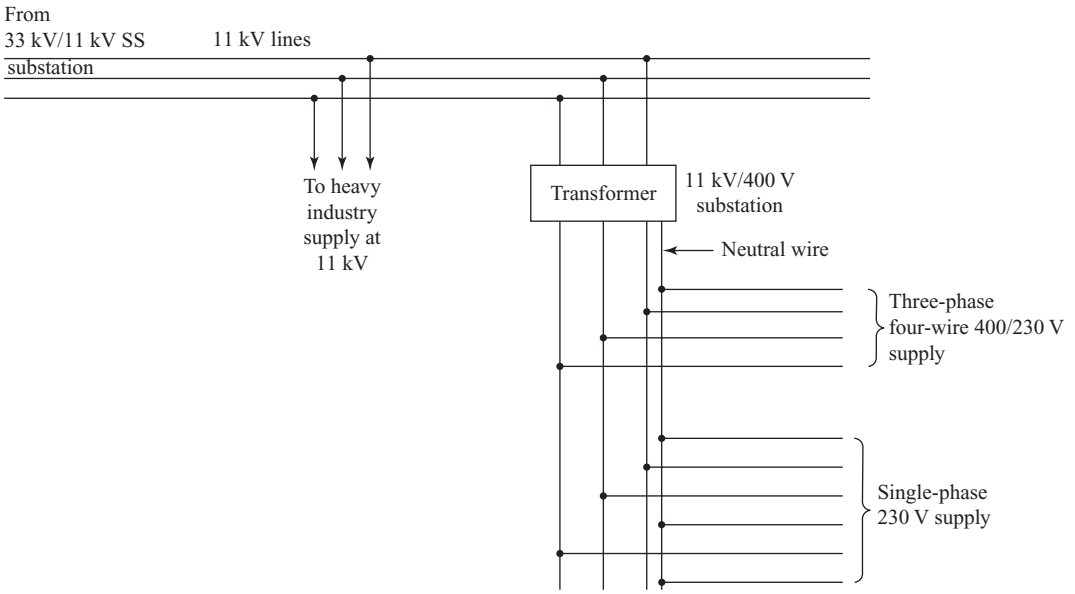


Figure 6.7 *Distribution of power to industries and residential complexes*

choice between overhead lines and underground cables depends upon a number of factors like initial cost, ease of location of any fault, safety consideration, maintenance cost, aesthetics, etc.

6.8.4 Connection Schemes of Distribution System

The distribution network consist of feeders, distributors, and service mains.

Feeder is a cable which feeds the distributor. From the distributor, connections are taken to supply the consumers. Such connections are called service mains. Two types of arrangements are made. These are called the radial system and the ring main system. Fig. 6.8 shows the difference between

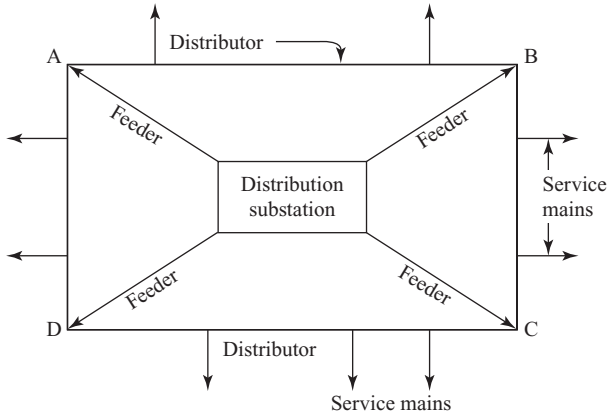


Figure 6.8 *Use of feeders, distributors, and service mains*

feeder, distributor, and service mains. Generally, no tappings are taken from the feeder. From the distributor, connections are taken through low-voltage cables to supply the consumers.

The radial system of distribution and the ring main system of distribution are illustrated through schematic diagrams in Fig. 6.9 (a) and (b).

The radial distribution system is commonly used in less populated areas where the primary feeder branches out to reach the total area to be served. Distribution transformers are connected to the feeders and laterals. CB represents the circuit breakers installed to disconnect the system in case of any fault.

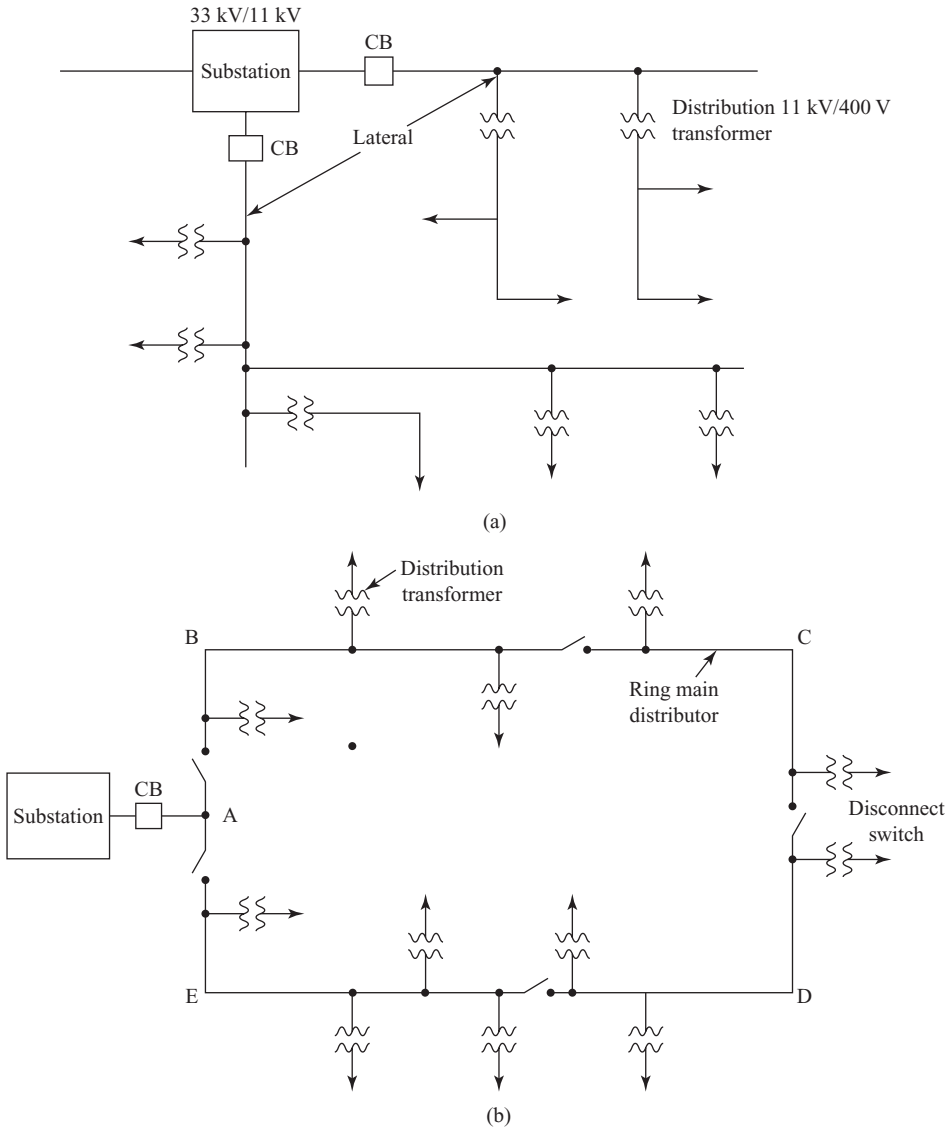


Figure 6.9 *(a) Radial distribution system; (b) ring main distribution system*

The service reliability of this system of distribution of electric power is low as in case of any fault on a particular line, the whole area gets effected, i.e., gets switched off. The advantage of the ring main system over the radial system is that it is more reliable. In case of any fault, an alternate route is available for the maintenance of power supply.

6.9 DOMESTIC WIRING

Electricity distribution authorities supply power to the consumers at the following two voltages. Single-phase supply: 230 V, 50 Hz, two-wire, three-phase supply: 415 V, 50 HZ, four-wire. In a two-wire single-phase supply there is one phase or live wire and the other is called the neutral wire. Single-phase supply is required for electrical appliances like fan, tube light, lamp, washing machine, refrigerator, electric iron, room heater, room air-conditioner, kitchen electrical appliances like mixer, grinder, microwave oven, etc.

In a four-wire, three-phase supply, power is supplied through three live wires and a neutral wire. The neutral wire is normally at zero potential and is earthed at the substation. Three-phase loads like three-phase induction motors used for water lifting are supplied with three-phase supply. In such a case all the phases get equally loaded. Three-phase supply can also be used to feed single-phase loads as shown in Fig. 6.10. Single-phase electrical loads are connected between the phase or live wires and the neutral wire in such a way that all the phases are equally loaded.

As shown in Fig. 6.10, supply from the secondary of a three-phase transformer is taken through feeder wires to the busbars (busbars are thick copper strips). From the busbars both single-phase supply and three-phase supply are taken out and are connected to the loads.

6.9.1 Service Connection

Electricity supply authorities supply power to the domestic loads, i.e., residential houses through a low-voltage three-phase four-wire distribution system. The distribution system is formed either through overhead lines or through underground cables. In modern cities, only underground cables are used for electric power distribution system. From the electricity supplier's distribution system, power is brought to the premises of the consumer through a cable called the service line or service connection. The service line is normally an underground PVC cable. The service cable is brought to the consumer's distribution board.

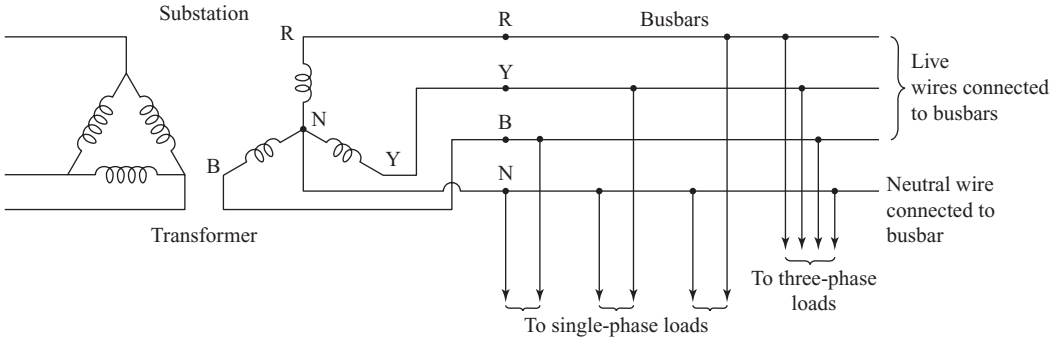


Figure 6.10 *Three-phase four-wire distribution system*

6.9.2 Service Mains

In the main distribution board installed in the premises of the consumer, an energy meter is connected so as to measure the amount of electricity consumed by the customer. Thus, the incoming service cable is connected to the energy meter at the distribution board. A cutout, i.e., a fuse wire is connected in the line which will blow off in case the consumer draws more current than the current for which he has been supplied with. In the cutout of proper rating is not provided, the meter will get burnt in case of heavy loads (more than the rated load) drawn by the consumer. For example, if the consumer install more air-conditioners in his house than permitted, the meter may get burnt. The cutout can only be replaced by the electricity supply authority. The consumer must not tamper with the cutout. The supplier may disconnect supply by removing the cutout in case the energy bill is not paid by the consumer.

6.9.3 Distribution Board for Single-phase Installation

The service mains is connected to the input terminals of the single-phase energy meter at the distribution board as shown in Fig. 6.11. Before the energy meter is placed the electricity supply authority's cut-out. The consumer's main switch-fuse is connected after the meter. From here the supply is fed to the distribution fuse board through a busbar and neutral link.

It is to be noted that all fuses should be placed only in the live or phase wire and not in the neutral wire. All switches should be connected on the live lines and not on the neutral line.

6.9.4 Neutral and Earth Wire

The neutral wire is taken out from the star point of the secondary of the distribution transformer. The star point is earthed. The lines are taken out from the terminals R, Y, B of the three-phase windings and the neutral point N. Each phase wire and the neutral wire constitutes single-phase supply.

In addition to the four wires taken out from R, Y, B, and N terminals, a fifth wire, called earth wire is provided. The earth wire is provided for the purpose of protection of persons using electrical appliances. In case of any leakage in the system, the persons using any equipment will not get any electric shock if the body of all electrical appliances are connected to the earth terminal.

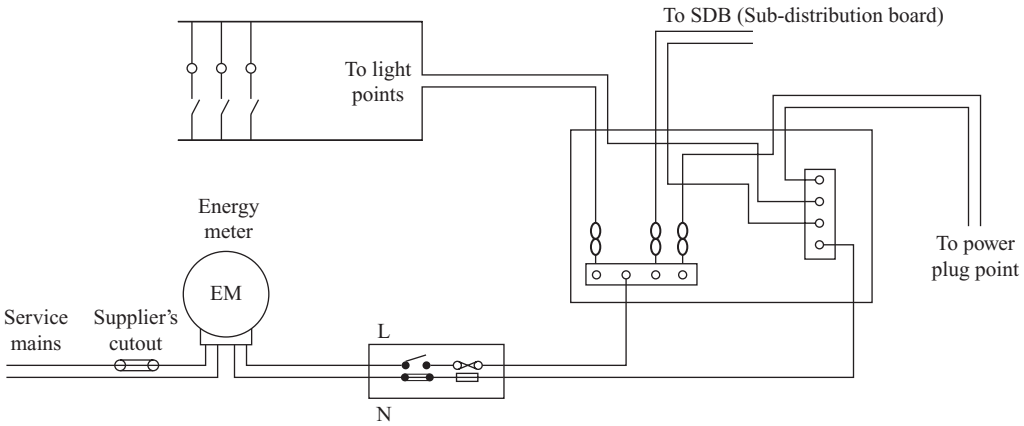


Figure 6.11 *Layout of distribution board for single-phase installation*

The neutral wire is connected to the neutral point of the transformer which is earthed at the substation itself. The earth wire starts at a solid earthing point at the substation and runs along the supply lines. This wire is earthed along its run once in every 1.6 km.

6.9.5 Earthing

All metal parts of electrical appliances are always connected to the earth through an earth wire. If the earth wire is not provided and any part of the current-carrying live wire touches a metal frame, the frame will acquire the same voltage as the live wire. Any person touching the body of the appliance will get a severe electric shock. If a low-resistance earth wire is provided, in case of any fault in the circuit, a large amount of current will flow through the circuit. This will cause the fuse provided in the live wire to blow and protect the circuit as well as the person touching the electrical appliance, gadget or equipment, as the case may be.

As shown in Fig. 6.12, the live and the neutral wires are used to supply power to an electric device, say a heater. The metal case of the heater is connected to the earth wire of a three-core cable supplying power to the heater. The earth wire from the distribution board is connected to the main earth terminal of the building as has been shown. This earth terminal in turn gets connected to the earth electrode at the substation through the general mass of the earth (earth is the conducting media). Thus, in case of any electric fault, if the live wire touches the metal case, it gets connected to the earth electrode at the substation. At the substation, the neutral point is earthed.

Method of earthing

Earthing means connecting the earth terminal solidly and securely on the ground through an electrode. Conductivity of earth depends on the moisture content of the soil and its chemical composition. The soil near the earth electrode should have low resistivity (i.e., high conductivity). Before placing the earth electrode, the soil around is made highly conducting by adding some agents like common salt, i.e., NaCl, calcium chloride, i.e., CaCl, sodium carbonate, i.e., Na_2Co_3, soft coke, etc.

Earth electrode

Places where there is a network of underground cables, the earth terminal is obtained by making connections with the lead sheath or steel armour of the underground cable. The lead sheath or the steel armour of the cable will serve as the earth electrode. However, where such underground cable is not available, earth electrodes are laid on the ground to get the earth terminal. To avoid corrosion, the material chosen for the earth electrode, either a pipe or a plate, should be made of copper or zinc-coated iron. Typical

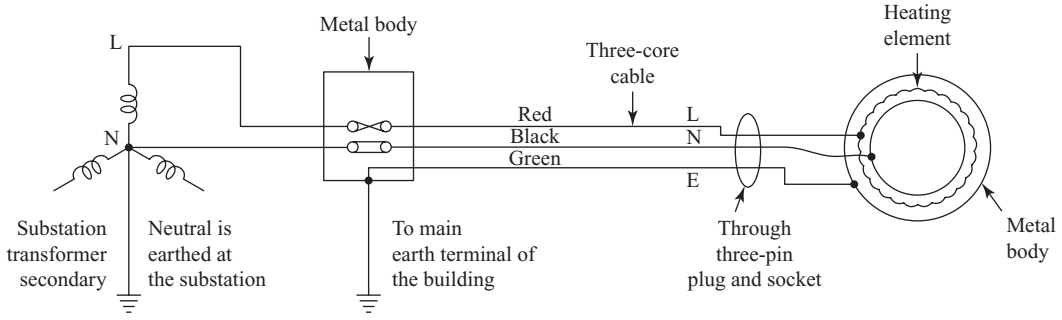


Figure 6.12 *Shows the use of earth wire as an essential requirement to protect against electric shock*

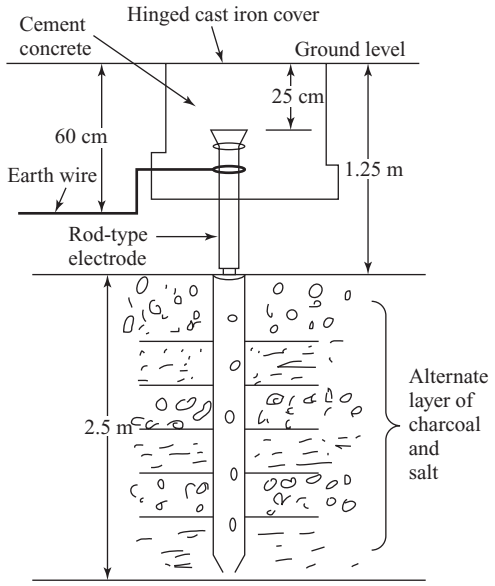


Figure 6.13 *Earthing terminal from a pipe or rod-type electrode*

installation of two types of electrodes, viz the rod or pipe electrode and plate electrode are made. A rod-type electrode has been shown in Fig. 6.13.

In the case of plate earthing, the plate electrode is made of galvanized iron or steel. The size of the plate electrode should be 60 cm × 60 cm and should be burried atleast 1.5 m below the surface of the soil.

All earth wires should be made of copper, galvanized iron, steel or aluminum. Interconnections of earth continuity conductors should be such that good electrical connections are permanently made. The neutral conductor should not be used as earth wire.

Neutral and earth wire

Low-voltage supply at 230 V for single-phase supply and 415 V for three-phase supply are obtained from the distribution transformer. The distribution transformer is a step-down transformer whose primary windings are delta connected and the secondary windings are star connected. The neutral point, i.e., the star point of the secondary is earthed and the neutral wire is taken out from the star point. Thus, the output from the distribution transformer is brought out by three live wires named as R, Y, B, and one neutral wire denoted as N. Supply is taken out through a four-core underground cable or through four-wire overhead conductors.

In addition to the four wires, a fifth wire, called the earth wire is also provided. This earth wire is denoted as E. It originates from a solidly made earth point at the substation. As per Indian Electricity Rules, the electricity supply authority will provide the earth wire to the consumers.

The earth wire while running along the overhead lines is not insulated from the poles. The earth wire originates from a solid earth at the substation and also earthed at not less than four equally spaced points every 1.6 km of the distribution line. The neutral wire is connected to the neutral point of the transformer which is earthed only at the substation. In a single-phase supply system, the neutral wire carries the return current. In a three-phase supply system the neutral wire does not carry any current if the load on all the phases are balanced. In case of unbalanced load on the three phases, the balance

current flows through the neutral wire. The earth wire, under normal conditions, does not carry any current. However, if any earth fault occurs, the earth wire will carry large current which will cause the fuse on the live wire to blow, thereby protecting the life of the operator and the equipment. The earth wire should never be used as neutral wire to supply any single-phase load.

6.9.6 System of Wiring

Electricity supply authority, i.e., the State Electricity Boards provide electric supply upto a point outside the consumer's premises.

From this point the consumer will take connection to his main switchboard. Insulated electrical wires will then be taken out to various places in the premises to supply power to different types of electrical loads like lights, fans, refrigerators, room coolers, heaters, etc. There are different types of wirings used. The choice of wiring will depend upon a number of factors.

The various types of internal wirings are

(i) cleat wiring
(ii) wood casing wiring
(iii) batten wiring
(iv) conduit wiring

Cleat wiring

Cleats are made of porcelain and are fixed on walls or ceiling at intervals of 0.6 m. The insulated wires, i.e., the cable is taken through the holes of each cleat. Thus, the cleats support the wire, such a cleat wiring is cheap and is used for temporary installation.

Wood casing wiring

In wood casing wiring, the cable is run through a wood casing having grooves. The wood casing of a required length is fixed on the walls or ceiling with screws. The cables are placed inside the grooves of the casing. A capping, also made of wood with grooves, is used to cover the cables.

The casing and capping are made from well-seasoned teak wood. The casing should be fixed with flat-headed wooden screws to wooden plugs at an interval of 90 cm. After all the insulated cables are laid inside the grooves of the casing, the capping should be attached to the casing by rust-resistant screws; care should be taken in fixing the screws on the cappings so that the insulation of the cables inside is not damaged. Wood casing–capping wiring system is used in dry places like Rajasthan.

Batten wiring

In batten wiring, insulated wires are run on wooden battens. PVC wires are run on well-seasoned straight teak wood battens. The battens are fixed on the walls or ceilings by plugs and screws. The cables are held on the batten by means of tinned bruss link clips. The clips are fixed on the battens with rust-resistant nails. Batten wiring is widely used for indoor installations. Batten wiring is cheap and takes comparatively less time to install.

Conduit wiring

Conduit wiring consists of PVC wires taken through either steel conduit pipes or through PVC conduit pipes. Conduits are run over the surface of walls and ceiling or are concealed under masonery work. When conduits are run over the surface of walls, the wiring is called surface conduit wiring. When the conduits are run inside the walls, the wiring is called concealed conduit wiring. Surface conduit wiring is used in factories for installation of heavy motors and other electrical equipment. The system is water proof and replacement of defective wires is easy.

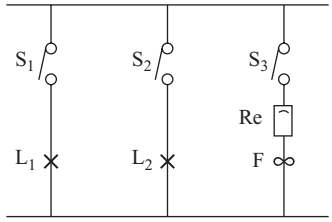


Figure 6.14 *Light and fan control circuit*

In concealed conduit wiring, a chase or groove is cut on the wall to place the conduit pipes. In case of buildings under construction the chase should be provided on the wall and ceilings for laying the conduit pipes before plastering of walls and ceiling is done. Suitable inspection boxes are provided to permit the inspection and replacement of wires, if necessary. Concealed conduit wiring is used in almost all modern residential, commercial, and public buildings. The appearance of buildings from inside look good with concealed conduit wiring as compared to batten wiring.

6.9.7 System of Connection of Lights, Fans and Other Electrical Loads

All electrical loads are connected in parallel and not in series. There are separate circuits for light and fan loads and for heavy electrical loads like heaters, air conditioners, etc.

Fig. 6.14 shows the connection scheme for two lamps and one fan circuit. Each is operated by an independent switch. The fan is controlled by a regulator to change its speed.

As per Indian Electricity Rules, the number of light and fan points that can be put in one circuit is restricted to eight. This includes the 5 A plug points which are provided to plug-in some small electrical loads like a battery charger, a mosquito repeller, a tape recorder, a TV, etc. For comparatively heavy loads, 15 A plug points are to be provided through separate power circuits. The wires used are thicker so that they are able to carry heavy currents. Some typical house wiring circuits are shown through a few examples.

Example 6.1 Draw the wiring diagram for a single tube light circuit.

Solution:

For a tube light circuit, in addition to an On/Off switch we will require a choke and a starter. The choke and the starter help in developing a high voltage across the tube during starting so that the flourescent tube gets illuminated.

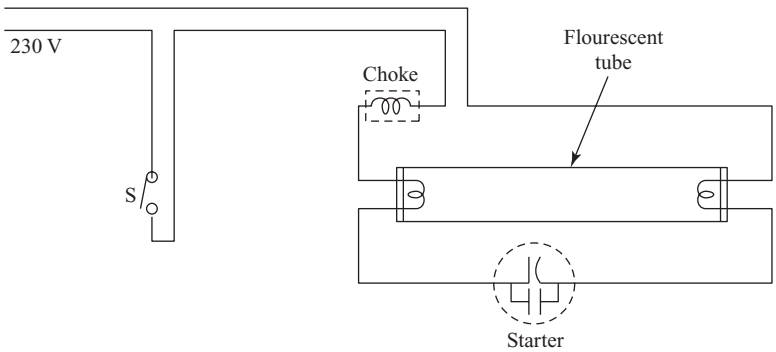


Figure 6.15 *A tube light circuit*

Example 6.2 Draw the circuit for a staircase lamp controlled from two positions. It should be possible to switch-on or switch-off the lamp by any of the two switches, one located upstairs and the other located downstairs.

Solution:

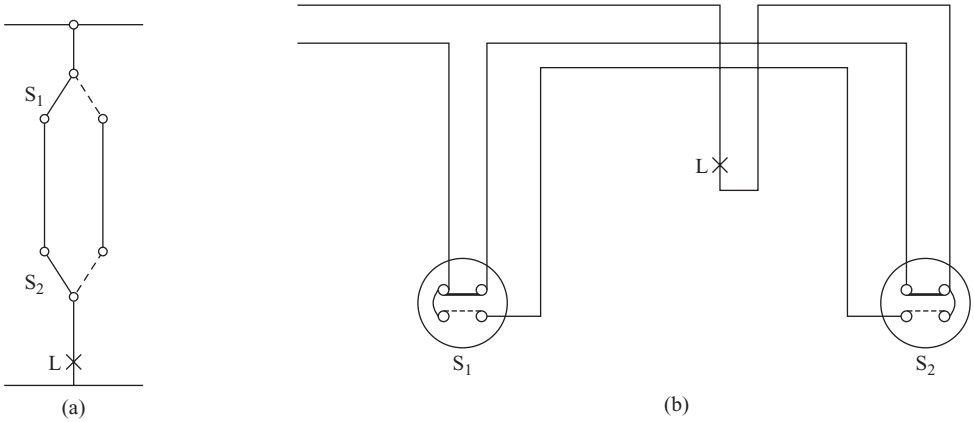


Figure 6.16 *Staircase lighting circuit*

Two two-way switches have been used to control the lamp L. Fig. 6.16 (a) shows the schematic diagram whereas Fig. 6.16 (b) shows the actual wiring diagram. Switch S_1 is fixed on the ground floor near the staircase while switch S_2 is fixed on the first floor. While going up S_1 is operated to switch on the light L as shown. After reaching the first floor, switch S_2 is operated. The switch contact moves to position shown by the dotted line. The lamp gits switched off as power supply to the lamp is now cut off.

Example 6.3 Two lamps and one fan are to be controlled by independent switches placed on a single switch board. Draw the schematic circuit diagram.

Solution:

The schematic circuit is shown in Fig. 6.17.

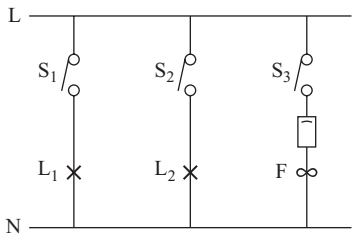


Figure 6.17 *Schematic diagram of a light and fan circuit*

Example 6.4 Draw the schematic diagram for one lamp to be switched on and off from any of the three positions.

Solution:

In this case we need to use a two two-way switch and an intermediate switch. S_1 and S_3 are the two-way switches and S_2 is an intermediate switch. The working of the single-way, two-way and intermediate switches have been shown in Fig. 6.18 (a), (b), and (c). These have been used to make the circuit as shown in Fig. 6.18 (d).

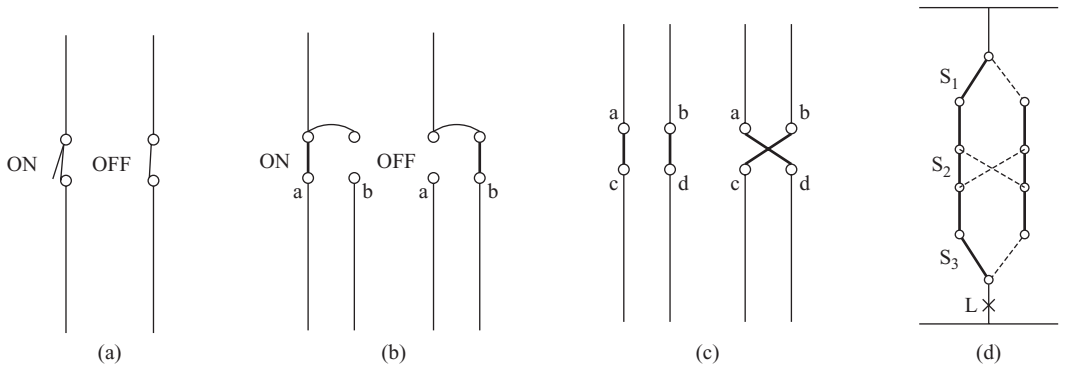


Figure 6.18 *(a) One-way switch; (b) two-way switch; (c) intermediate switch; (d) schematic diagram for a lamp controlled from any of the three positions*

6.10 CIRCUIT PROTECTIVE DEVICES AND SAFETY PRECAUTIONS

Circuits should be protected against any abnormal condition like overload and short circuit which may be due to any fault conditions or excess load connected to the circuit. Protective devices like fuses and circuit breakers are used, which help protect the circuit from burning out under abnormal conditions.

An important protective device used in all electrical installations is the fuse.

A fuse is a device that, by fusion of one of its specially designed and proportional component open the circuit in which it is inserted when the current through it exceeds a given value for a sufficient time.

Fuses are of two types, viz the *rewirable type* and the *cartridge type*.

A rewirable fuse consists of a length of wire made either of tinned copper or some other metal of such size that while it will carry the rated current, it will fuse, i.e., melt, and thus break the circuit if the current rises above the value for which the circuit is rated. Rewirable fuse has the disadvantage that the wire can be replaced by any other wire of any size, which will defeat the purpose for which a fuse is used. Often the user may replace a fuse by a thicker wire which is not desirable at all.

A rewirable fuse also gives an external flash when blown off.

The performance of a fuse is improved by having the fuse wire in a sealed cartridge packed with filler material. Such a fuse is called a cartridge fuse. There are two types of cartridge fuses, viz diazed-type fuse, or simply D-type cartridge fuse and high rupturing capacity, or HRC cartridge fuse. In cartridge fuses, the cartridge, which carries the fuse wire has to be replaced by another cartridge of the same rating. A cartridge of some other rating will not fit into the base. This restricts the user in replacing the fused cartridge by another cartridge of the same rating

For the protection of a power system against any fault condition, e.g., single-phase-to-ground fault or phase-to-phase fault or three-phase fault, protective devices like *relays* and *circuit breakers* are incorporated in the system.

A circuit breaker will break the circuit automatically under any fault conditions. Circuit breakers can be operated by remote control with the help of relays. Relays detect the fault and initiate tripping of the circuit breaker. Under fault conditions the electrical quantities like current, voltage, and frequency become abnormal and is sensed by the relay. There are various types of relays, like distance or impedance relay, induction-type over-current relay, differential relay, etc.

6.10.1 Safety Precautions in Using Electricity

Electricity has to be used very carefully. Careless use may lead to severe consequences to the person using electricity or to the system. Every circuit should be given supply through a switch–fire arrangement. The phase provided should be of the proper rating. No connection should be taken directly using naked wires. All metallic parts of electrical equipment should be earthed.

6.11 EFFICIENT USE OF ELECTRICITY

Electrical energy is used in almost every place like in running electric trains and metro-rails, heating and cooling of buildings, creating cold storage facilities, illuminating building interiors, street lighting, etc. The demand for electricity is increasing day by day. Non-renewable energy resources like coal and gas are getting depleted. It is therefore important that we save electricity by reducing losses and misuse.

Energy efficient technologies, gadgets, and appliances are to be developed and the less efficient ones be replaced. For example, gradually, electric filament lamps will be replaced by CFLs (compact fluorescent lamps). Electronic fan regulators have already replaced the traditional variable-resistance-type regulators. For saving energy on heating and cooling, the losses due to leakage have to be reduced. Further, through proper automatic control devices, constant temperature could be maintained irrespective of variation of outside temperature. This will ensure that over cooling and over heating is not done. The persons using electricity need to be made aware of the need for efficient use of electricity.

A subject with the name of Energy Management deals with the reduction of losses in the transmission and distribution system, use of energy efficient technologies, conservation of energy, more and more emphasis on non-conventional sources of energy for generation of electricity, and energy audit to reduce losses, etc.

6.12 REVIEW QUESTIONS

A. Short Answer Type Questions

1. What are the various sub-systems of an electrical power system? Explain the significance of each of them.
2. What are the various ways of generation of electricity from natural resources? What are their limitations?
3. Draw and explain how electricity is generated in a coal-fired thermal power plant.
4. Explain the basic principle of generation of electricity in a nuclear power plant.
5. What is meant by renewable energy sources? Give two examples.
6. What is the basic principle of solar electricity generation. What are the requirements and limitations?
7. Give brief descriptions of generation of electricity from non-conventional energy sources.
8. Explain how electricity is brought from the generating stations to the consumers of electricity.
9. Compare dc and ac transmission systems.
10. Compare overhead versus underground distributions systems.
11. Explain the function of neutral wire and earth wire in an electrical distribution system.
12. Explain the importance of earthing of electrical systems.
13. Explain the method of earthing.

14. Explain the various types of electrical wiring.
15. Draw the schematic diagram of stair case lighting, i.e., one lamp controlled from two positions. Also draw the wiring diagram.
16. Draw the wiring diagram of a tube light circuit.
17. What are the protective devices used in power systems?

B. Multiple Choice Questions

1. The voltage range at which power is generated in power stations is
   (a) 440 V to 1 kV (b) 11 kV to 30 kV
   (c) 33 kV to 66 kV (d) 66 kV to 132 kV.
2. The generated voltage is transmitted through transmission lines after stepping up the voltage so that
   (a) the voltage drop in the transmission lines is reduced
   (b) for the same power transmitted the current flowing through the lines is reduced, there by reducing the transmission losses
   (c) the transmission lines are kept far away from the ground level
   (d) large amount of power could be transmitted.
3. In India, transmission voltage level range is
   (a) above 1000 kV (b) 765 to 1000 kV
   (c) above 400 kV (d) 66 kV 400 kV.
4. Which of the following is used as a prime mover in a nuclear power station?
   (a) Water turbine (b) Gas turbine
   (c) Steam turbine (d) Diesel engine.
5. Which of the following is renewable energy?
   (a) Coal (b) Oil
   (c) Gas (d) Solar.
6. Which one of the following is not used as a source of heat energy in a thermal power station?
   (a) Gas (b) Oil
   (c) Uranium (d) Coal.
7. In which of the following places in India, a nuclear power plant has not yet been constructed?
   (a) Tarapur in Maharastra
   (b) Rawatbhata (near Kota) in Rajasthan
   (c) Mehrauli in Delhi
   (d) Kalpakkam in Tamil Nadu.
8. Cogeneration means
   (a) the system where electricity, steam, and hot water are simultaneously produced
   (b) the system of generating electricity using both coal and diesel oil
   (c) the system of converting heat into mechanical energy at a very high temperature
   (d) the system of generating electricity and other forms of mechanical energy from any primary source of energy.
9. Which of the following is not true for the amount of power generated, P, in a hydroelectric power plan?
   (a) P depends on the quantity of water discharged
   (b) P depends on the head of water being discharged
   (c) P depends on the specific weight of water
   (d) P depends on the capacity of the water reservoir.
10. Which of the following is not the location of a hydroelectric power station in India?
    (a) Salal in Jammu and Kashmir
    (b) Nagarjun Sagar in Andhra Pradesh
    (c) Dehar in Himachal Pradesh
    (d) Faridabad near Delhi on river Yamuna.
11. Commercial solar cells producing electricity has an efficiency of the range.
    (a) 10 to 15 per cent (b) 20 to 40 per cent
    (c) 50 to 70 per cent (d) above 70 per cent.
12. Which of the following is not a part of the electricity distribution network?
    (a) Feeder (b) Distributor
    (c) Optical fiber (d) Service mains.
13. Which of the following statement is not true?
    (a) The neutral wire is taken out from the star point of the secondary of the distribution transformer
    (b) The neutral wire is taken out from the star point of the secondary of a step-up transformer
    (c) Each phase wire and the neutral wire constitute single-phase supply
    (d) The neutral wire is taken out from the star point of the secondary of the distribution transformer and is earthed of the substation.

14. In a distribution transformer
    (a) the primary windings are star connected and the secondary windings are delta connected
    (b) both the primary and secondary windings are delta connected
    (c) the primary windings are delta connected and the secondary windings are star connected
    (d) both the primary and secondary windings are star connected.
15. Which of the following types of wiring is used for purely temporary installations?
    (a) Concealed conduit wiring
    (b) Surface conduit wiring
    (c) Wood casing–capping wiring
    (d) Cleat wiring.
16. The function of choke and starter in a tube light circuit is to
    (a) create a high voltage across the tube during starting
    (b) improve the power factor of the tube light circuit
    (c) reduce the power consumed by the tube light circuit
    (d) help draw very high current during starting.

Answers to Multiple Choice Questions

1. (b)	2. (b)	3. (d)	4. (c)	5. (d)	6. (c)
7. (c)	8. (a)	9. (d)	10. (d)	11. (a)	12. (c)
13. (b)	14. (c)	15. (d)	16. (a)		

7

Field-effect Transistors

TOPICS DISCUSSED

- Field-effect transistors
- Metal-oxide semiconductor FET (MOSFET)
- Complementary MOSFETs (CMOS)

7.1 INTRODUCTION

It is expected that the students have already studied bipolar junction transistors (BJTs) in detail. However, a brief review has been provided in this section to help recapitulate the concepts.

A BJT is a three-layer, two-junction semiconductor device. It is formed by sandwiching a p-type or an n-type material between two opposite types of material forming either an n–p–n or a p–n–p transistor. The three regions of a transistor are called emitter, base, and collector. The term bipolar refers to the use of both holes and electrons as charge carriers in the transistor. The representation of two types of transistors is shown in Fig. 7.1. One type consists of two n regions separated by a p region (n–p–n), and the other type consists of two p regions separated by an n region (p–n–p).

As shown in Fig. 7.1, the base region is very thin when compared to the emitter and the collector region. The base region is also lightly doped while the emitter and collector regions are heavily doped.

Doping

The conductivity of silicon and germanium (semiconducting materials) can be drastically increased and controlled if certain amount of material with impurities is added with them. This process of adding impure material (n-type or p-type) to an intrinsic semiconductor material is called doping. Doping increases the number of charge carriers (electrons and holes) in the material.

7.1.1 N–p–n and p–n–p Transistors

The symbols for n–p–n and p–n–p transistors are shown in Fig. 7.2(a). The emitter terminal has an arrow. The two p–n junctions in the transistor must be properly biased with external dc voltages

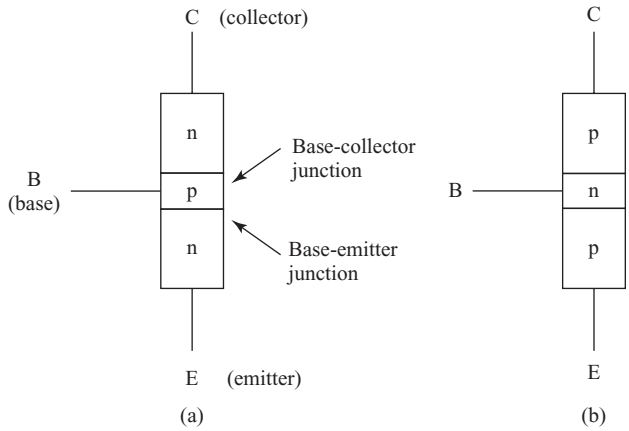


Figure 7.1 *Two types of bipolar junction transistors (a) n–p–n type (b) p–n–p type*

to operate a transistor as an amplifier. Figure 7.2(b) shows the bias arrangement for both n–p–n and p–n–p transistors.

In both the cases the base–emitter junction is forward-biased and the base–collector junction is reverse-biased. The biasing of the two types of transistors with dc voltages is shown in Fig. 7.3.

The directions of current in an n–p–n transistor and a p–n–p transistor have been shown. It is seen that in both the cases the emitter current is the sum of the collector current and the base current. That is,

$$I_E = I_C + I_B.$$

The transistor has two p–n junctions. They are like two diodes connected as shown in Fig. 7.4. The junction between the emitter and base may be called emitter–base junction or simply called emitter diode. The junction between the base and collector may be called collector–base junction or simply called collector diode.

The emitter diode is always forward biased and, hence, offers very small resistance. The collector diode is reverse biased and, hence, offers very high resistance.

With the dc bias, as mentioned above, a signal that may be very weak is introduced in the low resistance emitter–base circuit and the output is taken from the high resistance collector–base circuit.

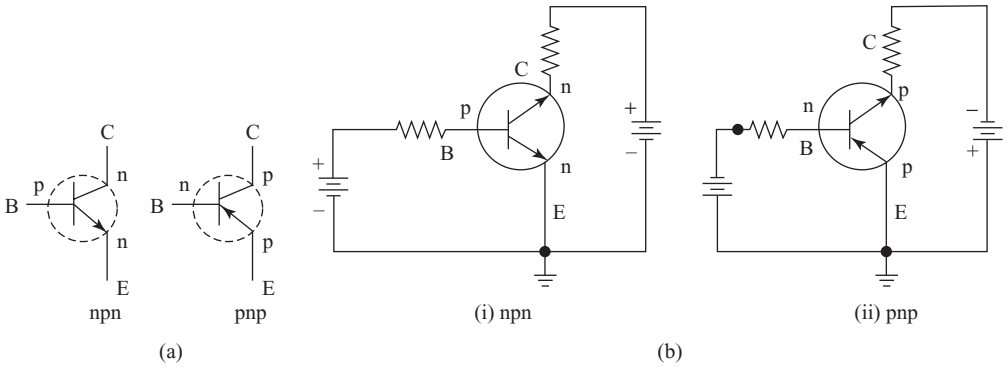


Figure 7.2 *(a) Transistor symbols, (b) forward–reverse biasing of BJT*

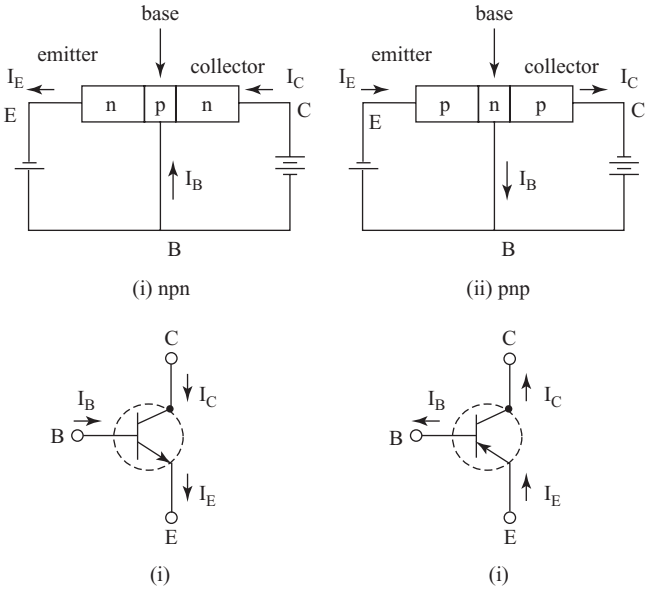


Figure 7.3 *Transistor biasing and transistor currents (i) n–p–n transistor (ii) p–n–p transistor.*

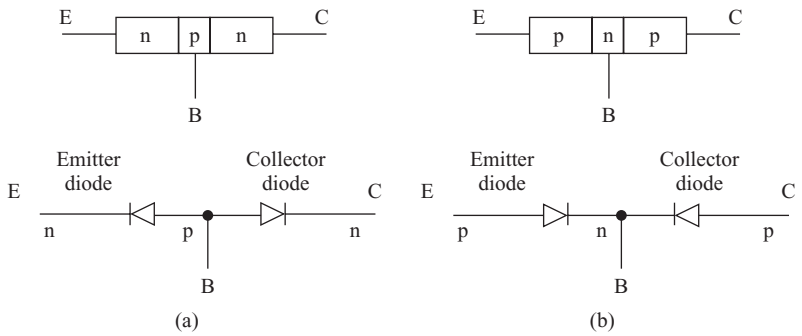


Figure 7.4 *Two-diode analogy of a transistor (a) npn transistor (b) pnp transistor.*

Therefore, a transistor transfers a signal from a low resistance circuit to a high resistance circuit. The name transistor has been given to this device by combining the two terms as

Transfer + Resistor → Transistor

7.1.2 Transistor Action

Let us consider an n–p–n transistor to explain the transistor action. The emitter–base junction is forward biased (i.e. p-region is connected to the positive terminal and n-region connected to the negative terminal of the battery). The collector–base junction is reverse biased with the dc voltage. The transistor is shown in an enlarged manner for the sake of clarity in Fig. 7.5.

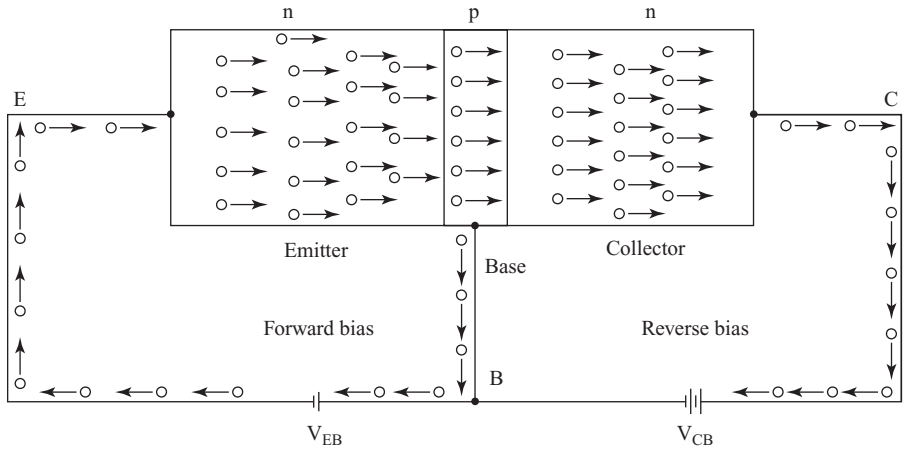


Figure 7.5 *Flow of charge carriers in a transistor when bias voltages are applied*

The forward bias causes electrons in the n-type emitter to flow towards the base. This constitutes emitter current, I_E. As these electrons flow through the p-type base, they tend to combine with the holes. As the base is lightly doped and is very thin, only a few electrons (less than 5 per cent) combine with the holes to constitute base current, I_B. The electrons that combine with the holes become valence electrons. Then as valence electrons they flow down through the holes and into the external base lead. This constitutes the base current.

The remaining (more than 95 per cent) electrons cross over into the collector region to constitute collector current I_C. Most of the electrons from the emitter region continue their journey through the base to the collector to form collector current because

(i) the base is lightly doped and is very thin. There are a few holes that find enough time to combine with electrons;

(ii) the reverse bias on collector is quite high and hence attracts more electrons towards the collector.

The input circuit comprises the emitter–base junction in a transistor. This junction offers very low resistance because it is forward biased. The output circuit comprises the collector–base junction, which is reverse biased. The output circuit has high resistance due to the reverse biased junction in the circuit.

We also know that most of the input–emitter current flows into the collector circuit. When an input ac signal is connected to the input circuit, the input signal from the low resistance circuit gets transferred to a high resistance collector circuit. *The amplifying capability of a transistor is dependent on the key factor that a signal gets transferred from a low resistance circuit to a high resistance circuit.*

In a transistor, the base current is very small as compared to emitter current.

$$I_E = I_B + I_C$$

When I_B is small, $I_E \simeq I_C$.

Note that the currents I_E, I_B and I_C expressed in capital letters are the dc values. These currents are related to two parameters, namely alpha dc (α_{DC}) and beta dc (β_{DC}).

$$\alpha_{DC} = \frac{I_C}{I_E} \text{ and } \beta_{DC} = \frac{I_C}{I_B}$$

β_{DC} is the dc gain and is usually designated as h_{FE} on the transistor data sheet.

$I_C = \alpha_{DC} I_E$ where α_{DC} has a value somewhat less than 1. Typically its value is between 0.950 and 0.995. If α_{DC} is taken approximately as 1, then $I_C \simeq I_E$.

The collector amount, $I_C = \beta_{DC} I_B$ where β_{DC} has a value between 20 and 200.

Transistor voltages

As shown in Fig. 7.6, the three voltages for a biased transistor are the emitter voltage, V_E, the collector voltage, V_C, and the base voltage, V_B. Note that these voltages are with respect to the ground potential, which is at 0 V.

The collector voltage,	$V_C = V_{CC} - I_C R_C$
The base voltage,	$V_B = V_E + V_{BE}$
Since the emitter, here, is common to ground,	$V_E = 0\text{V}$ and $V_B = 0.7$ V (given)

The two dc bias voltage sources can be replaced by a single dc voltage source using a resistive voltage divider as shown in Fig. 7.7.

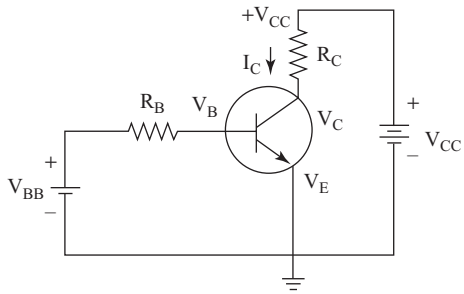


Figure 7.6 *Transistor bias voltages*

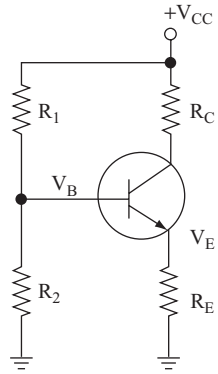


Figure 7.7 *Bias using voltage divider*

Two resistors, R_1 and R_2, form a voltage divider and provides base bias voltage. A resistor R_E is used to raise the potential of the emitter above ground potential.

$$V_B = \frac{R_2}{R_1 + R_2} \cdot V_{CC}$$

$$V_B - V_E = 0.7\,V$$

or,
$$V_E = V_B - 0.7\,V$$

Example 7.1 Calculate the values of V_B, V_E, V_C, V_{CE}, I_B, I_E and I_C in the transistor circuit shown in Fig. 7.8.

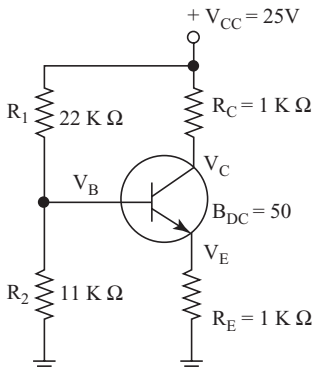


Figure 7.8

Solution:

Bias voltage,
$$V_B = \frac{R_2}{R_1 + R_2} \cdot V_{CC} = \frac{11}{22 + 11} \cdot 25 = 8.33\ V$$

$$V_B - V_E = 0.7\ V$$

or,
$$V_E = V_B - 0.7\ V = 8.33 - 0.7 = 7.63\ V$$

$$I_E = \frac{V_E}{R_E} = \frac{7.63}{1 \times 10^3} = 7.63 \times 10^{-3} A = 7.63\ mA$$

$I_E \simeq I_C$ since I_B is very small comparatively, i.e. α_{DC} is taken as 1.

$$I_C = 7.63\ mA$$

$$V_C = V_{CC} - I_C R_C = 25 - 7.63 \times 10^{-3} \times 1 \times 10^{-3}$$

or,
$$V_C = 17.37\ V$$

$$V_{CE} = V_C - V_E = 17.37 - 7.63 = 9.74 \text{ V}$$

$$\beta_{DC} = \frac{I_C}{I_B}$$

Therefore,

$$I_B = \frac{I_C}{\beta_{DC}} = \frac{7.63 \text{ mA}}{50} = 152.6 \times 10^{-6} \text{ A}$$

$$= 152.6 \ \mu\text{A}.$$

Note that the collector current is in mA and the base current is only in microamperes.

7.1.3 Transistor as an Amplifier

DC bias allows a transistor to work as an amplifier. A transistor can produce a larger ac signal using a smaller ac signal. Figure 7.9 shows the basic circuit of a transistor as an amplifier. The circuit produces an output signal, which is greater in amplitude than the input signal without any change in the signal waveform. The input signal in applied to the base. The collector voltage is the output voltage. In order to achieve gainful amplification, the input circuit should always remain forward biased. The magnitude of the dc voltage should be such that it always maintains the input circuit forward biased regardless of the polarity of the ac input signal. This means that the dc bias voltage at the base, $V_{EB,}$ should be higher than the maximum value of the ac input signal. The input signal voltage causes the base current to vary at the same frequency above and below its dc value.

A variation in base current causes a larger variation in the collector current. The variation in the collector current is much larger than the variation in the base current due to the current gain through the transistor. This current gain is designated as β_{ac} or h_{fc} as

$$\beta_{ac} = \frac{I_C}{I_B}$$

The value of β_{ac} is slightly different than β_{DC} for a given transistor.
Let us now calculate the voltage gain, A_v, of the amplifier.

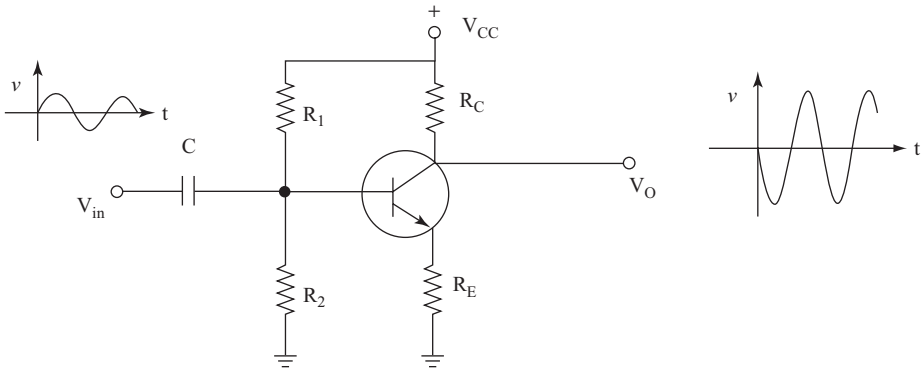


Figure 7.9 *Transistor as a voltage amplifier*

Voltage gain, $$A_v = \frac{V_C}{V_e} = \frac{I_C R_C}{I_e R_E} = \frac{R_C}{R_E}$$

$$(\text{since } I_c = I_e)$$

There is a phase difference between the input and output voltages by 180°. The input voltage appears inverted and magnified at the output. Note that I_c, I_b, etc. are the ac values for which lower subscripts have been used.

Example 7.2 Calculate the output voltage in the transistor amplifier circuit shown in Fig. 7.10. Also calculate the dc voltage at the collector on which the output signal will be superimposed.

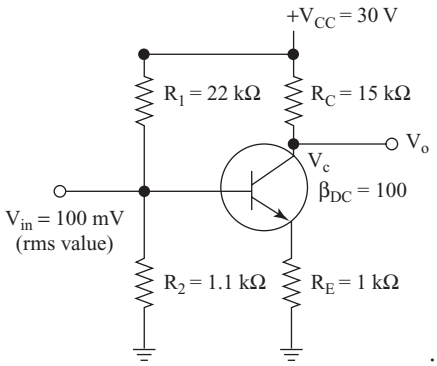


Figure 7.10

Solution:

Voltage gain, $$A_v = \frac{V_o}{V_{in}}$$

$$V_{in} = 100 \text{ mV}$$

$$A_n = \frac{R_C}{R_E} = \frac{15}{1} = 15$$

$$V_o = A_v V_{in} = 15 \times 100 \times 10^{-3} = 1.5 \text{ V (RMS value)}$$

$$= \sqrt{2} \times 1.5 \text{ V (Max value)}$$

$$= 2.12 \text{ V (Max)}$$

$$V_c = V_{cc} - I_c R_c$$

$$I_C \simeq I_E = \frac{V_E}{R_E}$$

$$V_E \simeq V_B$$

$$V_B = \frac{R_2}{R_1 + R_2} \cdot V_{CC} = \frac{1.1}{22 + 11.1} \cdot 30 = 1.428 \text{ V}$$

$$I_C \simeq I_E = \frac{V_E}{R_E} = \frac{V_B - 0.7}{R_E} = \frac{1.428 - 0.7}{1 \times 10^3} = 0.728 \text{ mA}$$

$$V_c = V_{cc} - I_c R_c$$

$$= 30 - 0.728 \times 10_{-3} \times 15 \times 10^3$$

$$= 30 - 10.9$$

$$= 19.1 \text{ V}$$

$V_c = 19.1$ V is the dc level of the output voltage. The ac output voltage will get super-imposed on this voltage as shown in Fig. 7.11.

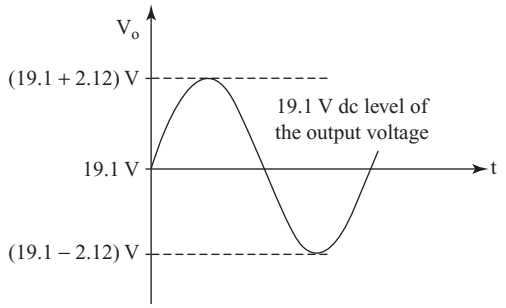


Figure 7.11 *Output amplifier voltage*

We have just seen how a transistor can be used as an amplifier. Transistor amplifiers can be made in common–emitter, common–collector and common–base configurations. Transistor amplifiers have two types of operations—class A and class B. In class A amplifiers, the transistor conducts for the full cycle of the input signal, i.e. for 360°.

In class B amplifiers, the transistor operates for 180° of the input cycle and is in cut-off state for 180°. The primary advantage of a class B amplifier over a class A amplifier is that a class B amplifier is more efficient, i.e. you can get more output for a given amount of input.

7.1.4 BJT as a Switch

When the base–emitter junction of a transistor is not forward biased, the transistor will not be in cut-off state and will work as an open switch as shown in Fig. 7.12(a). The transistor can be brought to saturation state by forward biasing the base–emitter and base-collector junctions and making the base current large enough to cause collector current reach saturation value. This condition represents a short between the collector and the emitter, which is equivalent to a closed switch as shown is Fig. 7.12(b).

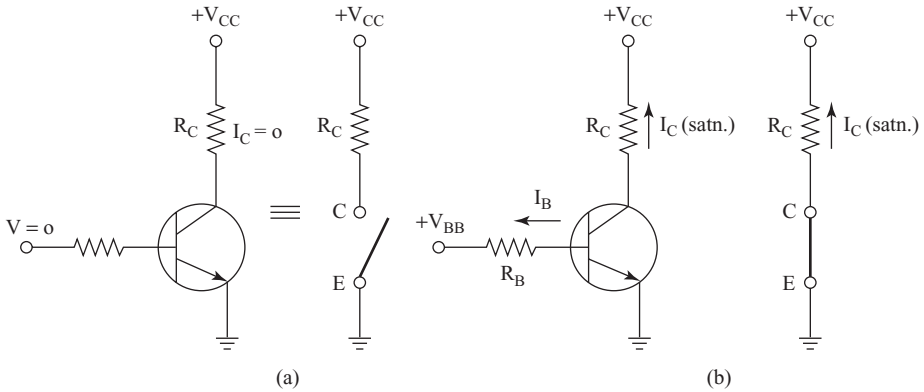


Figure 7.12 *Switching action of a transistor (a) Switch open (b) Switch closed*

The minimum value of base current, $I_{B,}$ to produce saturation in the collector current is given by the relation,

$$I_B(\text{Minimum}) = \frac{I_C(\text{Saturated})}{\beta_{DC}}$$

We have observed that a bipolar junction transistor (BJT) is a current-controlled device. The base current controls the magnitude of the collector current.

7.2 FIELD-EFFECT TRANSISTORS

The base current controls the amount of collector current in a bipolar transistor. A BJT is thus called a current-controlled device. In a field-effect transistor (FET) the voltage at the gate terminal controls the amount of current through the transistor. Hence, an FET is called a voltage-controlled device. In a BJT the gate is forward biased and has very low input impedance. FETs have high input impedance when compared to BJTs. This is because the gate of an FET is reverse biased.

There are two types of FETs, namely, the junction field-effect transistor (JFET) and the metal-oxide semiconductor field-effect transistor (MOSFET).

Like BJTs, FETs can be used in amplifiers and switching circuits. Since the gate of an FET is reverse biased, the device virtually requires no input current and its input impedance is very high. This is the most important advantage of the FET over BJT for certain applications.

7.2.1 Junction Field-effect Transistor (JFET)

A JFET is a three-terminal device in which conduction of current is caused by a single type of carrier, i.e. either electrons or holes. The current conduction is controlled by means of an electric field established between the gate electrode and the conducting channel of the device. The field is established due to the application of a negative voltage at the gate. JFETs are subdivided into n-channel and p-channel devices.

Construction details

A JFET consists of either a p-type or an n-type silicon bar (channel) containing two p–n junctions on either sides of the bar. The silicon bar forms the conducting channel for the charge carrier. If the

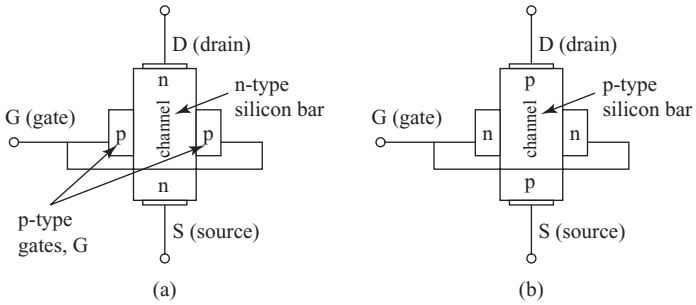


Figure 7.13 *Junction field-effect transistors (a) n-channel JFET (b) p-channel JFET*

silicon bar is of n-type, the device is called n-channel JFET and if the bar is of p-type, it is called p-channel JFET.

The two p–n junction forming diodes are connected internally and a common terminal called gate is taken out as shown in Fig. 7.13.

The n-type or p-type semiconductor material forms the channel as shown. The ends of the channel are designated as drain (D) and source (S). In an n-channel JFET, two pieces of p-type material are connected together and their terminal is named gate (G). Similarly, in a p-channel JFET, the two pieces of n-type material are connected together and their terminal is named gate (G). The channel is more lightly doped than the gate material.

n-Channel JFET

We will now examine how the gate voltage controls the current through the channel in an n-channel JFET. Let us assume that no voltage is applied to the gate, i.e. the voltage between the gate and the source, $V_{GS} = 0$. Let a drain-source voltage, $V_{DS,}$ be applied with positive at the drain and negative at the source. A drain current, $I_{D,}$ will flow as shown in Fig. 7.14(a). Next, when a gate voltage is applied with the gate negative with respect to the source, the gate-channel p–n junctions are reverse biased. Note that the two gate terminals are interconnected. So, there are now two reverse biased p–n junctions. With the increase of V_{GS}, the depletion layer of the p–n junctions will increase. As the channel is more lightly doped than the gate material, the depletion region will penetrate deep into the channel as shown in Fig. 7.14(b).

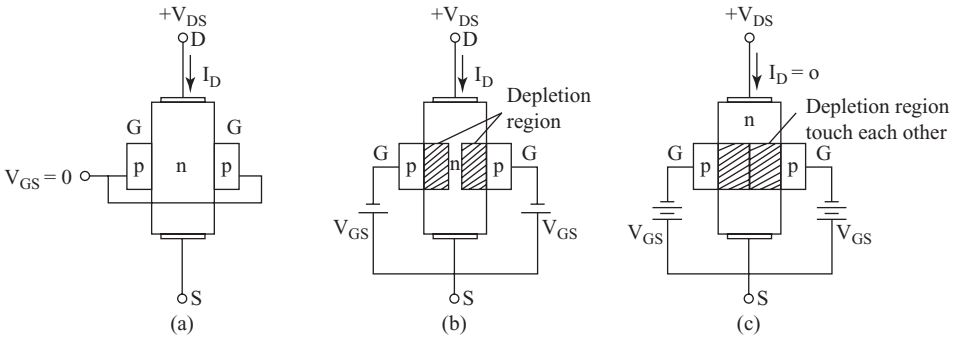


Figure 7.14 *Effect of change of gate voltage on current through the channel in an n-channel JFET. (a) No gate voltage (b) Negative gate voltage (channel width gets narrowed) (c) More negative gate voltage (channel closed)*

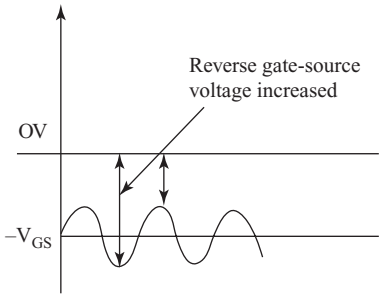


Figure 7.15 *Variation in gate-signal voltage when an ac signal is applied to the gate circuit*

Because the depletion region is depleted of charge carriers, they will behave as insulators. As a result, the channel width is narrowed and there is an increase in the resistance to the flow of current, I_D. Hence, I_D gets reduced.

When the negative gate-source bias voltage is further increased as in Fig. 7.14(c), the depletion regions are further increased and they will meet each other. The channel closes and when that happens no current will be able to flow. Hence, I_D will be zero.

If an ac signal is applied to the gate, the reverse gate-source voltage will increase when the signal goes negative and will decrease when the signal goes positive as shown in Fig. 7.15. The greater the negative voltage, the greater the widening of the depletion region (or narrowing the channel width). Hence, there will be an increase in the channel resistance. As a consequence, the drain current will get reduced.

As the signal becomes positive, the depletion region gets narrower and more current flows through the channel. This way the gate-source voltage, V_{GS}, controls the drain current.

The depletion regions in the channel in an FET are produced by the electric field at the reverse biased gate-channel junctions. That is why the name field-effect transistor was suggested.

Thus, it is seen that the junction field-effect transistor is always operated with reverse biased gate-to-source p–n junction. The reverse bias with a negative gate voltage produces depletion regions in the n-channel. The width of the depletion regions increases with the increase in the negative gate voltage. The channel width and channel resistance are controlled by the gate voltage. The amount of drain current, I_D is thus controlled by the gate voltage.

p-Channel JFET

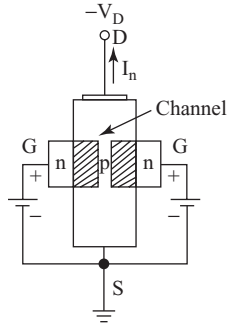


Figure 7.16 *P-channel JFET (Negative supply terminal is connected to terminal D and gate-source p–n junction is reverse-biased)*

Note that a p-channel JFET operates exactly the same way as an n-channel JFET except that current directions and voltage polarities are reversed as shown in Fig. 7.16.

7.2.2 JFET Symbols

The symbols for n-channel and p-channel JFETs have been shown in Fig. 7.17. The arrow-head for n-channel JFET is inward and for p-channel is outward.

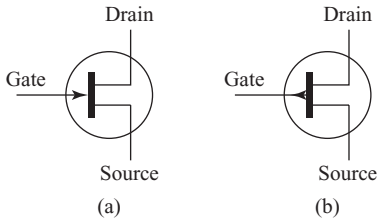


Figure 7.17 *Symbols of JFETs (a) n-channel (b) p-channel*

7.2.3 Characteristics of JFETs

We shall now study the drain characteristics and transfer characteristics of a JFET.

Drain characteristics (output characteristics)

This characteristic shows the relation between drain current, I_D and drain-source voltage, V_{DS}. This relationship is studied for different values of gate voltage, V_{GS}.

We will first draw the drain characteristics with $V_{GS} = 0$. Then we will apply V_{GS} gradually and in each case draw the I_D versus V_{DS} characteristics.

(a) Drain characteristics with VGS = 0

The relation between drain current, I_D, and drain-source voltage, V_{DS}, of a JFET at constant gate-source voltage, V_{GS}, is called the drain characteristics or output characteristics of the JFET. The circuit diagram for determining the output characteristics at no gate voltage is shown in Fig. 7.18.

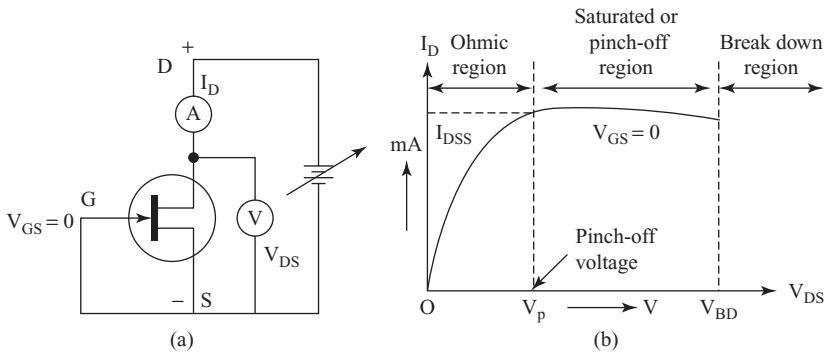


Figure 7.18 *(a) Circuit for obtaining I_D versus V_{DS} characteristics, (b) I_D/V_{DS} characteristic of an n-channel JFET with $V_{GS} = 0$.*

As can be seen from the I_D versus V_{DS} characteristic, when $V_{DS} = 0$, $I_D = 0$. With the increases in V_{DS}, the drain current increases linearly as the channel offers almost a constant value resistance for the current I_D. Current I_D produces voltage drops along the channel. As the gates are connected to the source, S, the gates become negative with respect to the potential of a point on the channel. This negative bias makes the depletion region of the p–n junction to penetrate into the channel. As a consequence, the channel resistance gets affected due to the depletion region. As V_{DS} is further increased, I_D increases at a reduced rate. However, increase in I_D causes further increase in the voltage drop in the channel and negative bias of the gate. The depletion layer further penetrates into the channel and channel resistance is increased. A saturation level of I_D is reached when further increase in V_{DS} will have no effect on the value of I_D. *The drain-source voltage at which the drain current reaches a saturation value with* $V_{GS} = 0$*, is called the pinch-off voltage,* V_P *as has been shown in Fig. 7.18(b). The drain-source saturation current is designated as* I_{DSS}. For the value of V_{DS} from 0 volt to V_P volts, the channel offers more or less a constant resistance to the current flow, which is thus called channel ohmic region. If the V_{DS} is increased beyond the pinch-off region [see Fig. 7.18(b)], the gates channel junction breaks down and I_D increases sharply to destroy the device.

The JFET operates at the pinch-off region at which $I_D = I_{DSS}$ with $V_{GS} = 0$.

(b) Drain characteristics (output characteristics) with negative VGS.

The circuit diagram for obtaining the I_D versus V_{DS} characteristics for an n-channel JFET at different V_{GS} is shown in Fig. 7.19(a). The gate voltage is set at increased negative voltages of –1 V, –2 V, –3 V and –4 V. The values of I_D for increasing values of V_{DS} is plotted as shown in Fig. 7.19(b).

A series of I_D versus V_{DS} characteristics can be drawn for different values of negative voltage as gate-source bias voltages.

To draw the characteristics, we have to keep V_{GS} fixed at some value, say –1 V and change the drain-source voltage, V_{DS}, in steps. Corresponding to each value of V_{DS}, the drain current, I_D, is noted. A plot of these values gives one output characteristic of JFET at $V_{GS} = -1$ V. A family of output characteristics at different gate-source voltages can be drawn by repeating this procedure.

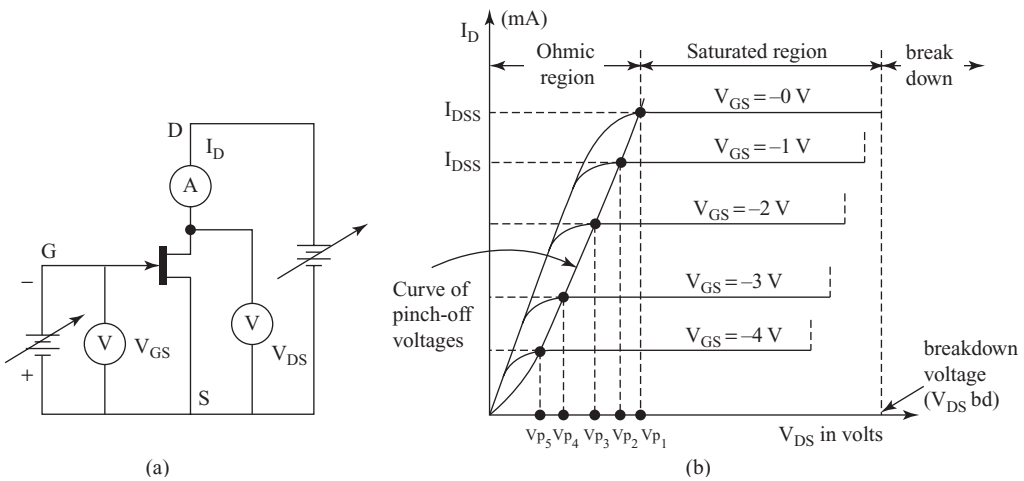


Figure 7.19 *Determination of* I_D *versus* V_{DS} *characteristics (i.e output characteristics) of an n-channel JFET at different values of* V_{GS}

Observations from the drain characteristics

The characteristics have been divided into three regions, i.e. ohmic region, saturation region, and breakdown region.

Ohmic region: The drain current, I_D, varies with the variation of the drain to source voltage, V_{DS}, in the ohmic region. The JFET is, therefore, said to be operating as a voltage variable resistance. The resistance offered by the JFET decreases with the increase in the negative bias voltage. The FET resistance in the ohmic region, i.e. resistance between the drain and the source, R_{DS}, is

$$R_{DS} = \frac{V_P}{I_{DSS}}$$

where V_P is the pinch-off voltage and I_{DSS} is the maximum saturated value of the drain current.

Saturation region: Saturation region is that region or portion of the output characteristic where the current I_D remains constant at I_{DSS} and does not change with the change in V_{DS}. A JFET is operated in the saturation region when used as an amplifier. The V_{DS} at which the saturation to drain current begins when $V_{GS} = 0$ is referred to as the pinch-off voltage. If now the $V_{Gs} = 0$ is made negative, the pinch-off will occur for smaller values of V_{DS} and I_{DSS} will be smaller. Pinch-off voltages for various values of V_{Gs} have been shown in Fig. 7.19(b). At V_{GS} equal to $-V_P$, the drain current will be zero and the device is cut-off.

FET breakdown Region: When a JFET is operating in the saturation region, I_D does not change with the change in the V_{DS} up to a certain value of V_{DS}. If V_{DS} is increased further, i.e. beyond this value of V_{DSbd} as shown in Fig. 7.19, the curve for $V_{GS} = 0$. The gate channel junction breaks down and the drain current shoots up suddenly to a very high value. This heavy current flow through the device may damage the transistor. The value of breakdown voltage is dependent on the negative value of V_{GS}. The breakdown voltage decreases with the increase in negative gate voltage V_{GS}.

Cut off condition: With the increase in the negative V_{GS} voltage, the channel width available for conduction decreases. At a certain voltage called V_{GS} (off), the depletion regions touch each other to close the channel completely. This is called cut-off stage when $I_D = 0$ and $V_{GS} > V_{GS}(off)$.

Transfer characteristics

The transfer characteristics for an n-channel JFET is the plot of output current I_D versus the input controlling voltage, which is V_{GS}. The gate-source voltage of an FET controls the magnitude of drain current and the characteristic shows how I_D is controlled by V_{GS}. The circuit for determining the transfer characteristic and the plot of I_D versus V_{GS} has been shown in Fig. 7.20. The two extremes of the transfer characteristic are $I_D = I_{DSS}$ at $V_{GS} = 0$ and $I_D = 0$ at $V_{GS} = -V_{GS}(off)$. The pinch-off voltage V_P is equal to $-V_{GS}(off)$.

If we have the family of drain characteristics, the transfer characteristic can be drawn from these drain characteristics.

The relationship between I_D and V_{GS} is defined by Shockley's equation, which states that

$$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_p \text{ or } V_{GS}(off)}\right]^2 \tag{7.1}$$

where I_D is the drain current at a given $V_{GS,}$ I_{DSS} is the maximum drain-source saturation current when $V_{GS} = 0$, V_P is the pinch-off voltage,V_{GS} (off) is the gate-source cut-off voltage and V_{GS} (off) = V_P.

The FET transfer characteristic approximately follows Shockley's equation as mentioned above.

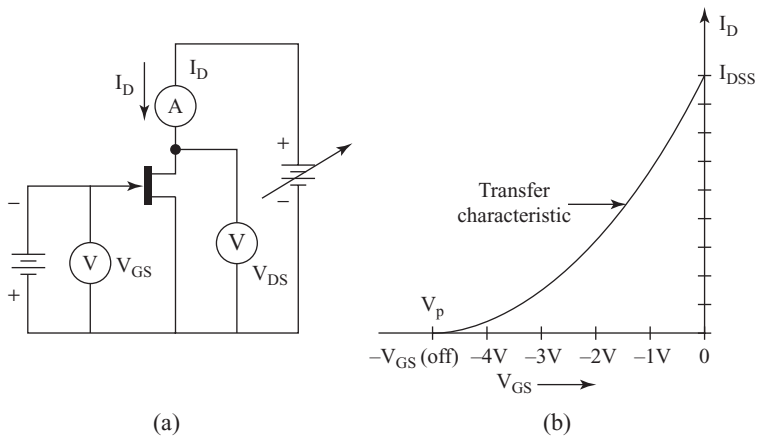


Figure 7.20 *(a) Circuit diagram for determining transfer characteristic (b) Transfer characteristic*

7.2.4. Parameters of JFET

The drain current is a function of gate-source voltage and drain-source voltage for an FET. These quantities vary by small amounts over their values when a small ac signal is applied. The ac component of the drain current, i_d, can be expressed in terms of the ac component of the gate voltage v_{gs} and the ac component of the drain-source voltage v_{ds} as

$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_d} \tag{7.2}$$

The three important parameters of a JFET are as follows:

(i) Drain resistance (r_d): It is defined as, $r_d = \dfrac{\partial V_{DS}}{\partial I_D}$ at constant V_{GS}. The value of r_d can be calculated from the output characteristics of a JFET.

(ii) Transconductance (g_m): It is defined as, $g_m = \dfrac{\partial I_D}{\partial V_{GS}}$ at constant V_{DS}. This value can be obtained from the transfer characteristics of the JFET.

(iii) Amplification factor (μ): It is defined as, $\mu = \dfrac{\partial V_{DS}}{\partial V_{GS}}$ at constant I_D.

The relation between the three parameters of a JFET is given as

Amplification factor = Drain resistance × Transconductance

i.e., $$\mu = r_d \times g_m \tag{7.3}$$

AC equivalent Circuit

From Eq. (7.2), we have $v_{ds} = i_d r_d - r_d g_m v_{gs}$

or, $$v_{ds} = i_d r_d - \mu v_{gs}. \tag{7.4}$$

The quantities related in Eq. (7.4) are shown as an equivalent circuit with a voltage source μv_{gs} in Figure 7.21.

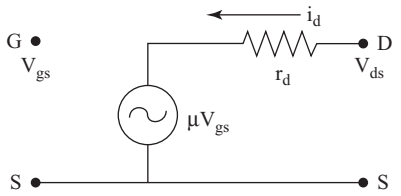


Figure 7.21

Example 7.3 An n-channel JFET has I_{DSS} = 12 mA and pinch-off voltage, V_p = –4V. Calculate the drain current for V_{GS} = –2 V. If the tranconductance, $g_{m,}$ of a JFET with the same I_{DSS} at V_{GS} = 0 is 4×10^{-3} mho, find the pinch-off voltage.

Solution:

From Shockley's equation we have,

$$I_D = I_{DSS}\left[1-\frac{V_{GS}}{V_P}\right]^2$$

Given, $I_{DSS} = 12$ mA, $V_p = -4$ V, $V_{GS} = -2$ V

Substituting the given values,

$$I_D = 12\left[1-\frac{(-2)}{(-4)}\right]^2 \text{ mA}$$

$$= 12\cdot\frac{1}{4}\text{ mA}$$

$$= 3 \text{ mA.}$$

$$g_m = \frac{\partial}{\partial V_{GS}}\bigg|_{V_{DS}=\text{Constant}}$$

$$= \frac{\partial}{\partial V_{GS}} I_{DSS}\left[1-\frac{V_{GS}}{V_P}\right]^2$$

$$= \frac{-2I_{DSS}}{V_P}\left[1-\frac{V_{GS}}{V_P}\right]$$

$$= g_{mo}\left[1-\frac{V_{GS}}{V_P}\right]$$

where $$g_{mo} = \frac{-2I_{DSS}}{V_P}$$

g_{mo} is the value of g_m when $V_{GS} = 0$.

Therefore, $$V_P = \frac{-2I_{DSS}}{g_{mo}} = -\frac{2 \times 12\,\text{mA}}{4 \text{ millimho}} = -6 \text{ volts.}$$

Example 7.4 Figure 7.22 shows the transfer characteristics of a JFET. Write the equation for the drain current.

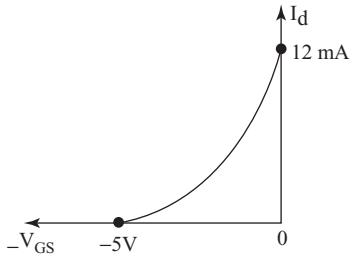


Figure 7.22

Solution:

Given $$I_{DSS} = 12 \text{ mA}$$

$$V_{GS} \text{ (off) or } Vp = -5 \text{ V.}$$

From Shockley's equation, $$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_P}\right]^2$$

Substituting given values, $$I_D = 12\left[1 + \frac{V_{GS}}{5}\right]^2 \text{ mA.}$$

Example 7.5 A JFET has a drain current of 5 mA. If $I_{DSS} = 10$ mA and $V_{GS}(off) = -6$V, find the value of V_{GS} and that of V_p.

Solution:

$$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_{GS}(off)}\right]^2$$

Substituting values, $$5 = 10\left[1 + \frac{V_{GS}}{6}\right]^2$$

$$\sqrt{\frac{5}{10}} = 1 + \frac{V_{GS}}{6}$$

or, $$1 + \frac{V_{GS}}{6} = 0.707$$

or, $$V_{GS} = 6(0.707 - 1)$$

or, $$V_{GS} = -1.76 \text{ V}$$

$$V_P = -V_{GS}\,(\text{off}) = 6 \text{ V}.$$

Example 7.6 When the V_{GS} of a JFET changes from –3.1 V to –3 V, the drain current changes from 1 mA to 1.3 mA. What would be the value of the tranconductance of the JFET?

Solution:
Given $\Delta V_{GS} = 3.1 - 3 = 0.1\text{V}$ (in magnitude)

$$\Delta I_D = 1.3 - 1.0 = 0.3\text{mA}.$$

Transconductance, $$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{0.3\,\text{mA}}{0.1\text{V}}$$

$$= 3 \text{ mA/V}$$

$$= 3 \text{ micro mho}.$$

Example 7.7 The following readings were obtained experimentally from a JFET:

V_{GS}	0 V	0 V	–0.2 V
V_{DS}	7 V	15 V	15 V
I_D	10 mA	10.25 mA	9.65 mA

Determine the (a) ac drain resistance; (b) transconductance and (c) amplification factor.

Solution:
(a) From the given readings we find that with constant V_{Gs} at OV,

$$\partial V_{DS} = 15 - 7 = 8 \text{ V}$$

$$\partial I_D = 10.25 - 10 = 0.25 \text{ mA}.$$

So, ac drain resistance,
$$r_d = \left.\frac{\partial V_{DS}}{\partial I_D}\right|_{\text{const.}V_{GS}}$$

$$= \frac{8 \text{ V}}{0.25 \text{ mA}} = 32 \text{ K}\Omega$$

(b) Again from the readings it is seen that with V_{DS} constant at 15 V,

$$\partial V_{GS} = 0.2 - 0 = 0.2 \text{ V}$$

$$\partial I_D = 10.25 - 9.65 = 0.6 \text{ mA}.$$

So, transconductance,
$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\text{const } V_{DS}}$$

Or,
$$g_m = \frac{0.6 \text{ mA}}{0.2 \text{V}}$$

$$= 3000 \ \mu \text{ mho}$$

(c) Amplification factor, $\mu = g_m \times r_d$

$$= (3000 \times 10^{-6})\ (32 \times 10^3)$$

Or, $\mu = 96.$

Example 7.8 An n-channel JFET has the following parameters: I_{DSS} = 10 mA and $V_p = -3.5V_p$. What will be the value of the drain current I_D when $V_{Gs} = -2.5$ V?

Solution:

$$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_P}\right]^2$$

$$= 10\left[1 - \frac{-2.5}{-3.5}\right]^2$$

$$= 10\ [1 - 0.714]^2$$

$$= 0.8\ 18 \text{ mA}$$

Example 7.9 The pinch-off voltage of a p-channel JFET is given as V_P = 5 V and $I_{DSS} = -40$ mA. The drain source voltage V_{DS} is such that a drain current $I_{DS} = -15$ mA is maintained. Find the gate-source voltage.

Solution:

We know,
$$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_P}\right]^2$$

Substituting values, $$-5 = -40\left[1-\frac{V_{GS}}{5}\right]^2$$

or, $$\left(1-\frac{V_{GS}}{5}\right)=\sqrt{\frac{15}{40}}$$

or, $$\frac{V_{GS}}{5}=1-\sqrt{\frac{15}{40}}$$

or, $$V_{GS}=5\left(1-\sqrt{\frac{15}{40}}\right)$$

or, $$V_{GS}= 1.94 \text{ V}.$$

Example 7.10 With constant V_{DS} when V_{GS} is changed from –1 V to –2 V, the drain current of an FET drops from 6 mA to 4 mA. If the ac drain resistance is 100 kΩ, calculate the amplification factor of the FET.

Solution:

The transconductance, $$g_m=\left.\frac{i_d}{v_{gs}}\right|V_{DS}$$

$$g_m=\frac{(6-4)\,\text{mA}}{-1-(-2)}=\frac{2}{-1+2}=2\,\text{mA/V}$$

Amplification factor, $\mu = g_m r_d$

or, $$\mu = 2 \text{ mA/V} \times 100 \text{ k}\Omega$$

or, $$\mu = 200.$$

7.3 METAL-OXIDE SEMICONDUCTOR FET (MOSFET)

The channel width in a JFET can only be decreased by reverse biasing the gate. With the gate reverse biased, the channel is depleted of charge carriers and hence is called depletion mode of operation. JFET can only be operated in depletion mode.

However, another category of FETs can be operated on both depletion mode and on enhancement mode.

Metal-oxide semiconductor field-effect transistor (MOSFET) falls under the second category of field-effect transistors. The MOSFET is also often known as the insulated-gate field-effect transistor (IGFET) and metal-insulator-semiconductor field-effect transistor (MISFET). Commercially, MOSFET is more important that JFET because MOSFETs are suitable for very large-scale integration (VLSI) fabrication.

The construction details of a MOSFET are different from a JFET. A MOSFET has no p–n junction between the gate and the channel. Instead, the gate of the MOSFET is insulated from the channel

by a layer of silicon dioxide (S_iO_2). Like JFETs, MOSFETs can be of n-channel type or p-channel type. There are two basic types of MOSFETs—depletion MOSFET (D-MOSFET) and enhancement MOSFET (E-MOSFET).

D-MOSFETs can be operated in both depletion-mode and enhancement mode whereas E-MOSFETs can be operated only in enhancement mode.

7.3.1 Depletion MOSFET

Figure 7.23 illustrates the basic structure of a D-MOSFET. The construction of n-channel and p-channel MOSFETs has been shown. The p-channel MOSFET consists of a lightly doped n-type semiconductor substrate into which two heavily doped p-regions are formed. These p-regions act as *source* and *drain,* respectively. The n-channel construction is the same except that a lightly doped p-type semiconductor substrate is used into which two heavily doped n-regions are formed

A thin layer of metal oxide (SiO_2) is deposited over a small portion of the channel. A metallic gate is formed over the oxide layer.

The gate becomes insulated from the channel as SiO_2 is an insulator. Since the gate is insulated from the channel, either a positive voltage or a negative voltage can be applied to the gate. This way a D-MOSFET can be operated in both depletion mode and enhancement mode.

The n-channel MOSFET operates in the depletion mode when a negative voltage is applied between the gate and the source. It operates in the enhancement mode when a positive gate to source voltage is applied. Figure 7.24 illustrates the operation of an n-channel MOSFET in depletion mode with negative V_{GS} and in enhancement mode with positive V_{GS}.

The silicon dioxide layer between the gate and the channel works as an insulator. The conducting gate, the channel and the insulating layer in between them forms a parallel plate capacitor. The SiO_2 layer is the dielectric.

We will now examine how an n-channel D-MOSFET works in both depletion mode and in enhancement mode.

Depletion mode of operation

We apply positive voltage at the drain-source terminal but negative voltage at the gate terminal. When a positive voltage is applied to the drain with respect to the source, a drain current will flow with no

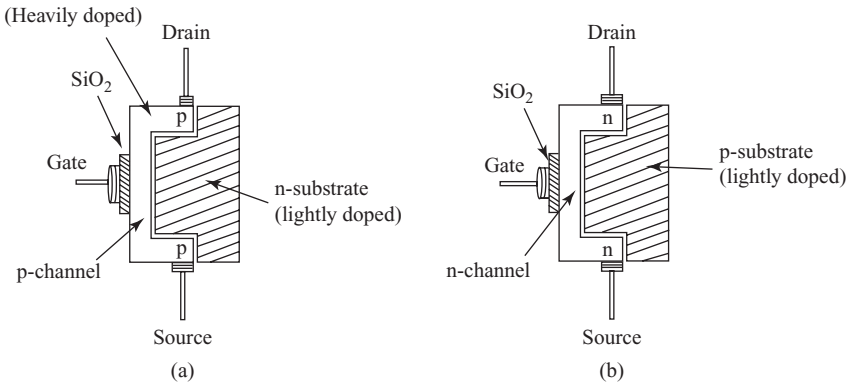


Figure 7.23 *Basic structure of D-MOSFET (a) p-channel D-MOSFET (b) n-channel D-MOSFET*

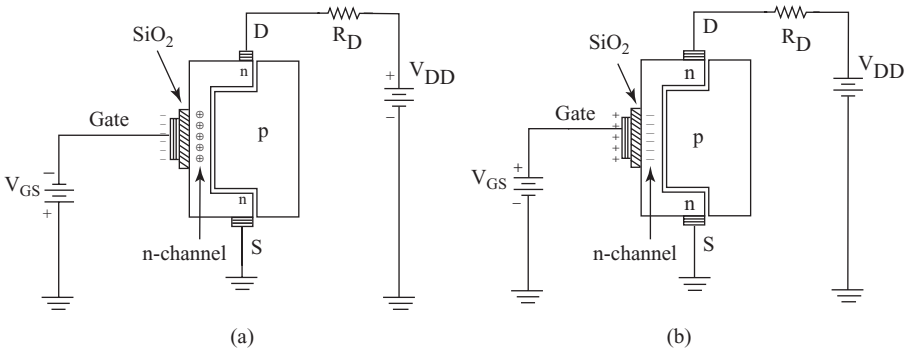


Figure 7.24 *Operation of n-channel D-MOSFET in depletion mode and enhancement mode respectively. (a) V_{GS} negative leads to depletion mode of operation (b) V_{GS} Positive leads to depletion mode of operation*

voltage applied to the gate, i.e. with $V_{GS} = 0$. The charge carriers in the channel will be electrons. When a negative voltage is applied to the gate, negative charges get accumulated on the gate. Induced positive charges arise in the channel due to capacitor action. The induced positive charges will deplete the majority carriers, i.e. the electrons in the channel. The conductivity of the channel is reduced due to depletion of electrons in the channel. As the gate voltage is further increased, there will be a greater depletion of n-channel electrons and a further reduction in drain current. At a further reduced gate-source voltage V_{GS} (off), there will be complete depletion of charge carriers and the drain current will be reduced to zero.

Enhancement mode of operation

A depletion MOSFET can be in enhancement mode as shown in Fig. 7.24(b). The gate is positively biased. There will be induced negative charges on the channel due to capacitor action. As a result, the channel conductivity is increased.

The increase in gate voltage increases the channel conductivity. This increase in conductivity is called enhancement and hence the name enhancement mode of operation. A p-channel MOSFET will also work in the same way as an n-channel MOSFET. However, the polarities of voltage and direction of currents will be reversed in the case of p-channel MOSFET.

We have seen how a D-MOSFET can be operated in both depletion mode and in enhancement mode.

The symbols for D-MOSFET of both n-channel and p-channel type are shown in Fig. 7.25.

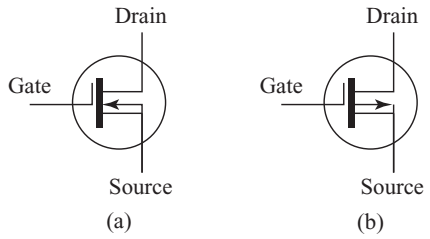


Figure 7.25 *Symbols for D-MOSFETs (a) n-channel type (b) p-channel type*

Note that the substrate is indicated by an arrow connected internally to the source. The arrow-head is inward for n-channel and outward for p-channel D-MOSFETs.

The input impedance of a D-MOSFET is very high due to the presence of an insulating layer of SiO_2 (silicon dioxide) between the gate terminal and the channel.

Since the depletion MOSFET described here can be operated in both depletion mode and in enhancement mode, these MOSFETs may also be referred to as depletion-enhancement MOSFETs or DE-MOSFETs. However, the term D-MOSFET is normally used instead of DE-MOSFET.

7.3.2 D-MOSFET Characteristics

The following two characteristics will be drawn, namely drain characteristics and transfer characteristics.

The drain characteristic for an n-channel depletion-enhancement MOSFET (D-MOSFET) is shown in Fig. 7.26(a). The transfer characteristic is shown in Fig. 7.26(b)

Let us assume that the gate voltage $V_{GS} = 0$ V. When positive voltage is applied to the drain terminal, free electrons from the channel are attracted to the drain and drain current stats flowing. With $V_{GS} = 0$, the drain current I_D saturates at I_{DSS} (see Figure 7.26(a)).

When negative voltage is applied to the gate, i.e. when V_{GS} is negative, the gate will tend to repel the free electrons of the channel and attract holes from the substrate. This will cause recombination inside the channel and hence the channel current will get reduced. There will be reduced I_D for the same V_{DS} when V_{GS} becomes more and more negative.

Thus, at constant V_{DS}, the value of I_D decreases with the increase in negative value of V_{GS}. This is the depletion mode of operation of the MOSFET. If the gate voltage is made positive with respect to the source, then the positive V_{GS} will draw additional electrons form the p-type substrate due to reverse leakage current. These accelerated electrons will knock out more electrons due to collision. The drain current will increase with the increase in positive gate voltage. This mode of operation with positive gate voltage is called enhancement mode of operation. The transfer characteristic showing the relation between V_{GS} both –vc and +vc and the drain current I_D has been presented in Fig. 7.26(b).

The device operates in the depletion mode when V_{GS} is negative and in the enhancement mode when V_{GS} is positive.

Figure 7.27 shows a comparison between enhancement mode and depletion mode D-MOSFET.

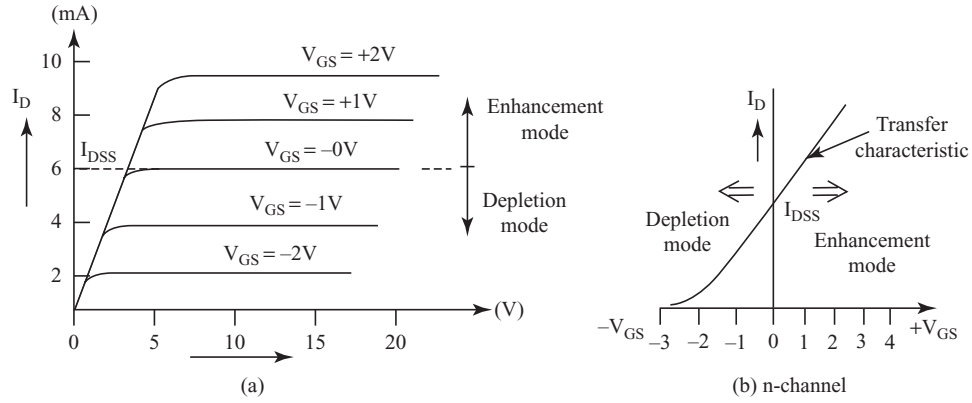


Figure 7.26 *(a) Drain characteristics of a D-MOSFET (b) Transfer characteristics*

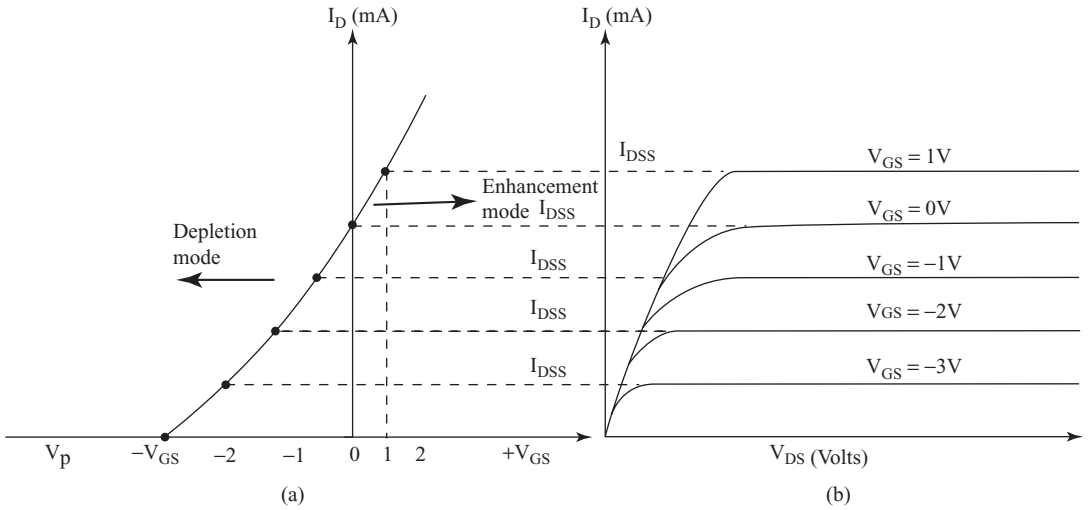


Figure 7.27 *D-MOSFET characteristics (both depletion mode and enhancement mode) (a) Transfer characteristic (b) Output or drain characteristic*

The positive values of V_{GS} produce enhancement mode and negative value of V_{GS} produce depletion mode of operation. The transfer characteristic indicated in Fig. 7.26(b) is for a constant value of V_{DS}. It can be observed from Fig. 7.26(b) that as V_{GS} is made more and more negative at a constant value of V_{DS}, the drain current goes on decreasing.

7.3.3 Enhancement MOSFET

A p-type semiconductor is used as the substrate in an enhancement MOSFET. Two blocks of heavily-doped n-type material are diffused into the substrate and then a very thin layer of silicon dioxide is coated on the surface of the substrate. Holes are made through the silicon dioxide layer to make contact with the diffused n-type material. By depositing metals through the holes so made the source terminal and drain terminal are made as shown in Fig. 7.28. A metal plate is deposited on the surface of the SiO_2 layer to form the gate terminal. The operation of the device with no gate voltage is shown in Fig. 7.28. No voltage is shown applied to the gate terminal, G. The drain terminal is made positive with respect to the gate due to the voltage applied between the drain and the source as has been shown. The two p–n junctions at the drain and the source are seen connected back to back through the resistance, R_e of the

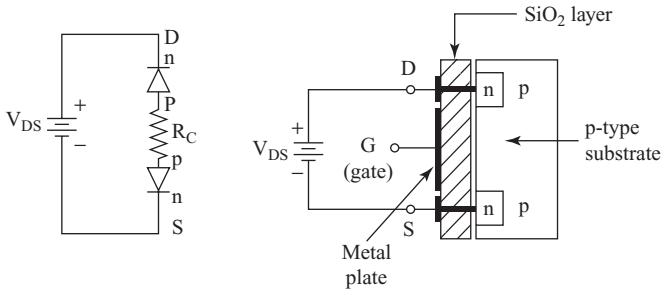


Figure 7.28 *E-MOSFET*

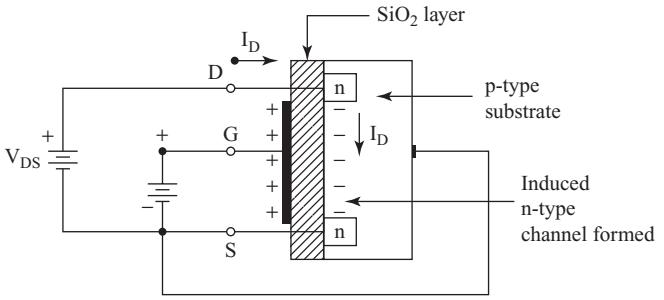


Figure 7.29 *An enhancement MOSFET with $+V_{DS}$ and $+V_{GS}$ applied to the drain and the gate*

channel. Since one of the p–n junctions is reverse biased, only a very small reverse leakage current will flow from the drain, D to the source, S. Note that no channel has been made into this device.

Now let us examine the working of the device when a positive voltage is applied to the gate. As shown in Fig. 7.29, the source terminal S is connected to the p-type substrate and a positive voltage is applied to the gate. Positive charges accumulate on the plate of the gate.

The minority charge carriers (electrons) from the p-type substrate get attracted to the positively charged plate, i.e. the gate. These electrons cannot cross the SiO_2 layer, which comes in-between. Hence, they accumulate close to the gate on the surface of the substrate. These accumulated electrons form the n-type channel between the drain and the source. As the gate voltage is increased, more and more electrons gather in the channel area and as a consequence the resistance of the channel gets reduced. The channel resistance is controlled by the gate voltage. The drain is connected to the positive polarity of V_{DS}. Hence, the drain current I_D will flow. The drain current is controlled by the gate-source voltage. The device is known as enhancement mode MOSFET since the channel conductivity gets enhanced due to the positive gate voltage.

7.3.4 Characteristics of E-MOSFETs

The drain characteristics and transfer characteristics for an n-channel E-MOSFET are shown in Fig. 7.30.

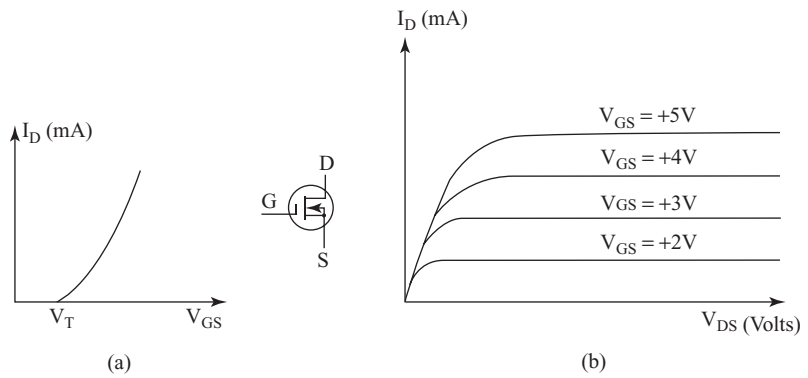


Figure 7.30 *Drain and Transfer Characteristics of an n-Channel E-MOSFET. (a) Transfer characteristics (b) Drain characteristics*

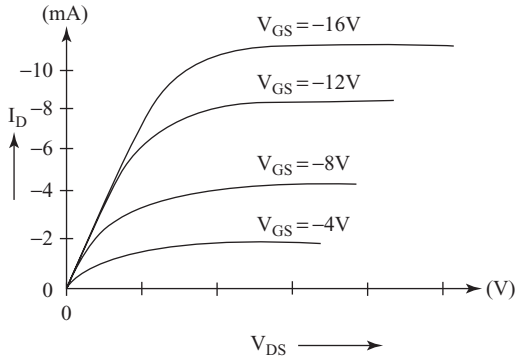


Figure 7.31 *Drain characteristics of a p-channel E-MOSFET*

It can be seen that at a constant V_{DS}, the drain current increases with the increase in positive gate-source bias voltage. Gate-source leakage current does not arise because the gate is insulated from the channel with a layer of SiO_2. The device has very high input impedance. It can be seen from the transfer characteristics that when $V_{GS} = 0$, $I_D = 0$ because of the absence of a channel. It is important to note that no channel has been made in the device. Due to the application of positive V_{GS}, channel is induced in the substrate. If there is no V_{GS} applied, there will be no channel and no channel current, i.e. I_D is zero. The minimum value of V_{GS} at which channel is formed and conduction starts is called threshold voltage, V_T. Threshold voltage (V_T) of a MOSFET is the value of gate-source voltage at which the drain current reaches some defined small value of the range of some μA.

The positive potential at the gate attracts the minority charge carrier of the p-type substrate, i.e. the electrons towards the gate. As we increase the positive gate bias further, more and more electrons will gather near the gate, i.e. by the side of the SiO_2 layer. As mentioned earlier, an induced channel will be created, which will connect the n-type doped regions. The value of V_{GS} at which the conduction begins is called threshold voltage V_T or V_{GS} (Ths).

The increase in V_{DS} will reduce the channel width to a point of pinch-off and the saturation condition for I_D will be created.

p-Type drain and source blocks are diffused into an n-type substrate for constructing a p-channel E-MOSFET. The characteristics of the device will be similar to the characteristics of n-channel E-MOSFETs with the difference that all the voltage polarities and current direction will be opposite as shown in Fig. 7.31.

7.3.5 Advantages and Disadvantages of MOSFETs

The advantages of MOSFETS are as follows:

- Extremely high input impedance to the order of $10^{15}\,\Omega$ or greater.
- Better thermal stability
- Small size
- Low internal noise
- Can operate at very high frequency

The disadvantages of MOSFETS are as follows:

- High input capacitance
- Large on-state voltage drop
- More power loss

7.3.6 Comparison between BJTs and FETs

At this stage, we are in a position to make a comparison between BJTs and FETs. Only the salient points are mentioned below.

1. Two types of charge carriers, i.e. electrons and holes take part in conduction in BJTs. Only one type of charge carrier is involved in FETs.
2. FETs offer higher input impedance when compared to BJTs.
3. A BJT is a current controlled device while an FET is a voltage controlled device.
4. FETs are less noisy.
5. FETs can be operated at higher frequencies. The performance of FETs in high frequency switching function is superior when compared to BJTs.
6. FETs are more stable thermally than BJTs.
7. FETs require less space when fabricated as integrated circuits than BJTs. They are also easier to fabricate.

7.3.7 MOSFET Configurations

The three terminals of a MOSFET, namely the source, gate, and drain can be connected in a circuit in the following three configurations or ways.

(1) **Common Source (CS) Configuration**: In CS configuration, the source terminal is kept common to the input and the output circuits as shown in Fig. 7.32

The CS configuration is the most widely used connection as it provides high input impedance, good voltage gain, and moderate value of output impedance. It is equivalent to a CE amplifier. It provides phase reverse of 180°.

(2) **Common Gate (CG) Configuration**

In this configuration the gate is kept common to the input and the output circuits as shown in Fig. 7.33(a).

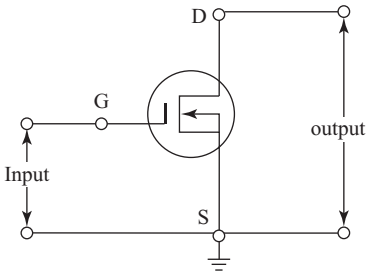


Figure 7.32 *n-Channel D-MOSFET in CS Configuration*

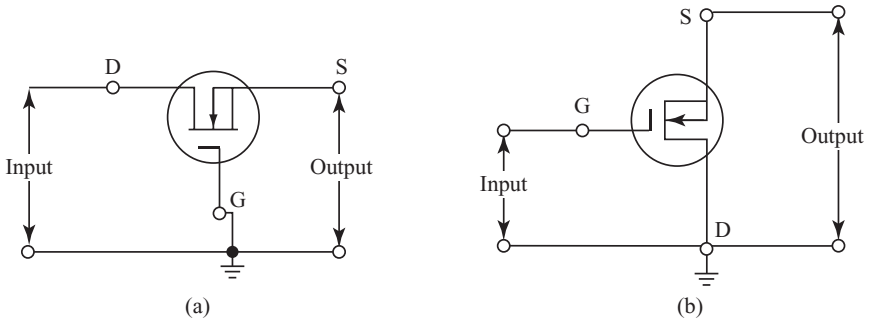


Figure 7.33 *CG and CD configuration of a MOSFET (a) n-channel D-MOSFET in CG configuration (b) n-channel D-MOSFET in CD configuration*

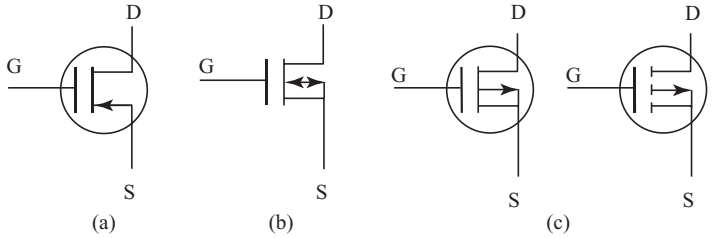


Figure 7.34 *(a) Symbol for p-channel MOSFET either DMOSFET or EMOSFET (b) Symbol for n-channel MOSFET either DMOSFET or EMOSFET (c) Symbol for n-channel enhancement MOSFET (EMOSFET).*

(3) **Common Drain (CD) Configuration**
In this case, the drain is kept common to the input and the output circuits as shown in Fig. 7.33(b)

7.3.8 Circuit Symbols for Different Types of MOSFETs

The symbols for p-channel and n-channel MOSFETs are shown in Fig. 7.34 (a) and (b), respectively. Note that the two symbols are similar except that the arrow head is in opposite directions.

7.4 COMPLEMENTARY MOSFETS (CMOS)

The process of very large-scale integration (VLSI) has enabled micro miniaturization of integrated circuits (ICs) where as many as 10^4 to 10^9 components are accommodated in a single chip. VLSI circuits play an important role in digital electronics and superfast computers. VLSI circuits use n-channel MOS (NMOS) and complementary MOS (CMOS) devices.

CMOS uses both n-channel MOS (NMOS) and p-channel MOS (PMOS) transistors fabricated in the same substrate. CMOS technology has become very popular because of its insignificant power consumption, high input impedance, bidirectional operation, and low noise level during operation.

As compared to bipolar technology, CMOS technology has the following advantages:

- Extremely low power consumption
- CMOS devices require very low driving current because of high input impedance

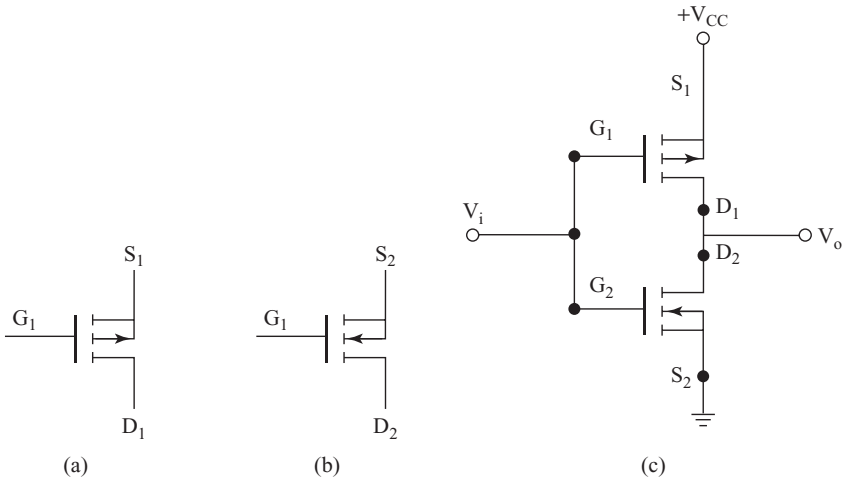


Figure 7.35 *A CMOS is obtained by connecting a p-channel and an n-channel MOSFET in series. (a) p-channel MOSFET (b) n-channel MOSFET (c) CMOS*

- The CMOS circuit is bidirectional since drain and source terminals are interchangeable
- Noise level of CMOS devices is very low
- CMOS technology enables higher density fabrication. However, bipolar transistors have higher gain and better high frequency performance than CMOS devices.

A complementary MOSFET (CMOS) is made by connecting a p-channel and an n-channel MOSFET in series.

The symbol for a p-channel and an n-channel MOSFET and that of a CMOS are shown in Fig. 7.35. These MOSFETs can be either of depletion type or enhancement type. Figure 7.35(c) represents an enhancement type MOSFET.

The input voltage, $V_{i,}$ is applied to the common gate connection formed by joining G_1 and G_2. The two drain terminals are connected together and the output, $V_{o,}$ is taken from the common point.

In CMOS both p-channel and n-channel MOSFETs are used and are fabricated on the same silicon chip.

MOSFETs have become extremely popular for logic circuits due to their low power dissipation and high-density fabrication (enabling very large number of the device fabricated in a small area).

The fabrication of enhancement mode p-channel and n-channel MOS devices on the same chip forms a complementary MOSFET and the logic based on this device is called CMOS logic. Because of the extremely low power consumption CMOS devices are most suitable for battery operated systems. The basic CMOS logic circuit is an inverter. More complex function can be realized by using a combination of inverters.

The principle of working of a CMOS transistor is more or less similar to that of an enhancement MOSFET.

7.4.1 CMOS Inverter

CMOS is mainly used as in inverter circuit as shown in Fig. 7.36 where the source terminal of the p-channel CMOS is connected to the supply voltage $+V_{cc}$ having a common input-gate connection and

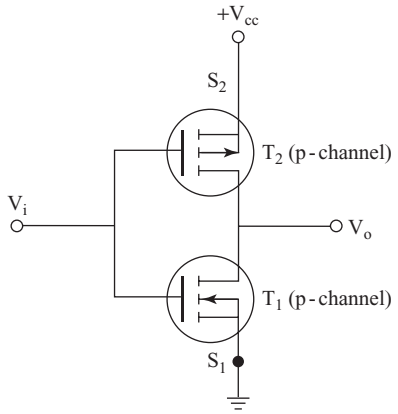


Figure 7.36 *A basic CMOS inverter*

common drain-output connection. The source of the n-channel is grounded. A typical logic high of 5 V and logic low of 0 V is used. Here V_{cc} is the logic high (logic 1).

When $V_i = V_{cc}$, transistor T_1 is ON and T_2 is OFF. Therefore, output $V_0 \simeq 0$.

When $V_i = 0$ V, T_1 is OFF and T_2 is ON and the output voltage V_o is approximately equal to V_{cc} (logic 1).

Logic gates like the NAND, NOR, AND and OR gates can be made to perform logic functions using CMOS inverters.

Different CMOS integrated circuits have been developed and they are part of the CMOS family. For example, 4000-series CMOS is the first commercially successful CMOS family introduced. Then its equivalent 1400 CMOS-series was introduced. 74 AHC/AHCT-series is an advanced CMOS series, which consists of high-speed low power dissipation devices.

Interfacing of CMOS and TTL is done to achieve optimum performance in digital systems. CMOS logic ICs can be used in the part or the system where low power consumption is essential and TTL can be used where high speed operation is required.

7.4.2 MOSFET Equations

The current–voltage characteristic (I_D versus V_{DS} characteristic) can be divided into three regions, namely cutoff regions, triode or linear region, and saturation region

Cut-off region

We know that the threshold voltage, V_T of a MOSFET is the minimum gate-source voltage, V_{GS} at which the drain current I_D (appreciable value) starts flowing.

Now, for $V_{GS} < V_T$, $I_D = 0$.

Triode or linear region

$$I_D = \frac{\mu_n C_{OX} W}{L}\left(V_{Gs} - V_T - \frac{1}{2}V_{DS}\right)V_{DS}$$

if $V_{DS} \le (V_{Gs} - V_T)$

Saturation region

$$I_D = \frac{\mu_n C_{qx} W}{2L}(V_{GS} - V_T)^2$$

Or,

$$I_D = K(V_{GS} - V_T)^2$$

$$\text{If } V_{DS} \geq (V_{GS} - V_T)$$

Where μ_n is the electron mobility, C_{qx} is the gate capacitance per unit area, W is the width of the channel, and L is the length of the channel.

Example 7.11 The saturation drain current of an enhancement MOSFET is 6 mA when gate-source voltage is 5 volts. The threshold voltage is 3 volts. Calculate the saturation drain current for a gate source voltage of 10 volts.

Solution:

The drain current, $I_D = K (V_{GS} - V_T)^2$

From the given data,

$$K = \frac{I_D}{(V_{GS} - V_T)^2} = \frac{6 \text{ mA}}{(5-3)^2}$$

$$= 1.5 \text{ mA/V}^2$$

When $V_{GS} = 10$ V,

$$I_D = K(V_{GS} - V_T)^2 = (1.5 \text{ mA/V}^2)(10 - 3)^2 \text{ V}^2$$

or,

$$I_D = 73.5 \text{ mA}.$$

Example 7.12 In the MOSFET circuit shown in Fig. 7.36, calculate the drain current and the drain voltage for (a) $V_{GS} = 0.5$ V, $R_D = 1$ kΩ and (b) $V_{GS} = 3$ V, $R_D = 1$ Kω. The threshold voltage is 2 V and value of $k = 1.5 \text{mA/V}^2$.

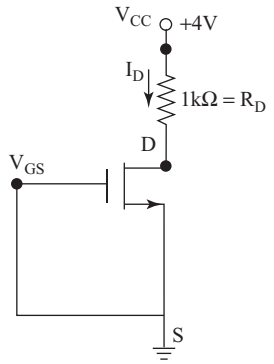


Figure 7.37

Solution:

(a) $V_{GS} = 0.5$ V, $V_T = 2$ V
Since V_{GS} is less than V_T, the transistor is in cutoff region and hence, $I_D = 0$.

(b) Let us assume at this point that the transistor is in the saturation region, then

$$I_D = K(V_{GS} - V_T)^2 = 1.5 \times (3-2)^2 = 1.5 \text{ mA}$$

$$V_{DS} = V_{CC} - I_D R_D = 4 - (1.5 \times 10^{-3} \times 1 \times 10^3)$$

$$= 2.5 \text{ V}$$

To check that the transistor is in saturation, we use the reduction $V_{DS} \geq (V_{GS} - V_T)$ i.e. $(V_{GS} - V_T) \leq V_{DS}$ or $(3-2) \leq 2.5$. So, our assumption is correct and the transistor is in the saturation region.

Example 7.13 An n-channel enhancement MOSFET is biased using the voltage divider method as shown in Fig. 7.38. Given the threshold voltage, $V_T = 3$ V and $k = 0.6$ mA/V^2, calculate the drain current I_D, V_{GS} and V_{DS}. Show that the device is operating in the active region.

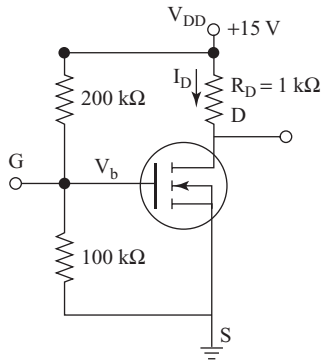


Figure 7.38

Solution:

$$V_{GS} = \frac{V_{DD} \times 100 \text{ K}\Omega}{(200 + 100) \text{ K}\Omega}$$

$$= \frac{15 \cdot 1}{3} \text{V} = 5 \text{ V}$$

$$I_D = K(V_{GS} - V_T)^2 = 0.6 \text{ mA/v}^2 (5-3)^2$$

or,
$$I_D = 2.4 \text{ mA}.$$

$$V_{DS} = V_{DD} - I_D R_D = 15 - 2.4\text{mA} \times 1\text{k}\Omega = 15 - 2.4 = 12.6 \text{ V}$$

The condition for operation in active region is $V_{DS} > (V_{GS} - V_T) = 5 - 3 = 2$ V.
Since, $V_{DS} = 12.6$ V, the operation is in the active region.

7.5 REVIEW QUESTIONS

A. Short Answer Type Questions

1. Sketch the structure of an n-channel JFET and explain the details. Show the circuit symbol.
2. Draw and explain the output characteristics of a JFET.
3. Define transconductance, drain resistance, and amplification factor of a FET. Show their relationship.
4. Draw and explain the transfer characteristic of a JFET.
5. What are the parameters of a JFET? What is the relationship between them?
6. Explain the following: (a) pinch-off voltage, (b) breakdown voltage of a FET, (c) ohmic region of an FET characteristic.
7. Draw and explain the basic structure of a D-MOSFET.
8. What are the advantage and disadvantages of MOSFETs.
9. Make a comparison between BJTs and FETs.
10. Show the various MOSFET configurations.
11. What is an enhancement MOSFET? Show its structure.
12. Draw and explain E-MOSFET characteristics.
13. Write a short note on complementary MOSFET.
14. Show two biasing conditions for a JFET.
15. Draw a CMOS inverter and explain its operation.
16. Explain why CMOS has become extremely popular for logic circuits.
17. Make a comparison between a BJT and a FET.
18. Justify the statement that a BJT is a current-controlled device while an FET is a voltage-controlled device.
19. How can the drain current in a MOSFET be enhanced by negative gate voltages.
20. Draw the volt-ampere drain characteristic and transfer characteristic of an n-channel MOSFET operated in both enhancement and depletion modes.

B. Numerical Problems

21. At a constant value of V_{DS} when V_{GS} of a FET is charged from –1.5 V to –2.5 V, its drain current changes from 8 mA to 5 mA. Calculate (a) the transconductance and (b) the amplification factor when the drain resistance is 100 kilo ohm.

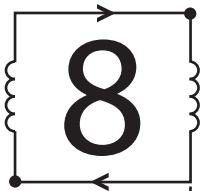
    (Ans. 3 mA/V, 300)

22. The I_{DSS} is 10 mA and V_P is –4 V for an n-channel JFET. Find the value of the saturation drain current I_{DSS} when V_{GS} is equal to –3c V.

    (Ans. 0.62 mA)

23. The drain current of a JFET changes by 20 micro ampere if V_{DS} is changed by 2 volts with a constant value of V_{GS},. What is the value of ac drain resistance.

    (Ans. 100 KΩ)



Feedback Amplifier

TOPICS DISCUSSED

- Concept of feedback
- Types of feedback
- Representation of a feedback amplifier
- Topologies of feedback amplifier
- Effect of negative feedback on performance parameters
- Effect of positive feedback

8.1 INTRODUCTION

While using amplifiers, one has to consider the parameters of the amplifier such as voltage gain, input impedance, output impedance, bandwidth, linearity and noise level. Feedback is a technique by which these performance parameters can be changed so as to get the desired performance of the amplifier. By feedback we mean that some amount of output is returned back or fed back to the input either positively or negatively through a feedback loop or circuit. The amplifier works as an open-loop system when we do not use a feedback loop. The amplifier works as a closed-loop system when feedback is used. The voltage gain of an amplifier is the ratio of output voltage, V_o, and the input voltage, V_s. When feedback is used, the effective input voltage is either increased or decreased because of the feedback voltage, $\pm V_f$. The input voltage, V_i, of a feedback amplifier becomes $V_i = V_s \pm V_f$. The open-loop gain, A_{OL} and closed-loop gain A_{CL} of an amplifier is expressed as

$$A_{OL} = \frac{V_o}{V_s} \text{ and } A_{CL} = \frac{V_o}{V_i} = \frac{V_o}{V_s \pm V_f}.$$

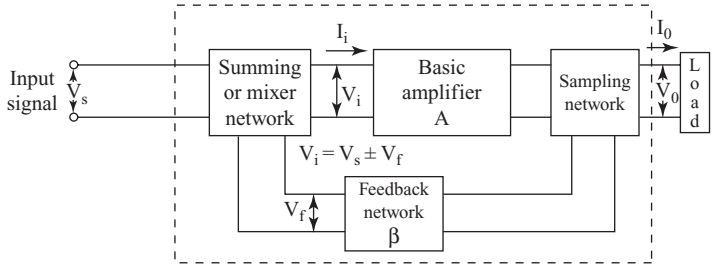


Figure 8.1 *Block diagram of a basic feedback amplifier*

8.2 CONCEPT OF FEEDBACK

In a feedback system, a fraction of the output of the system is fed back (returned back) to its input. This fraction of the output may be added to or subtracted from the input. The system performance gets modified in this way. For example, if a fraction of the output signal of an amplifier is fed back to its input signal, the performance of the amplifier changes considerably. The changes achieved through feedback are largely desirable changes. Feedback makes a system self-regulating, i.e. able to correct itself. The block diagram representation of a feedback amplifier is shown in Fig. 8.1.

The block diagram shows a basic amplifier and a feedback network. The input to the amplifier due to feedback is $V_i = V_s \pm V_f$. The open-loop gain, i.e. gain without feedback of the amplifier is A. The transfer gain or gain of the amplifier with the feedback is A_f.

$$A = \frac{V_o}{V_i} \text{ ; and } A_f = \frac{V_o}{V_s}$$

where V_i is the input voltage to the amplifier, V_o the output voltage and V_s is the input signal.

As shown in Fig. 8.1, the output voltage is sampled by a sampling network and through a two-port feedback network. At the input side, the feedback signal, V_f, is combined with the source signal, V_s, through a summing or mixer network and is fed to the basic amplifier. The components of the feedback amplifier are explained below.

Input signal: It is either a voltage or current signal, which is required to be amplified. It could be a voltage source, V_s, with a series resistance or a current source, I_s, with a parallel resistance.

Output signal: The output signal may be a voltage, V_o, across a load or current, I_o, flowing through the load. It is desirable that the output signal be independent of load variation or variation of parameters of the basic amplifier.

Transfer ratio or transfer gain of the feedback amplifier: The ratio of V_o/V_s is the transfer ratio or transfer gain of the feedback amplifier. This is also the closed-loop gain of the amplifier.

Open-loop gain or gain without feedback: The ratio of V_0/V_i is called the open-loop gain of the amplifier.

Sampling network: This network samples the output voltage and generates a signal, which is proportional to the output.

Feedback network: This is usually a passive two-port network consisting of certain circuit elements. Most often it is simply a resistive circuit. The output of the feedback network is V_f, which is equal to βV_o. β is the feedback fraction.

Mixing network: The mixing network either adds or subtracts the feedback signal, V_f, from the input signal, V_s, through series–input or shunt–input connections.

8.3 TYPES OF FEEDBACK

The voltage that is to be fed back to the input could be added or subtracted from the input voltage. If the feedback voltage is in phase with the input voltage, it will be added to the input signal. If the feedback voltage is out of phase by 180º, i.e. is in phase opposition to the input voltage, it will be subtracted from the input voltage. When feedback voltage is added to the input signal, it is called positive feedback. The feedback is negative when it is subtracted from the input signal. Therefore, feedback can be of two types—negative feedback and positive feedback. We will study both types of feedback systems and their effect on the performance parameters of the amplifier in this chapter.

8.3.1 Negative Feedback and its Effects

Feedback is called negative feedback when the feedback signal reduces the magnitude of the input signal. Negative feedback is also called inverse or degenerative feedback. Negative feedback reduces the transfer gain, i.e. the closed-loop gain of the amplifier. However, negative feedback has a number of advantages despite the reduction in the overall gain of the amplifier. The following are the effects of negative feedback:

(i) The overall gain of the amplifier is reduced but it also stabilizes against any variations in the transistor parameters used in the amplifier.
(ii) The bandwidth of the amplifier increases.
(iii) The noise level is reduced.
(iv) The phase distortion is reduced.
(v) Normally, the input impedance of the amplifier is very high and output impedance is very low. By the proper use of negative feedback, the input impedance of a voltage amplifier can be made higher and output impedance can further be made lower.
(vi) There is a significant improvement in the frequency response and the linearity of operation of a feedback amplifier when compared to a non-feedback amplifier.

8.3.2 Positive Feedback and its Effects

If the feedback signal increases or enhances the magnitude of the input signal, i.e. if $V_i = V_s + V_f$, the feedback is called positive feedback. Positive feedback is also called regenerative feedback. Positive feedback increases the overall gain of the amplifier but makes the system unstable.

8.4 REPRESENTATION OF A FEEDBACK AMPLIFIER

8.4.1 Block Diagram

Figure 8.2 represents the block diagram of an ideal feedback amplifier with input signal X_s and output signal X_o.

Input to the basic amplifier, $X_i = X_s \pm X_f$

The plus sign is for positive feedback and the minus sign is for negative feedback.

8.4.2 Loop Gain

Figure 8.2 indicates that the input of the basic amplifier, X_i, is multiplied by A when passing through the amplifier so that its output becomes AX_i. This amount is multiplied by β in passing through the

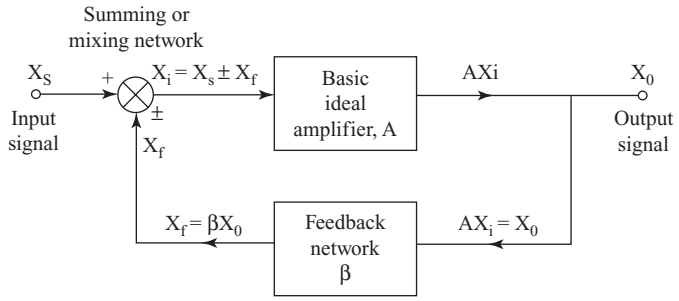


Figure 8.2 *Block diagram of an ideal feedback amplifier*

feedback network to make it $A\beta X_i$. This is multiplied by –1 in the mixer (also called differencing) network so that the product becomes $(-A\beta)\ X_i$. Thus, the input to the amplifier, i.e. X_i is multiplied by a factor $-A\beta$ when a path from the input terminal is taken around the loop consisting of the input. This is the reason why the term $-A\beta$ is called the **loop gain** or feedback factor, or loop transmission feedback factor or return ratio.

Assuming negative feedback, the input to the feedback amplifier, $X_{i,}$ is the difference of X_s and $X_{f,}$ i.e. $X_i = X_s - X_f$, input multiplied by gain as the output, and β times output as the feedback amount. Here β is the feedback fraction. Thus,

$$X_i = X_s - X_{f;} \qquad X_o = AX_i; \qquad X_f = \beta\, X_{o.}$$

$$X_o = AX_i = A(X_s - X_f)$$

$$= AX_s - AX_f$$

or, $$X_o = AX_s - A\beta\, X_{o.}$$

or, $$X_o(1 + A\beta) = AX_{s.}$$

or, $$\frac{X_o}{X_s} = \frac{A}{1+A\beta}.$$

Amplifier gain with feedback, $$A_f = \frac{X_o}{X_s} = \frac{A}{1+A\beta} \tag{8.1}$$

The general expression for the gain of a feedback amplifier is $A_f = \dfrac{A}{1 \pm A\beta}$. The positive sign is for negative feedback and negative sign is for positive feedback. If there is no feedback, $\beta = 0$. In which case,

$A_f = A$ where A is the gain without feedback. The factor A is also called the open-loop gain whereas A_f is called the closed loop gain or transfer gain with feedback.

8.4.3 Feedback Factor

The factor $(-A\beta)$ is called the loop gain because it is the product of the gains of the closed loop. A is the gain of the basic amplifier and β is that of the feedback network. The product $(-A\beta)$ is also called

the feedback factor or return ratio. β is the feedback ratio or feedback fraction. From Eq. (8.1), it is seen that if the magnitude $|1 + A\beta| > 1$ then $|A_f| < |A|$.

$|A_f| < |A|$ indicates negative feedback. If $|A_f| > |A|$, the feedback is positive. Again, if

$$|1 + A\beta| < 1, \text{ we get } |A_f| > |A|.$$

$|A_f| > |A|$ indicates positive feedback. Negative feedback reduces the gain of the amplifier whereas positive feedback increases the gain.

Writing the expression for closed-loop gain again, we get, $A_f = \dfrac{A}{1+A\beta}$

$$\text{If } |1 + A\beta| = 0, |A_f| = \infty$$

Infinite amplifier gain implies that even when there is no input signal there will be an output signal. The amplifier then will function as an oscillator**.**

8.4.4 Amount of Feedback

The amount of feedback introduced into a feedback amplifier is often expressed in decibels (dB) as

$$F = 20 \log \left|\frac{A_f}{A}\right| = 20 \log \left|\frac{1}{1+A\beta}\right|.$$

The value of F will be negative in case of negative feedback.

8.5 TOPOLOGIES OF FEEDBACK AMPLIFIER

We will describe the different types of feedback amplifiers in this section. Both voltage and current can be fed back to the input either in series or in parallel. This would result in four basic types of feedback topologies. They are

1. Voltage–shunt feedback or shunt–shunt feedback
2. Voltage–series feedback or shunt–series feedback
3. Current–shunt feedback or series–shunt feedback
4. Current–series feedback or series–series feedback

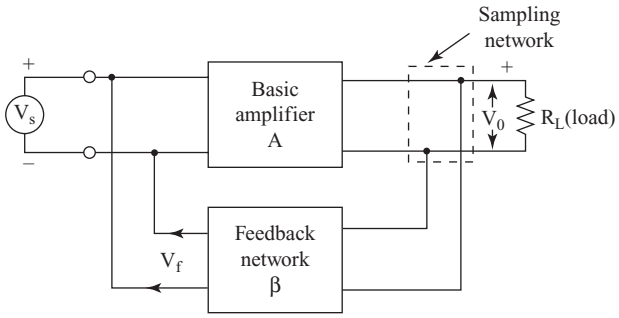
These are described with the help of block diagrams:

8.5.1 Voltage–Shunt and Voltage–Series Feedback

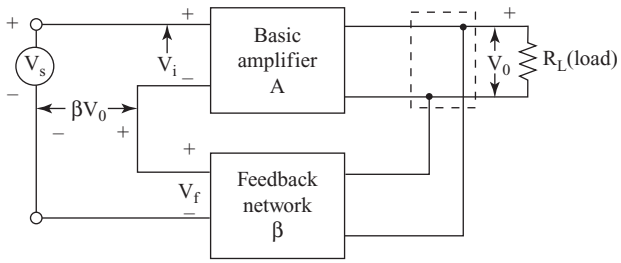
In a voltage–shunt feedback a voltage proportional to the output voltage is taken from the sampling network. The feedback network is connected in shunt or parallel across the output terminals as shown in Fig. 8.3(a).

It is desirable that the input impedance of the feedback network is much higher than the load impedance so that the output of the amplifier does not get loaded. Figure 8.3(b) shows the block diagram of a voltage–series feedback topology or configuration. The sampled signal from the output for the feedback network is a voltage. This voltage feedback signal is connected to the input signal voltage in series.

Thus, the feedback circuit consists of voltage–series or shunt–series feedback configurations. Remember, when we take voltage feedback, we make a parallel or a shunt connection with the output terminals. This voltage is fed back and connected to the input signal either in shunt or in series. Thus, the two configurations i.e. voltage–shunt and voltage–series may also be called shunt–shunt and shunt–series feedback, respectively.

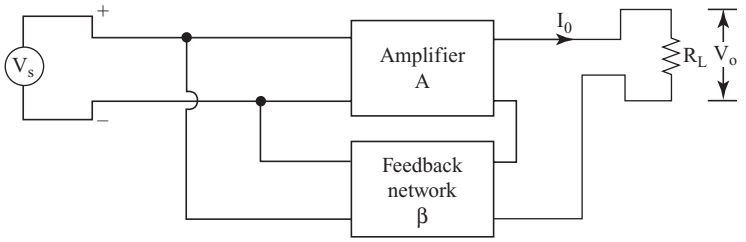


(a) Block diagram of a voltage–shunt feedback

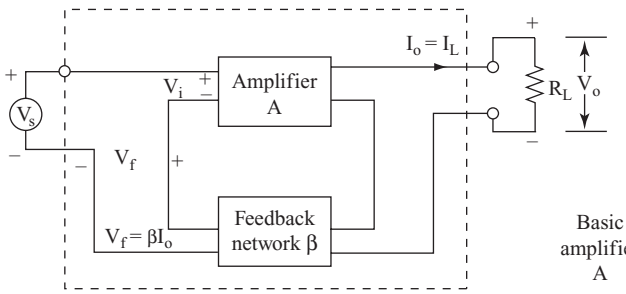


(b) Block diagram of a voltage–series feedback

Figure 8.3 *Feedback amplifier connections (a) Block diagram of a voltage–shunt feedback (b) block diagram of a voltage–series feedback*



(a)



(b)

Figure 8.4 *(a) Block diagram of current–shunt feedback (b) block diagram of current–series feedback*

8.5.2 Current–Shunt or Series–Shunt Feedback

The block diagram of a current–shunt feedback has been shown in Fig. 8.4(a). Figure 8.4(b) shows the block diagram of a current–series feedback. The current from the output, i.e. from the load is to be taken to the feedback network through series connection. The feedback may either be connected in parallel or in series with the input signal. Hence, these circuits can also be called series–shunt and series–series feedback, respectively.

8.5.3 Practical Examples of Feedback Circuits

(a) Current–series or series–series feedback

Figure 8.5 shows the block diagram representation and Fig. 8.6 a practical circuit for current–series negative feedback.

The circuit shown in Fig. 8.6 is basically a common-emitter amplifier. Since the emitter of a transistor is made common between the input terminals in this circuit, it is called common-emitter (CE) amplifier.

V_s is the input signal that is to be amplified while V_{CC} is the dc supply used to provide biasing to the amplifier. Voltage divider biasing is used in this type of amplifier.

R_1 and R_2 are the two resistors used in this biasing circuit. They are also known as biasing resistors. R_E is used to provide stabilization to the amplifier. The current between the collector and base will increase due to the rise in temperature,. This will increase I_C and I_E. As a result, the voltage drop, I_E $R_{E,}$ will also increase. Thus, the collector current I_C is brought to its original value. As R_E is connected between the inputs as well as the output, a feedback may occur through it.

Now refer Fig. 8.6. The signal to be amplified is applied between the base and emitter through a capacitor C_{in}. We already know that since capacitors have very high impedance for dc, it blocks dc and passes only ac. This capacitor is used so that the dc current does not flow through the transistor. $+V_{CC}$ will pass a current through R_1 and R_C. The current through R_C is I_C and the current to R_1 and R_2 is I. The current will be divided into the two resistors. In an amplifier without feedback, C_E (emitter capacitor) is provided near R_E so that it can pass the ac coming from the emitter and ground it because there is a requirement that it should prevent any ac feedback to the input. Hence, if we wish to provide feedback, we will remove $C_{E.}$ After this, an ac current will flow through R_E and flow to the input and join the input current.

The output of an amplifier may be taken:

(a) Across the collector and emitter

(b) Across collector resistance R_C

Now when the output is taken across the collector and emitter, the output and input are 180º out of phase. However, if the output is taken across $R_{C,}$ the output and input are in phase. Therefore, if

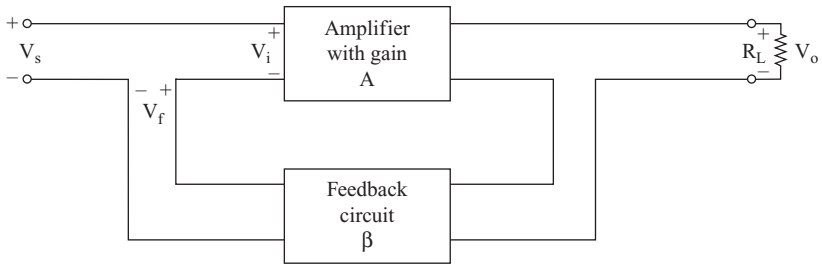


Figure 8.5 *Block diagram of current–series feedback connection*

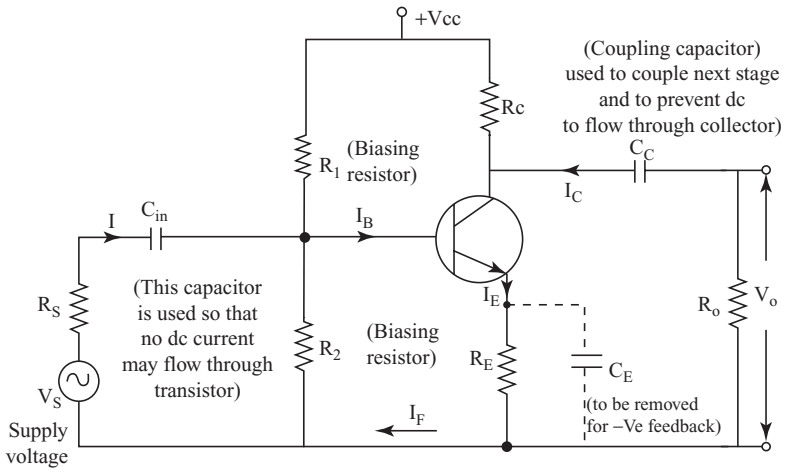


Figure 8.6 *Circuit diagram for current–series feedback.*

we wish to provide negative feedback, the output and the feedback current should be taken across the emitter collector as shown in Fig. 8.6.

(b) Current–shunt feedback circuit

Figure 8.7 shows the block diagram and Fig. 8.8 shows the practical circuit for a current–shunt negative feedback amplifier. A current–shunt feedback amplifier is basically a current amplifier in which two CE (common-emitter) amplifier stages are cascaded by a C_C (coupling capacitor), and R_f is used to provide feedback. R_f is the feedback resistor.

The working of a CE amplifier has been discussed in the previous section. In this circuit, two CEs are cascaded to increase the amplitude of the signal at the output.

There are two stages in the process. When a voltage signal arrives at the input, it gets amplified by the first transistor stage. R_1 and R_2 provide biasing by flowing current through emitter and base. The amplified output of the first stage is then passed to the second stage through a coupling capacitor C_C. The output from the R_C of the first stage will be sent to the second transistor amplifier stage. It will again amplify the signal. The necessary biasing is provided by R_3 and R_4. R_e provides stabilization to the circuit. C_e emitter capacitor will be removed so that ac may not be bypassed (grounded). Then this ac current flows through the R_f (feedback resistor), which will then reach the input and mix with the input by using parallel connection. Hence, the current feedback is provided in parallel in this amplifier circuit.

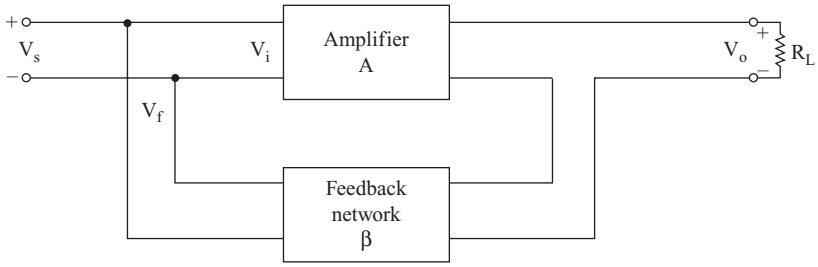


Figure 8.7 *Block diagram of current–shunt feedback*

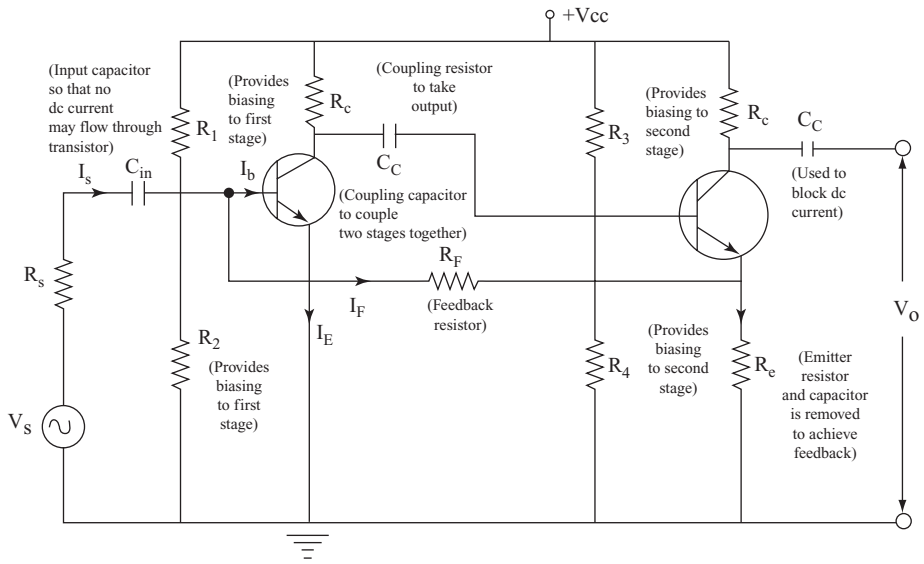


Figure 8.8 *Practical circuit for current–shunt feedback*

(c) Voltage–series feedback circuit

Figure 8.9 shows the block diagram and Fig. 8.10 shows a practical example of a voltage–series feedback circuit. An emitter-follower circuit is the best example of this type of circuit. It is ideal as it increases the input impedance and decreases the output impedance. R_1 and R_2 provide biasing to the transistor by the voltage-divider method. R_e provides stabilization as discussed earlier.

The AC output voltage is taken across R_e. No emitter capacitor is used with R_e (emitter resistor) so that AC voltage cannot be bypassed/grounded. Signal voltage, i.e. V_s is applied through the coupling capacitor (C_C) to the base.

Due to biasing, the AC emitter current produce an output voltage across the emitter resistor (R_E), which is then feed back to the input signal voltage and both voltages mix at the input.

(d) Voltage–shunt feedback

Figure 8.11 shows the block diagram and Fig. 8.12 shows the practical circuit diagram for a voltage–shunt feedback amplifier.

It is a common-emitter amplifier circuit in which the capacitor, $C_{e,}$ which lies near R_e is removed so that ac is not bypassed. Now input signal is given at the input, i.e. V_S. The ac signal will flow through

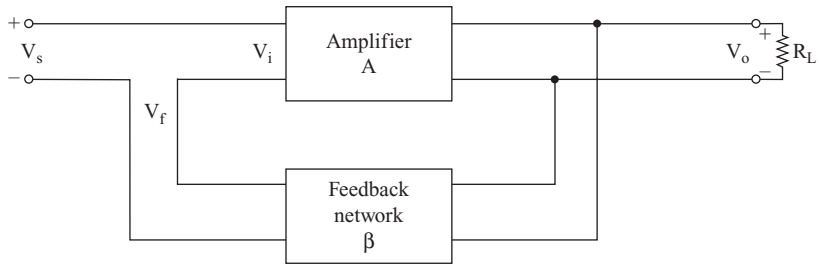


Figure 8.9 *Block diagram for voltage–series feedback*

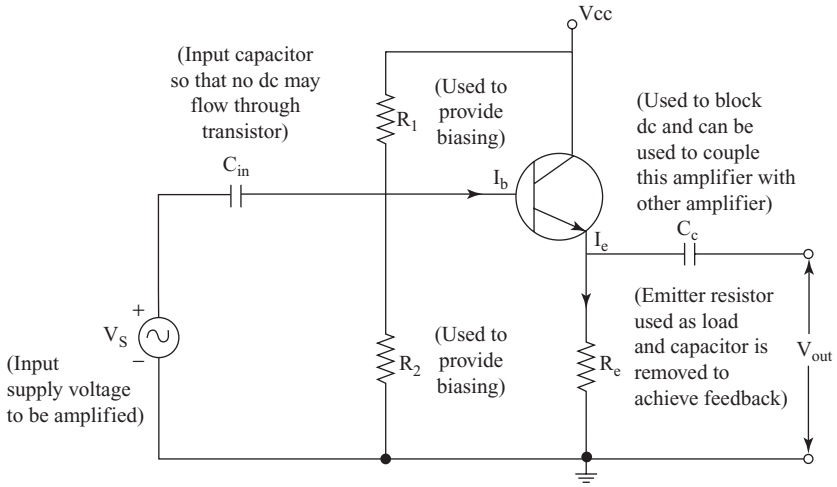


Figure 8.10 *Circuit diagram showing voltage–series feedback*

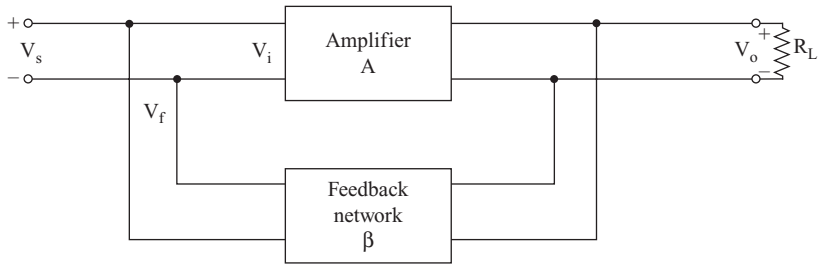


Figure 8.11 *Block diagram for voltage–shunt feedback*

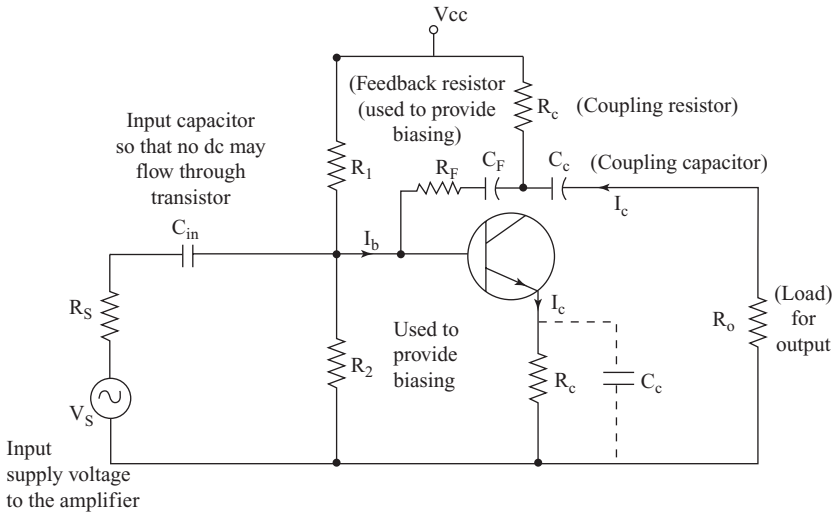


Figure 8.12 *Circuit diagram for voltage–shunt feedback*

the C_{in} to the base, which is used so that the dc does not flow through the transistor. The biasing to the transistor is given by the voltage divider circuit, which comprises R_1 and R_2.

The working of the voltage divider circuit is also explained in the previous section. A current will flow to the base, i.e. I_b. Base current, I_C will flow through the collector terminal and I_C (emitter current) flows through the emitter terminal and R_e (emitter resistor). Now, the signal reaches the base junction, which is to be amplified. It is passed to the emitter. C_e is removed so that ac is not bypassed and will pass through the load. DC output is taken by the load while the ac voltage is sent further through the feedback capacitor (C_f). C_f is used here so that the AC feedback voltage may pass easily through the feedback resistor (R_p) and join with the signal voltage and reach the base terminal for further amplification.

8.6 EFFECT OF NEGATIVE FEEDBACK ON PERFORMANCE PARAMETERS

Negative feedback is obtained by feeding a portion of the amplifier output back to the input. The feedback quantity is applied in opposition to the basic input signal so that the effective input signal is reduced ($X_i = X_s - X_f$). The feedback voltage, V_f, is out of phase with the input signal voltage, V_s. The effects of negative feedback are explained in the following sections.

8.6.1 Effect of Negative Feedback on Overall Gain

If A is the gain of the basic amplifier, the gain of the feedback amplifier, A_f, is expressed as

$$A_f = \frac{A}{1+A\beta}.$$

Let the gain of the basic amplifier, A = 10,000 and the value of β is 0.01. The gain of the feedback amplifier with negative feedback is

$$A_f = \frac{A}{1+A\beta} = \frac{10,000}{1+10,000\times 0.01} = \frac{10,000}{101}$$
$$= 99$$

This shows that the gain of the feedback amplifier has been reduced. Thus, we can state that negative feedback reduces the gain of the amplifier**.** Although the gain of a negative feedback amplifier is reduced, negative feedback has a number of advantages with respect to the performance of the amplifier. These advantages are explained in the following sections.

8.6.2 Effect of Negative Feedback on Gain Stability

One of the important advantages of negative feedback is that the overall gain of an amplifier can be made independent of the variations of the parameters of the transistor amplifier. The variation due to ageing, temperature, replacement of components, etc. of the circuit and transistor characteristics is reflected in the lack of stability of the amplifier gain, which is not desirable. Amplifier gain stabilization is the most important advantage of negative feedback amplifiers.

Consider the expression for gain of a feedback amplifier,

$$A_f = \frac{A}{1+A\beta}.$$

If $|A\beta|$ is made much greater than 1, then $A_f = \frac{A}{A\beta} = \frac{1}{\beta}$.

The gain, A_f, will depend only on the feedback network and not on the basic amplifier gain, A. This way the gain of the feedback amplifier is made independent of any variations in the parameters of the basic amplifier. The gain of the feedback amplifier, thus, gets stabilized.

From,
$$A_f = \frac{A}{1+A\beta}$$

The fractional change in amplification with feedback as related to the fractional change of amplifier gain without feedback is obtained by differentiating the above expression as,

$$\left|\frac{dA_f}{A_f}\right| = \left|\frac{1}{1+A\beta}\right| \left|\frac{dA}{A}\right|$$

The condition for gain stability with negative feedback is $|1+A\beta| >> 1$.

Example 8.1 Calculate the closed-loop gain of a negative feedback amplifier with A = 10,000 and β = 0.01. Show that the closed-loop gain does not change much even when the gain of the basic amplifier is changed by ± 50 per cent.

Solution:

$$A_f = \frac{A}{1+A\beta}$$

Substituting values,
$$A_f = \frac{10{,}000}{1+10{,}000\times 0.01}$$
$$= \frac{10{,}000}{101} = 99.00$$

For the changed condition of A from 10,000 to 15,000,

$$A_f = \frac{15{,}000}{1+15{,}000\times 0.01} = \frac{15{,}000}{151} = 99.33$$

For changed condition A from 10,000 to 5,000,

$$A_f = \frac{5{,}000}{1+5{,}000\times 0.01} = \frac{5{,}000}{51} = 98.03$$

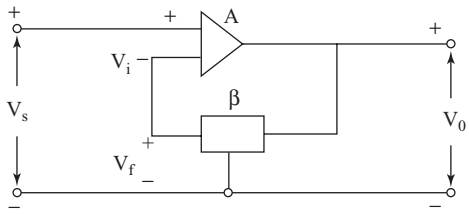


Figure 8.13

This shows that when the open-loop gain, i.e. the gain of the basic amplifier, A changes by ± 50 per cent, the gain of the feedback amplifier changes by a very small percentage. Thus, the negative feedback stabilizes the amplifier voltage gain.

The condition for stabilization of negative feedback amplifier gain is

$$|A\beta| >> 1$$

i.e., $$A_f = \frac{A}{1+A\beta} \rightarrow \frac{1}{\beta}.$$

Again, $$A\beta >> 1$$

or, $$A >> \frac{1}{\beta}$$

This means that $A >> A_f$.

Thus, for satisfactory operation of the negative feedback amplifier (with stabilized gain), the open-loop voltage gain must be much higher than the required closed-loop gain.

Example 8.2 The open-loop gain of an amplifier changes by 10 per cent due to the effect of temperature, ageing, etc. A change in gain of not more then 2 per cent is allowed. If the amplifier gain with feedback is 100, find the minimum value of feedback ratio.

Solution:

We know that negative feedback stabilizes the voltage gain of an amplifier. Therefore, negative feedback will be used here.

We know, $$A_f = \frac{A}{1+\beta A}$$

or $$\frac{dA_f}{A_f} = \frac{dA}{A} \times \frac{1}{1+\beta A}$$

given, $$\frac{dA}{A} = 10 \text{ per cent}$$

and $$\frac{dA_f}{A_f} = 2 \text{ per cent.}$$

Therefore, $$\frac{\frac{dA}{A}}{\frac{dA_f}{A_f}} = 1 + \beta A$$

or, $$\frac{10}{2} = 1 + \beta A$$

or, $$1 + \beta A = 5 \qquad (8.2)$$

$$A_f = \frac{A}{1+\beta A}$$

or, $$A = A_f(1 + \beta A) = 100 \times 5 = 500. \quad (8.3)$$

[From Eqs. (8.2) and (8.3), $1+ \beta \times 500 = 5$]

or $$\beta = \frac{4}{500} = 0.008$$

Example 8.3 The open-loop gain of an amplifier is 10,000. The feedback fraction, β is 0.01. The gain of the amplifier changes by 50 per cent due to ageing and temperature effect. What will be the change in the closed-loop gain?

Solution: Open-loop gain, $A = 10{,}000$

and $\beta = 0.01$

$$A_f = \frac{A}{1+\beta A} = \frac{10{,}000}{1+0.01\times 10{,}000} = \frac{10{,}000}{101}$$
$$= 99$$

Let us calculate the value of A_f when A changes by 50 per cent. The new value of A will be 15,000.

$$A_f' = \frac{A'}{1+\beta A'} = \frac{15{,}000}{1+0.01\times 15{,}000}$$
$$= \frac{15{,}000}{151} = 99.33$$

The percentage change in A_f is calculated as $\frac{99.33-99}{99}\times 100 = 0.33$ per cent.

This small change shows that negative feedback stabilizes the voltage gain of an amplifier.

Example 8.4 The open-loop voltage gain of an amplifier is 10,000. The feedback fraction is 0.01. Calculate (a) the amount of feedback introduced in the feedback amplifier in dB; (b) the feedback factor; (c) the feedback voltage; and (d) the output voltage for an input voltage of 100 mV.

Solution:

$$A_f = \frac{A}{1+\beta A} = \frac{10{,}000}{1+0.01\times 10{,}000}$$
$$= \frac{10{,}000}{101} = 99.$$

(a) Let the amount of feedback in dB be N.

$$N = 20 \log_{10} \left|\frac{A_f}{A}\right| = 20 \log_{10} \frac{99}{10{,}000}$$
$$= 20 \left[\log_{10} 99 - \log_{10}\ 10{,}000\right]$$

$$= 20[1.9956 - 4.0]$$
$$= 20 \times -2.0044 = -40.08 \text{ dB}$$

(b) Feedback factor = $A\beta = 10{,}000 \times 0.01 = 100$.
(c) Feedback voltage = $V_f = \beta V_o = 0.01 \times 9.9 = 0.099$ V.
(d) Output voltage = $A_f \times V_s = 99 \times 100 \times 10^{-3}$ V = 9.9 V.

Example 8.5 An amplifier has an open-loop voltage gain of 1,000. The feedback ratio is 0.01. Calculate the (a) gain of feedback amplifier; (b) amount of feedback in decibel; (c) feedback factor (d) output voltage; and (e) feedback voltage of the feedback amplifier for an input voltage of 10 mV.

Solution:

Given, $A = 1{,}000, \beta = 0.01$

(a) $A_f = \dfrac{A}{1+\beta A} = \dfrac{1{,}000}{1+0.01\times 1{,}000} = \dfrac{1{,}000}{11} = 99.9$

(b) Amount of feedback, N, is expressed as

$$N = 20 \log_{10} \left|\frac{A_f}{A}\right| = 20 \log_{10} \frac{99.9}{1{,}000} = -20.08 \text{ db.}$$

(c) Feedback factor = $-A\beta = -1{,}000 \times 0.01 = -10$
(d) Output voltage , $V_o = A_f V_i = 99.9 \times \text{mV} = 999 \text{ mV} = 0.999$ V
(e) Feedback voltage, $V_f = \beta V_o = 0.01 \times 0.999$ V
$= 0.01 \times 0.999 \times 10^3$ mV
$= 9.99$ mV.

Example 8.6 An amplifier has an open-loop voltage amplification of A = 1000. A fraction of the output voltage is fed back in opposition to the input. If the feedback fraction is 0.01, calculate the percentage change in gain of the feedback amplifier if the amplifier gain, A gets reduced by 6 dB due to ageing.

Solution:
Gain of the feedback amplifier, A_f, is calculated as

$$A_f = \frac{A}{1+A\beta} = \frac{1000}{1+1000\times 0.01} = \frac{1000}{11} = 90.99$$

Let the open-loop gain, A change to A_1 due to ageing. This change has been expressed in dB.

Therefore, $$20 \log_{10} \left|\frac{A}{A_1}\right| = 6$$

$$\log_{10} \left|\frac{A}{A_1}\right| = \frac{6}{20} = 0.3$$

or, $$\frac{A}{A_1} = 2$$

or, $$A_1 = \frac{A}{2} = \frac{1{,}000}{2} = 500$$

Now, $$\text{let } A_{f_i} = \frac{A_1}{1 + A_1\beta} = \frac{500}{1 + 500 \times 0.01} = \frac{500}{6} = 83.33$$

The percentage change in gain of the feedback amplifier is calculated as

$$\text{Percentage change} = \left(\frac{A_f - A_{f_i}}{A_f}\right) \times 100$$

$$= \left(\frac{90.99 - 83.33}{90.99}\right) \times 100$$

$$= 8.4.$$

8.6.3 Effect of Negative Feedback on Input Impedance

A negative feedback amplifier with voltage series feedback is shown in Fig. 8.14. The input impedance of a BJT amplifier without feedback is seen as the impedance looking into the base of the amplifier.

The input impedance without feedback is $Z = \frac{V_i}{i_i}$

The input impedance with negative feedback is, $$Z_i = \frac{V_s}{i_i} = \frac{V_s \times Z}{V_i} \tag{8.4}$$

Again, $$V_s + V_s - V_i - V_f = 0$$

or, $$V_i = V_s - V_f \tag{8.5}$$

$$V_f = \beta V_o \tag{8.6}$$

$$V_i A = V_o \tag{8.7}$$

$$V_s = V_i + V_f = V_i + \beta V_o$$

$$= V_i + \beta A V_i$$

or, $$V_s = V_i (1 + \beta A)$$

$$Z_i = \frac{V_s}{V_i} \times Z$$

$$= \frac{V_i(1+\beta A)}{V_i} \times Z$$

or, $$Z_i = (1+\beta A)\, Z \tag{8.8}$$

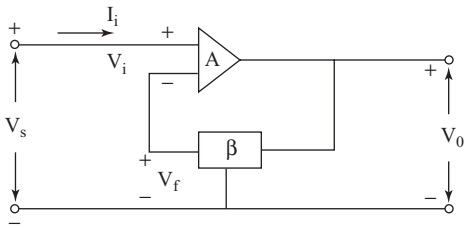


Figure 8.14 *Input impedance of an amplifier with negative feedback*

This shows that the input impedance due to voltage series feedback increases the input impedance by a factor of $(1+\beta A)$. It is desirable to have high input impedance for an amplifier. Thus, feedback amplifier is more advantageous than a non-feedback amplifier.

Example 8.7 A negative voltage feedback amplifier has been shown in Fig. 8.15. Calculate the overall voltage gain if the gain of the amplifier without feedback is 10,000.

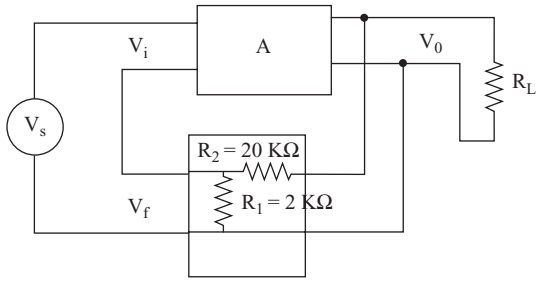


Figure 8.15

Solution:

$$V_f = \beta V_o$$

$$= \left[\frac{R_1}{R_1+R_2}\right] V_o.$$

$$\beta = \frac{R_1}{R_1+R_2} = \frac{2\times10^3}{(2+20)10^3} = \frac{2}{22} = 0.09$$

$$A_f = \frac{A}{1+A\beta} = \frac{10,000}{1+10,000\times0.09}$$

$$= \frac{10,000}{901} = 11.09$$

Example 8.8 The input impedance of an amplifier without feedback is 1 KΩ. The open-loop voltage gain of the amplifier is 20,000. Calculate the value of input impedance when negative feedback is used.

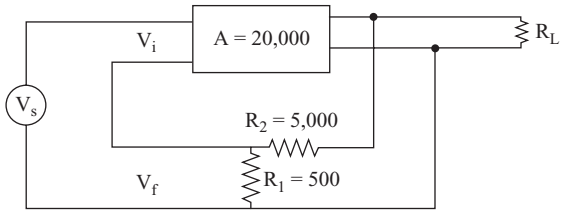


Figure 8.16

Solution:

$$\beta = \frac{R_1}{R_1 + R_2} = \frac{500}{500 + 5,000} = \frac{500}{5,500} = 0.09$$

$$Z_i = (1+A\beta)Z = (1 + 20,000 \times 0.09) \times 1 \times 10^3$$

$$= 1,801 \times 1 \times 10^3 = 1,801\ K\Omega$$

It is seen that the input impedance has increased from 1 KΩ to 1,801 KΩ.

8.6.4 Effect of Feedback on Output Impedance

Let Z_C b_c be the output impedance of the amplifier without feedback and Z_o be the output impedance with negative feedback.

With the input voltage source short circuited, the amplifier circuit is shown as in Figure 8.17

The output impedance of the negative feedback amplifier is z. The output voltage V_o produces a feedback voltage V_f where $V_f = \beta V_o$. This feedback voltage V_f generates a voltage AV_f.

Applying Kirchhoff's voltage law around the output circuit we can write

$$+V_o - I_o Z_e - AV_f = 0$$

or,
$$V_o = I_o Z_e - AV_f \tag{8.9}$$

or,
$$I_o Z_c = V_o + AV_f$$

$$= V_o + A\,\beta V_o$$

or,
$$I_o Z_c = V_o\,(1 + A\beta) \tag{8.10}$$

From Eqs. (8.9) and (8.10)

$$Z_o = \frac{V_o}{I_o} = \frac{V_o \times Z_c}{V_o(1+A\beta)} \tag{8.11}$$

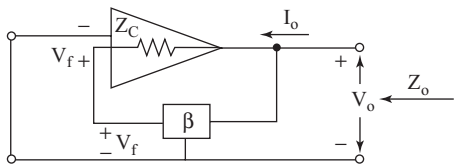


Figure 8.17 *Output impedance of an amplifier with negative feedback*

This shows that the output impedance of the feedback amplifier gets reduced by a factor $(1+A\beta)$.

It is interesting to note that because of negative feedback the following happens:

(i) Gain of the amplifier gets reduced by a factor $(1+A\beta)$;

(ii) Input impedance increases by the factor $(1+A\beta)$; and

(iii) Output impedance decreases by the same factor $(1+A\beta)$.

8.6.5 Effect of Negative Feedback on Bandwidth

Bandwidth is defined as the difference between the higher and lower frequencies at 3 dB cut-off frequencies in the gain–frequency response of an amplifier. Negative feedback increases the overall bandwidth of an amplifier.

Figure 8.18 shows the amplifier frequency response with and without negative feedback. It is seen that negative feedback increases the amplifier bandwidth.

The open-loop gain is A and closed-loop gain is A_f. The open-loop bandwidth = $f_2 - f_1$ while the closed-loop bandwidth = $f_2' - f_1'$ where f_1 and f_2, f_1' and f_2' are the cut-off frequencies on open-loop and closed-loop conditions, respectively.

The upper cut-off frequencies are f_2 and f_1' and f_2' in non-feedback and feedback amplifiers, respectively. The upper cut-off frequencies for an amplifier is usually in KHz range while the lower cut-off frequency is somewhat less than 100 Hz, i.e. $f_2 >> f_1$ or $f_2' >> f_1'$.

Amplifier bandwidth, then, can be taken as equal to upper cut-off frequency. The product of amplifier gain and upper cut-off frequency is called gain–bandwidth product and is a constant quantity.

According to gain–bandwidth product as constant quantity,

$$A \times f_2 = A_f \times f_2'$$

i.e. Open-loop gain $\times BW_{(OL)}$ = Closed-loop gain $\times BW_{(CL)}$

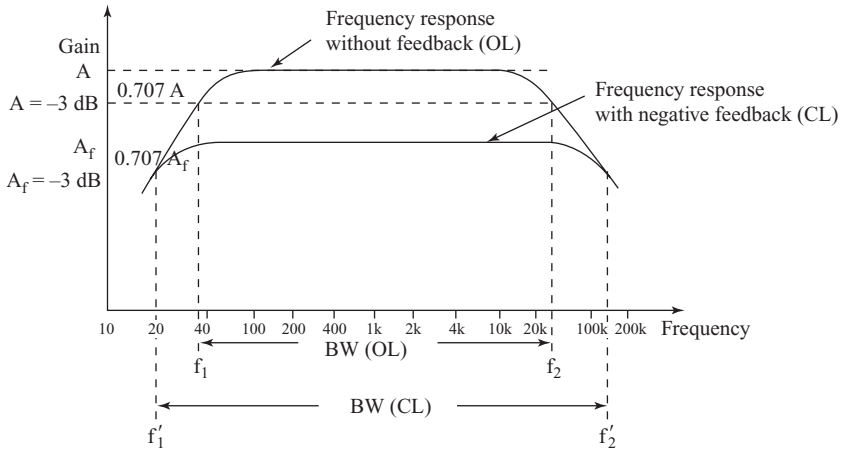


Figure 8.18 *Frequency response of an amplifier without feedback and with negative feedback*

8.6.6 Effect of Negative Feedback on Noise

Negative feedback reduces the circuit humming noise. It can be shown that noise introduced in an amplifier gets multiplied by a factor $1/(1+A\beta)$ if feedback is used (see Fig. 8.19). The factor $(1+A\beta)$ is larger than unity and hence there will be considerable reduction in noise.

If a large amplitude signal is applied to the amplifier input, then the non-linearity in the transfer characteristic of the amplifier active device distorts the output signal.

In Fig. 8.19,

V_s = Input voltage without feedback
V_0 = output voltage without feedback
V_i = Input voltage with feedback

Let the amplifier produce distortion D at the output without feedback. If feedback is present, a part of the output voltage and distortion is sent back to the input. The gain with feedback is A_f and total distortion at the output with feedback is D'. The feedback network provides a feedback of $D' = \beta(AD')$

$$D' = D - A\beta D'$$

$$D' + A\beta D' = D$$

$$D'(1 + A\beta) = D$$

$$D' = \frac{D}{(1+A\beta)}$$

Hence, distortion reduces by a factor $(1+A\beta)$ due to feedback circuit.

Let the noise voltage at the input of an amplifier be V_N.

The gain of amplifier is A in the absence of a feedback circuit. Therefore, noise without feedback at the output V_oN will be

$$V_{ON} = AV_N$$

where V_{ON} is the noise voltage at the output of the amplifier without feedback. Now, if negative feedback circuit is present with amplifier then gain $A_f = \frac{A}{1+A\beta}$. The output noise due to feedback is

$$V_{ONF} = \left(\frac{A}{1+A\beta}\right)V_N \qquad (\text{Also } AV_N = V_{NO})$$

$$V_{ONF} = \frac{AV_N}{1+A\beta}$$

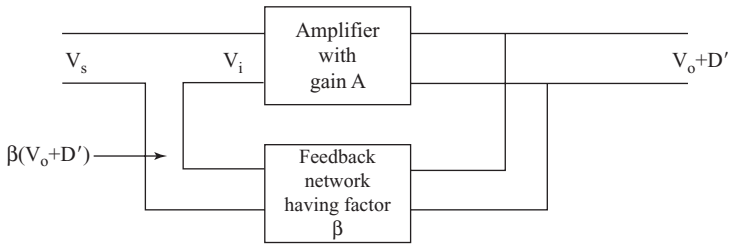


Figure 8.19 *Effect of negative feedback on distortion*

$$V_{ONF} = \frac{V_{ON}}{1+A\beta}$$

$$\text{i.e. Output noise with feedback} = \frac{\text{Output noise without feedback}}{(1+A\beta)}$$

Therefore, output noise voltage will decrease by a factor (1+Aβ) due to negative feedback.

8.6.7 Effect of Negative Feedback on Harmonic Distortion

Harmonics are undesirable components of the output voltage. Only the fundamental voltage wave is to be retained and the harmonics are to be reduced or eliminated. The harmonics generated are reduced by using negative feedback by a factor (1+βA). Harmonic distortion is generally expressed as a percentage of the RMS value of the output voltage. Thus, we can state that negative feedback reduces harmonic distortion. The percentage distortion is expressed as

$$\text{Percentage distortion with negative feedback} = \frac{\text{Percentage distortion without negative feedback}}{(1+A\beta)}$$

Effect of negative feedback on distortion is further illustrated as follows

Example 8.9 A negative feedback amplifier has an open-loop gain of 50,000. The open-loop upper cut-off frequency (i.e. BW) is 100 KHz. Calculate the closed-loop upper cut-off frequency (i.e. BW) if negative feedback of 2 per cent is introduced. Also, calculate the total harmonic distortion with feedback if there is a 15 per cent distortion without feedback.

Solution:

Given,

$$BW_{(OL)} = 100 \text{ KHz}$$

$$A = 50{,}000.$$

$$\beta = 0.02$$

$$A_f = \frac{A}{1+\beta A} = \frac{50{,}000}{1+0.02\times 50{,}000} = \frac{50{,}000}{1{,}001}$$

$$= 49.95.$$

Using the relation, gain–bandwidth product as constant quantity,

Open-loop gain × $Bw_{(OL)}$ = Closed-loop gain × $Bw_{(CL)}$

or,
$$50{,}000 \times 100 \times 10^3 = 49.95 \times BW_{(CL)}$$

or,
$$BW_{(CL)} = \frac{50{,}000 \cdot 100 \cdot 10^3}{49.95}$$

$$= 100{,}100 \times 10^3 \text{ Hz}$$

$$= 100.1 \text{ Hz}$$

Since the value of the lower cut-off frequency is low, the upper cut-off frequency is approximately taken as bandwidth.

Example 8.10 The open-loop gain of an amplifier is 10,000. The input and output impedances without feedback are 100 KΩ and 100 Ω, respectively. The feedback fraction is 0.01. Calculate (a) the gain with feedback; (b) input impedance with feedback; and (c) output impedance with feedback.

Solution:
Given,

$$A = 10{,}000$$

$$\beta = 0.01$$

$$Z_{in} = 100\ K\Omega$$

$$Z_o = 100\,\Omega$$

$$A_f = \frac{A}{1+\beta A} = \frac{10{,}000}{1+0.01\times 10{,}000} = \frac{10{,}000}{101}$$

$$= 99$$

$$Z_{in} \text{ with feedback} = (1+\beta A)\, Z_{in}$$

$$= (1+0.01\times 10{,}000)\times 100\ K\Omega$$

$$= 101\times 100\ K\Omega$$

$$= 10{,}100\ K\Omega$$

$$= 10.1\ M\Omega$$

$$Z_o \text{ with feedback} = \frac{Z_o}{1+\beta A} = \frac{100}{1+0.01\times 10{,}000}$$

$$= \frac{100}{101} = 0.99$$

The result shows that the value of input impedance has become very high due to the introduction of negative feedback and the value of output impedance has become quite low. This is the advantage of negative feedback. Amplifier gain has, however, been reduced.

Example 8.11 The open-loop gain of a transistor amplifier is 1,000. The input and output impedances are 5 kΩ and 100 Ω, respectively. Calculate the voltage gain and input and output impedances when one percent negative feedback is incorporated.

Solution:
Given,

$$A = 1{,}000$$

$$Z_{in} = 5\ K\Omega,\ Z_o = 100\,\Omega$$

$$\beta = 0.01$$

$$\text{Closed-loop voltage gain, } A_f = \frac{A}{1+\beta A} = \frac{1{,}000}{1+0.01\times 1{,}000}$$

$$= \frac{1{,}000}{11} = 90.9$$

$$Z_{in} \text{ with feedback} = Z_{in}(1 + \beta A)$$
$$= 5(1 + 0.01 \times 1{,}000)\ \text{K}\Omega$$
$$= 55\ \text{K}\Omega$$

$$Z_o \text{ with feedback} = \frac{Z_o}{1+\beta A} = \frac{100}{1 + 0.01 \times 1{,}000} = \frac{100}{11}$$
$$= 9.9\ \Omega$$

Example 8.12 The input and output voltages of an amplifier are 2 mV and 2V, respectively. The input and output impedances of the amplifier without feedback is 5 KΩ and 100 Ω, respectively. The closed-loop gain of the amplifier with negative feedback is 100. Calculate the input and output impedances with feedback.

Solution:

$$Z_{in} = 5\ \text{K}\Omega$$
$$Z_o = 100\,\Omega$$
$$A_f = 100$$
$$A = \frac{V_o}{V_i} = \frac{2}{2 \times 10^{-3}} = 1{,}000$$

We know,

$$A_f = \frac{A}{1+\beta A}$$

Substituting values,

$$100 = \frac{1{,}000}{1 + \beta \times 1{,}000}$$

or,
$$1 + 1{,}000\ \beta = 10$$

or,
$$\beta = \frac{9}{1{,}000} = 0.009$$

$$Z_{in} \text{ with feedback} = (1 + \beta A)\ Z_{in} = (1 + 0.009 \times 1{,}000) \times 5\ \text{K}\Omega$$
$$= 50\ \text{K}\Omega$$

$$Z_o \text{ with feedback} = \frac{Z_o}{1+\beta A} = \frac{100}{(1 + 0.09 \times 1{,}000)} = 10\ \Omega.$$

8.6.8 Effect of Negative Feedback on Sensitivity

We have seen from the examples discussed earlier that even when there is a large change in the value of the gain of an amplifier due to temperature, ageing, etc., the change in the closed-loop gain is only marginal.

Sensitivity is expressed as the ratio of the fractional change in the gain with feedback to the fractional change in the gain without the feedback.

The gain sensitivity, S, is expressed as

$$S = \frac{\frac{dA_f}{A_f}}{\frac{dA}{A}}$$

We had,

$$A_f = \frac{A}{1+\beta A}$$

Differentiating with respect to A,

$$\frac{dA_f}{A_f} = \frac{A}{1+\beta A}\ \frac{dA}{A}$$

or,

$$\frac{\frac{dA_f}{A_f}}{\frac{dA}{A}} = \frac{1}{1+\beta A}$$

Therefore, sensitivity, $S = \frac{1}{1+\beta A}$.

The inverse of sensitivity is called de-sensitivity.

Example 8.13 An amplifier has an open-loop gain of 10,000. The open-loop gain changes by 20 per cent due to ageing. Calculate the percentage change in the closed-loop gain for a feedback ratio of 0.1.

Solution:

Taking,

$$\frac{dA}{A} = 20 \text{ per cent}$$

$$A = 10{,}000$$

$$\beta = 0.1$$

$$A_f = \frac{A}{1+\beta A}$$

$$\frac{dA_f}{A_f} = \frac{dA}{A} \times \frac{1}{1+\beta A}$$

or,
$$\frac{dA_f}{A_f} = \frac{20}{1 + 0.1 \cdot 10,000} = \frac{20}{1001} = 0.019 \text{ per cent}$$

Therefore, the percentage change in the closed-loop gain is very small. This shows that negative feedback stabilizes the amplifier voltage gain.

Example 8.14 The open-loop gain of an amplifier changes by 20 per cent due to ageing, temperature effect, etc. A negative feedback of 20 dB is used to have a reasonably stable closed-loop gain. Calculate the percentage change in the gain of the negative feedback amplifier for an open-loop gain change of 20 per cent.

Solution:
The amount of feedback expressed in decibels is

$$20 \log_{10} \left| \frac{A_f}{A} \right| = -20$$

or,
$$\log_{10} \left| \frac{A_f}{A} \right| = -1$$

$$A_f = \frac{A}{1 + \beta A} .$$

$$\frac{A_f}{A} = \frac{1}{1 + \beta A} .$$

Therefore,

$$\log_{10} \frac{1}{1 + \beta A} = -1$$

$$\log_{10} (1 + \beta A) = 1$$

$$(1 + \beta A) = 10$$

Again,

$$A_f = \frac{A}{1 + \beta A}$$

$$\frac{dA_f}{A_f} = \frac{dA}{A} \times \frac{1}{1 + \beta A}$$

$$= 20 \cdot \frac{1}{10}$$

or, v

$$\frac{dA_f}{A_f} = 2 \text{ per cent.}$$

The change in gain with negative feedback is reduced to 2 per cent only.

8.7 EFFECT OF POSITIVE FEEDBACK

A positive feedback amplifier produces a feedback voltage, $V_{f,}$ which is in phase with the original input signal. For a positive feedback circuit, $|1+\beta A|<1$. For a negative feedback circuit, we had seen earlier that $|1+\beta A|>1$. For negative feedback, the gain of the amplifier gets reduced; whereas for positive feedback, the gain of the amplifier increases. For positive feedback system, $A_f > A$

or,

$$A_f = \left|\frac{A}{1-A\beta}\right| > A$$

Positive feedback or regenerative feedback is an effective means of increasing the gain of an amplifier. However, this method is not used in practice because of the reduced stability of the enhanced gain with positive feedback. Positive feedback increases the amplifier gain but makes the performance of the amplifier unstable. It, therefore, does not make any sense in increasing the gain of the amplifier with positive feedback and making the system unstable. If we try to increase the amplifier gain by making $|A\beta|$ almost equal to unity, there will be a spontaneous oscillation of the output voltage. Oscillations may persist at the output even with no input signal.

8.7.1 Sinusoidal Oscillators

We need to generate different types of waveforms such as sinusoidal, square, pulse, or triangular of varied frequency and amplitude for use. Oscillators generate repetitive waveforms of fixed amplitude and frequency without any external signal. Oscillators are also called waveform generators. Oscillators may be sinusoidal or non-sinusoidal (relaxation) type. Relaxation oscillators generate different types of waveforms using multivibrators. Multivibrators employ operational amplifiers (op-amps) and logic gates. Sinusoidal oscillators generate sinusoidal waveforms. One type of sinusoidal oscillator is a feedback oscillator.

A feedback sinusoidal oscillator has an amplifier and a feedback circuit. The feedback signal should be in phase with the input to the amplifier.

8.7.2 Principle of Sinusoidal Feedback Oscillation

We have seen earlier that the overall gain of a negative feedback amplifier, $A_{f,}$ is

$$A_f = \frac{A}{1+\beta A}$$

where A is the open-loop gain of the amplifier and β is the feedback ratio. If the feedback aids the input signal, i.e. if the feedback is positive, the overall gain can be expressed as

$$A_f = \frac{A}{1-\beta A} \ .$$

where βA is the loop gain.

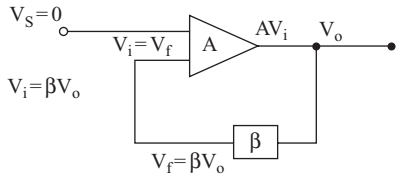


Figure 8.20 *Condition of oscillations illustrated*

If βA is made equal to 1 (i.e. unity loop gain), then A_f will be infinity. Therefore,

$$A_f = \frac{V_o}{V_s} = \infty$$

For $A_f = \infty$, V_s must be equal to zero. The amplifier will then give an output voltage without requiring any input voltage. The amplifier will become an oscillator. The initial signal to trigger the oscillations is provided by the transient noise signal appearing at the input terminal.

To clarify the concept of sustained oscillation, let us assume that a noise signal gives an output voltage, V_o, without any input voltage as shown in Fig. 8.20.

The feedback voltage, $V_f = \beta V_o$ is fed back to the input such that $V_i = V_f = \beta V_o$. The amplifier output is $AV_i = A\beta V_o$. If this signal is equal to V_o, then $A\beta V_o = V_o$ or $A\beta = 1$, where Aβ is the loop gain. With unity loop gain, an output voltage is available with $V_s = 0$, i.e. with no signal at the input. There will be a regeneration of the output voltage and the amplifier will provide an oscillatory output, i.e. the amplifier will oscillate.

8.7.3 Conditions for Oscillation

The conditions for oscillation are

(i) The loop gain must be unity, i.e. $|A\beta| = 1$.

(ii) The phase angle of Aβ must be zero or an integral multiple of 360°. This implies that the feedback must be regenerative. (Note that Aβ is a complex quantity and is a function of frequency)

When V_f is made equal to V_s, the amplifier cannot distinguish the source of its input, i.e. whether it is from an external source or from the feedback path. The condition for $V_f = V_s$ is that the loop gain must be unity.

The gain of the feedback amplifier with positive feedback loop is

$$A_f = \frac{A}{1 - \beta A}.$$

Note that both A and β are functions of frequency.

If at a particular frequency, say W_0, the loop gain Aβ is unity, then A_f will be infinity. Again, since $A_f = \frac{V_o}{V_s}$, the amplifier will have finite output for zero input. The amplifier with positive feedback will work as an oscillator. Since A and β are complex quantities, two conditions of oscillations emerge, i.e.

$$|A\beta| = 1 \text{ and } \angle A\beta = 0.$$

The magnitude condition states that the feedback signal to the input must be the same as the input signal. The second condition is that the feedback should be positive with zero phase shift. Thus, at a particulars frequency, ω_0 the phase of the loop gain should either be zero or multiples of 2π and the magnitude should be unity. This is known as Barkhausen Criterion.

8.7.4 The Barkhausen Criterion

The condition of unity loop gain $|A\beta|=1$ and that the phase shift of $A\beta$ is zero is called the Barkhausen criterion. The frequency at which these conditions are satisfied is the frequency at which the oscillator will operate. We can also state that the frequency at which the sinusoidal oscillator will oscillate is determined by the condition of loop-phase shift to be zero or multiples of 360°.

The oscillations will not be sustained (i.e. continue) if the loop gain is less than unity.

The Barkhausen criterion for oscillation is summarized as

(i) positive feedback is to be used

(ii) The loop gain, $|A\beta|$ must be equal to 1, i.e. be unity. Consequently, the feedback gain for a positive feedback amplifier will be infinity as

$$A_f = \frac{A}{1 - A\beta} .$$

If $A\beta = 1$,

$$A_f = \frac{A}{0} = \infty.$$

(iii) The loop phase shift is to be zero or multiples of 360°.

In an oscillator if $|\beta A|$ becomes less than unity, the oscillation will die out soon. If $|\beta A|$ is greater than unity, the amplitude of oscillation will become larger and larger due to regenerative action. The oscillation will continue to increase. However, the increase will be limited by the saturation effect (non-linearity of operation). An oscillator in which the loop gain, i.e. $|\beta A|$ is exactly unity is unrealizable in practice. The values of circuit components change due to temperature and other reasons. For the loop gain not to fall below unity and thereby stop oscillation, the loop gain of a practical oscillator is kept somewhat higher (say 5 per cent) than unity. The increased amplitude of oscillation will be limited by the non-linearity of operation.

8.7.5 Instability and Nyquist Stability Criterion

In an oscillating circuit, oscillations will die out due to dissipation of energy in the resistance elements contained in the circuit. To sustain oscillation, therefore, virtual negative resistances should be used. However, the same objective is achieved by using positive feedback such that the overall gain reaches infinity. The initial triggering signal is received from the noise voltage appearing at the input. The ac power of the output is supplied from the dc supply to the amplifier. An amplifier is stable if a transient input signal gives a response (i.e. output) that diminishes with time. The amplifier is unsta-

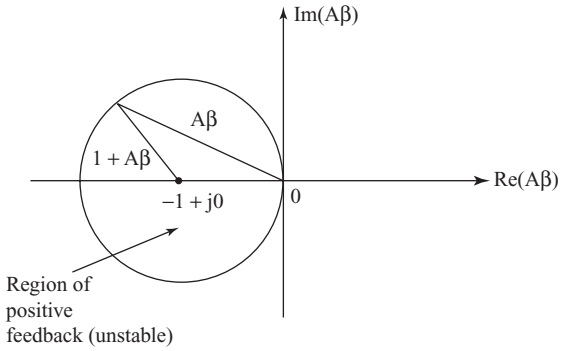


Figure 8.21 *Nyquist stability criterion*

ble if the response increases with time. The amplifier is critically stable if the response continues to persist indefinitely.

Nyquist in 1932 developed a criterion to determine the condition for stability of a feedback amplifier or any control system. The criterion has been developed from the frequency response characteristic of the loop gain Aβ. The product Aβ is a complex number and is a function of frequency.

The real component of Aβ is taken on the X-axis, i.e. the real axis and the imaginary component is taken on the imaginary axis. For all values of frequency ω from 0 to ∞, the values of Aβ (both real component and the imaginary component) are plotted. By joining all the plotted points we will get the locus. Here the locus will be a closed circle as shown in Fig. 8.21.

The Nyquist criterion states that an amplifier is unstable if (Aβ) plot encloses (−1 + j0) in the complex plane. The amplifier will be stable if the curve does not enclose the point (−1 + j0).

The criterion for positive and negative feedback can also be represented in the complex plane.

The condition for negative feedback, we have seen earlier, is that $|1+ A\beta|=1$ for all frequencies.

It is observed from Fig. 8.21 that $|1 + A\beta|>|$ represents a circle of radius equal to 1 and the centre of the circle at (−1 + j0). For $|1 + A\beta| >1$ the feedback is negative, and the vector Aβ will extend beyond the circle of unit radius. The area beyond the circle represents negative feedback. If Aβ lies within the circle, then $|1 + A\beta|<1$ and the feedback is positive.

8.8 REVIEW QUESTIONS

A. Short Answer Type Questions

1. With the help of block diagrams show positive and negative feedback in amplifiers.
2. Show the effect of positive feedback on the voltage gain of an amplifier.
3. Show the effect of negative feedback on the voltage gain of an amplifier.
4. Explain the principle of a feedback amplifier with the help of a block diagram. Find an expression for voltage gain with feedback.
5. Draw the four feedback topologies.
6. State the effects of negative feedback in an amplifier.
7. Explain why negative feedback is used in amplifiers although it reduces the gain of the amplifier.

8. Show how negative feedback improves the stability of the gain of an amplifier.
9. Show that negative feedback can change the input and output impedances of an amplifier.
10. Show the effect of positive feedback in an amplifier on voltage gain and its stability.
11. Explain the effect of negative feedback on gain stability.
12. Show the effect of negative feedback on bandwidth.
13. Explain the effect of negative feedback on noise.
14. What is a sinusoidal oscillator?
15. Explain the principle of sinusoidal feedback oscillations.
16. What are the conditions for sustained oscillations in a feedback amplifier?
17. State the Barkhausen criterion for sustained oscillation in an oscillator.
18. Briefly explain the Nyquist stability criterion.

B. Numerical Problems

19. An amplifier has a voltage gain of –100. The feedback ratio is –0.04. Calculate the voltage gain with feedback and the amount of feedback in dB.

    [Ans. $A_f = -20$; $F = -13.98$]

20. The open-loop voltage gain of an amplifier changes by 10 per cent due to temperature effect on its components. However, a change in gain of only 2 per cent is allowable. State the kind of feedback that has to be employed. For an amplifier gain with feedback of 20, calculate the minimum value of feedback ratio.

    [Ans. Negative feedback, 0.04]

21. The open-loop voltage gain of an amplifier changes by 5 per cent. Calculate the percentage change of the closed-loop gain for a 10 dB negative feedback.

    [Ans. 1–6 per cent]

22. The closed-loop gain of an negative feedback amplifier is 99.7 when feedback factor is 0.01 and is 297 when feedback factor is 0.0033. What is the value of the open-loop gain?

    [Ans. 33,233]

23. Calculate the closed-loop gain for a negative feedback amplifier when open-loop gain of the amplifier is increased by 20 per cent from its value of 50,000. Consider $\beta = \frac{1}{100}$.

    [Ans. 99.83]

24. For a negative feedback amplifier, A = 30,000 and $\beta = \frac{1}{200}$. Determine the closed-loop gain of the amplifier. Also find the open-loop gain of the amplifier when $\beta = \frac{1}{100}$.

    [Ans. 198.67, 247.92]

Integrated Circuits and Operational Amplifiers

TOPICS DISCUSSED

- Basics of integrated circuits
- Operational amplifiers
- Basic op-amp circuits

9.1 INTRODUCTION

Diodes, transistors, resistors and capacitors are components that are connected in a circuit to perform certain functions thereby forming a functional unit. These individual circuit components are called discrete components.

In linear integrated circuits (ICs) many discrete components such as diodes, transistors, resistors and capacitors are fabricated using special fabrication techniques on a single silicon chip. An operational amplifier (op-amp) is an integrated circuit silicon chip packaged in a single casing. The operational amplifier is the best example of a linear integrated circuit. The significance of the term operational amplifier is that it can perform many operations such as summation, subtraction, multiplication, differentiation, integration, etc. Such operations are used in analog computers. Operational amplifiers can also be used in signal amplification, active filters, oscillators, voltage regulators, analog to digital and digital to analog converters, etc. The diverse applications of an op-amp make it an invaluable IC in the field of electronics, instrumentation and control engineering. The advantage of using an op-amp is that negative feedback can be applied and the use of negative feedback makes the operation of the circuit stable.

9.2 BASICS OF INTEGRATED CIRCUITS

The demand for the manufacture of electronic devices and circuits, which are smaller in size and consume less power, has led to the development of integrated circuits (ICs). An integrated circuit contains a large number of discrete components such as diodes, transistors, resistors, and

capacitors, which are interconnected together so as to perform some function. ICs can be classified into *monolithic* and *hybrid*. They are also classified as *linear* and *digital*. Monolithic is a Greek word that means single stone or single piece. In monolithic ICs, the circuit components and their interconnections are formed on a single piece of crystalline silicon. Very thin, defect-free, flat silicon circular sheets called *wafers* are used to fabricate monolithic ICs. The wafers are approximately 1mm thick. A wafer is divided into a large number of identical areas. Each small area will contain an integrated circuit along with connections. A small area in a wafer containing an integrated circuit is called a *chip*. Thus, a large number of chips are formed on a single wafer. The miniaturization is such that 60 to 70 circuit components of transistors, diodes, capacitors, resistors, etc. with their in connections can be formed on a single chip of area 1.65 mm × 1.65 mm. The miniaturization effect has continued with the advancements in technology and it is now possible to develop memory chips containing a million transistors.

A hybrid IC is formed by interconnecting a number of monolithic ICs. As mentioned earlier, ICs can be linear and digital. In a linear IC, the input–output relation is linear. Digital ICs often perform bistable functions and work as switches ('on' or 'off'). Linear ICs are used in amplifiers, oscillators, voltage regulators, etc. Digital ICs find applications in logic circuits and computers.

Figure 9.1 shows a silicon wafer with a large number of square-shaped chips on it. A sample circuit to be constructed on a chip has also been shown. After the fabrication of a circuit on a chip it is packaged.

ICs can be used to perform different functions and can be replaced as a whole in case defects occur in the functioning of the IC. Integrated circuit technology has advanced from small-scale integration to very large-scale integration (VLSI).

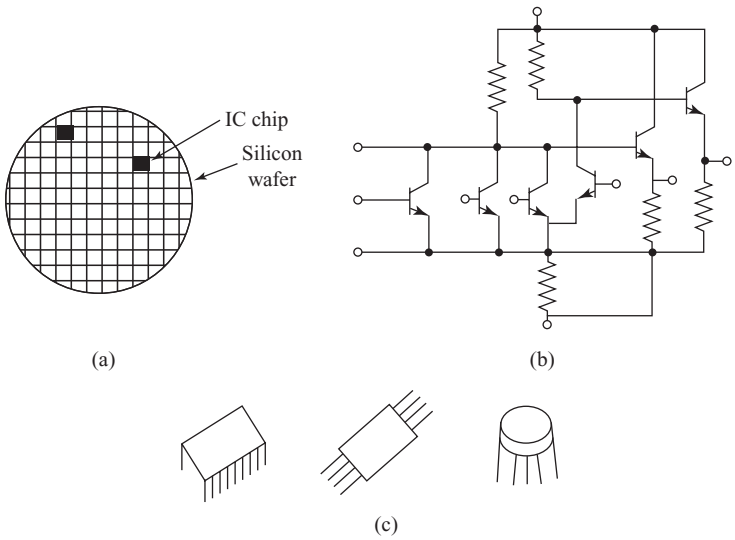


Figure 9.1 *(a) IC chips made from a thin silicon wafer; (b) logic circuit to be fabricated on a chip; (c) IC packages of various types*

Fabrication of monolithic ICs

The fabrication of ICs involves a very special and sophisticated technology. The steps followed are mentioned below.

(i) **Wafer preparation:** A wafer is a very thin surface of p-type silicon, which provides the base or substrate on which the circuit elements are grown or developed. Wafers are made by slicing a p-type round silicon material of certain diameter. These wafers are polished to mirror finish surface.

(ii) **Epitaxial growth:** An n-type layer is grown on the p-type substrate by placing the wafer in a furnace in an atmosphere of phosphorous gas at 1200°C. This way, an epitaxial layer of n-type material is formed on the p-type wafer.

(iii) **Diffused isolation:** A thin layer of silicon dioxide is formed on the n-type layer by oxidation method, i.e. by oxidizing the wafer in dry oxygen. A thin coat of a chemical called photo resist is then made on the silicon dioxide layer. After these, use is made of a mask, ultraviolet exposure, etching, scrubbing and diffusion. Thus, the monolithic process of IC fabrication involves wafer preparation, epitaxial growth, diffused isolation, base and emitter diffusion, etching, metallization (i.e. making of contacts and interconnections), checking for circuitry, separating the wafer into individual chips, mounting, packaging, sealing, and testing. The components of integrated circuits such as resistors, capacitors, diodes, transistors, etc. are made on the monolithic, i.e. single chip structure. For example, resistors are made using the resistivity of the diffused areas. Diodes are made utilizing the p–n junctions available in the diffused structure. Both p–n–p and n–p–n transistors are made on the substrate by the diffusion process.

Advantages of ICs over discrete circuits

Integrated circuit technology has brought in a number of advantages over the fabrication of electronic circuits using individual circuit components, i.e. discrete circuits. The advantages are highlighted as follows:

(i) Miniaturization of circuits, batch production, i.e., a large number of identical circuits can be produced together.

(ii) Reduction of cost of production.

(iii) Smaller size and less weight, suitable for space applications like in aircraft, space vehicles, etc.

(iv) Maintenance is easy, the whole of the IC is replaced in case of any fault. There is no need for repair of components.

(v) Less likely to be faulty. Since ICs are manufactured in monolithic structure, there is very little scope for the circuit to go faulty except for misuse.

(vi) Performance of ICs are better than discrete circuits in high-frequency applications.

Digital ICs are also classified according to the number of components or gates placed on one chip. Large-scale integration (LSI) will have more than 100 components and very large-scale integration (VLSI) will have more than 1,000 components in a single chip.

Integrated circuits are manufactured by various manufacturers like Fairchild, Motorola, Texas Instruments, National Semiconductor, Signetics, etc. They put their brand name along with a particular number on each type of IC that they produce. For example, operational amplifiers, manufactured by various manufacturers are known as:

Motorola	MCI 741
National Semiconductor	LM 741
Texas Instruments	SN52 741

The last three digits in each case is 741. Irrespective of who manufactures an operational amplifier, the specification for an IC 741 is the same. We shall discuss in brief the IC 741, which is an op-amp, the IC 555 which is a timer, and the IC 78XX series which consists of voltage regulators.

9.3 OPERATIONAL AMPLIFIERS

An operational amplifier is abbreviated as op-amp. An op-amp is the best-known linear integrated circuit. Originally op-amps were meant for performing operations like integration, subtraction, differentiation, etc., and hence the name was given.

Such operations are useful in analog computers. However, op-amps can also be used in signal amplification, filters, oscillators, voltage regulators, analog to digital (A to D) and digital to analog (D to A) converters, etc. Manufacturers provide a number of informations of the IC they supply including their pin diagrams. The informations provided include intended applications, maximum ratings, electrical characteristics, performance limitations, equivalent circuit, etc. Maximum ratings may include supply voltage, internal power dissipation, temperature range, etc. The pin diagrams of an IC 741 in eight-pin metal-can package and eight-pin mini dip package are shown in Fig. 9.2.

An op-amp is a high-gain directly coupled linear differential amplifier. The performance of an op-amp is controlled by negative feedback from the output to the input. It is called a differential amplifier since it amplifies the difference between the two input signals. An op-amp amplifies both ac and dc input signals.

The circuit symbol of an op-amp has been shown in Fig. 9.3. The input terminals are marked a and b. Terminal a is marked negative while terminal b is marked positive. Terminal a is the inverting terminal. The input provided at this terminal appears inverted at the output with amplification. Terminal b is non-inverting. The output for input at b appears non-inverted but amplified . When signals are provided at both a and b terminals, the output at terminal c is proportional to the difference of the two signals.

The internal circuit is quite elaborate. But the user of an op-amp need not go to the details of the functioning of the op-amp circuit. The user has to know the basic specifications and the terminals so as to make connections with external circuit components for a purpose.

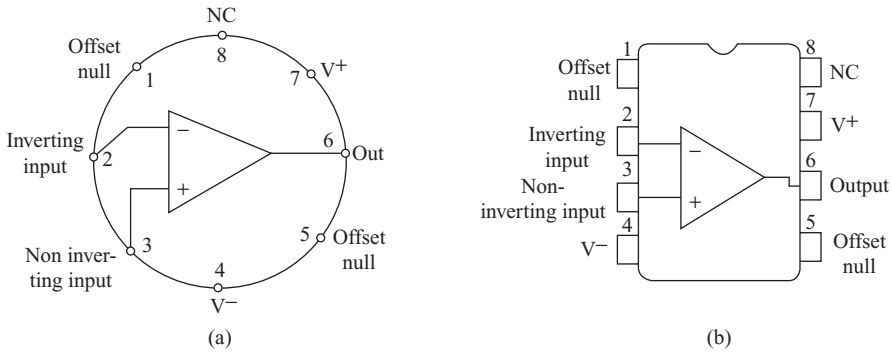


Figure 9.2 *Pin diagrams of IC op-amp 741. (a) 8-pin mini-dip packaging; (b) 8-pin metal-can packaging*

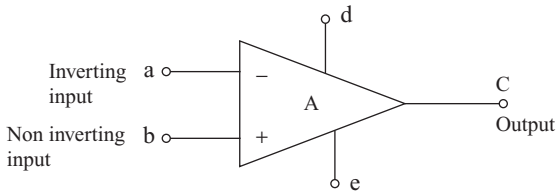


Figure 9.3 *Symbolic representation of a basic op-amp*

The power supply voltages which are usually balanced with respect to the ground are applied to terminals 7 and 4, i.e. to terminals d and e as in Fig. 9.2 and Fig. 9.3. Terminals d and e are often not shown in the circuit diagrams using op-amps.

An ideal op-amp has the following characteristics:

(i) Infinite input impedance
(ii) Zero output impedance
(iii) Infinite voltage gain
(iv) Infinite bandwidth
(v) Perfect balance (i.e., output is zero when both the inputs are equal)

A practical op-amp will have finite but very high input impedance, very high voltage gain, finite bandwidth and low output impedance. A practical op-amp may not have perfect balance. The output is fed back to the inverting input terminal to provide negative feedback for the op-amp.

9.3.1 The Practical Op-amp

An ideal op-amp is not possible to realize in practice. The characteristics of a practical op-amp are high voltage gain, high input impedance and low output impedance.

An op-amp normally is made up of three types of amplifier circuits, i.e. a differential amplifier, a voltage amplifier, and a push–pull amplifier. The differential amplifier is the input stage of the op-amp. It has two input terminals. It provides amplification of the difference of the two input voltages. Next is the voltage amplifier, which is normally a class A amplifier. This provides additional gain to the op-amp. In some op-amps more than one voltage amplifiers are used to obtain sufficient gain of the op-amp. A push–pull amplifier is used for the output stage as has been shown in the form of a block diagram in Fig. 9.4. A practical op-amp has been shown in Fig. 9.5.

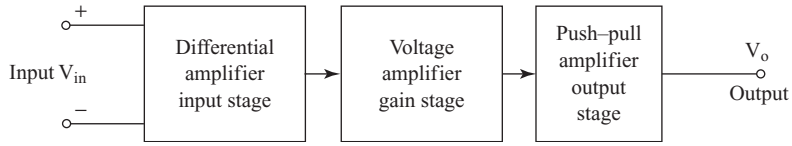


Figure 9.4 *Block diagram representation of the internal arrangement of an op-amp*

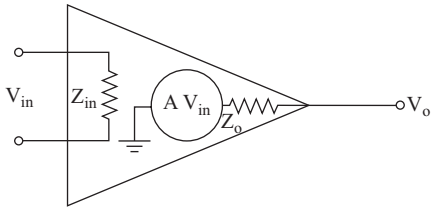


Figure 9.5 *Representation of a practical op-amp*

9.3.2 The Differential Amplifier

An op-amp may have more than one differential amplifier stage. However, we will consider one to explain the basic operation. A differential amplifier which is a part of the op-amp at the input stage has two input terminals and two output terminals. There is also a positive and negative supply voltage as shown in Fig. 9.6. Two bipolar junction transistors (BJTs) have been used. FETs can also be used if very high input impedance is desirable. Note that a differential amplifier has two input terminals and two output terminals whereas an op-amp has two input terminals and one output terminal.

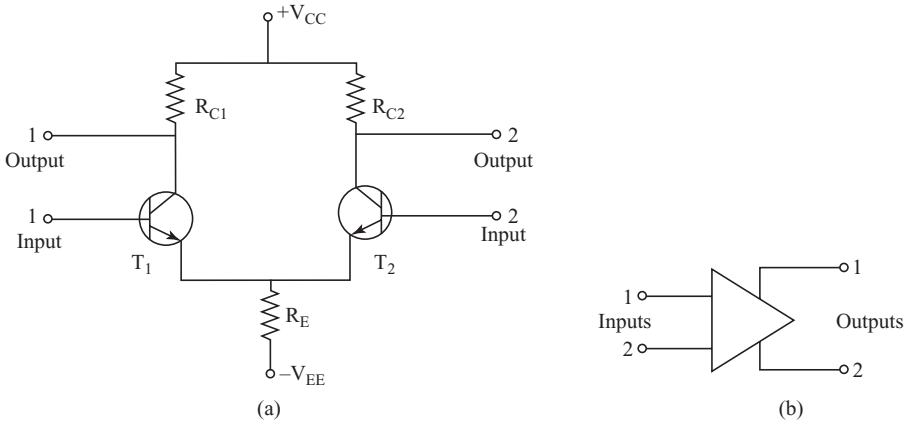


Figure 9.6 *(a) Circuit diagram of a differential amplifier (b) symbol of a differential amplifier*

Modes of operation

The response of a differential amplifier when operated with one input and with the other end grounded is called single-ended mode of operation. The single-ended input operation of a differential amplifier is shown in Fig. 9.7.

When a single voltage is applied to terminal 1, an inverted amplified voltage will appear at output terminal 1 as shown in Fig. 9.7(a). A non-inverted amplified signal voltage will appear at the output terminal 2. When the signal voltage is applied to input terminal 2 with input terminal 1 grounded, an inverted amplified signal voltage will appear at output terminal 2 and a non-inverted amplified signal voltage will appear at the output terminal 1 as shown in Fig. 9.7 (b).

We will now examine the differential mode of operation where two signals of opposite polarity are applied simultaneously to the two input terminals. The outputs due to such differential inputs are shown in Fig. 9.8.

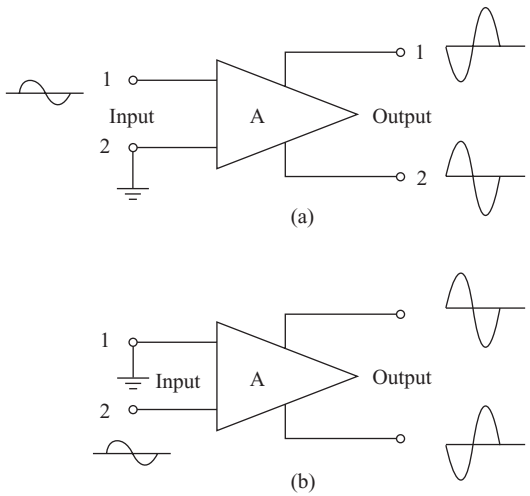


Figure 9.7 *Single-ended mode of operation of a differential amplifier: (a) input at terminal 1 (b) input at terminal 2*

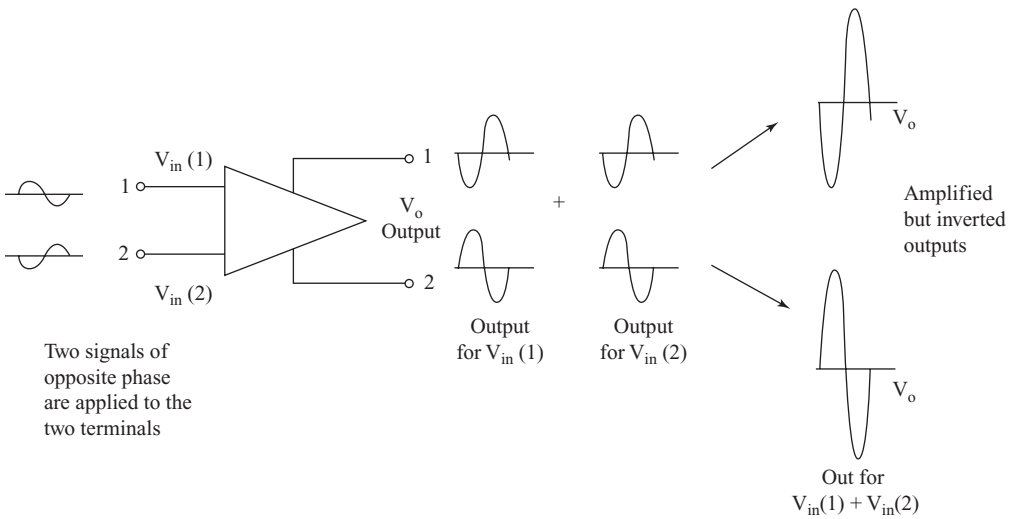


Figure 9.8 *Differential operation of a differential amplifier*

The outputs at terminals 1 and 2 are due to the sum of outputs due to the input at terminal 1 and 2. We have superimposed the outputs at terminal 1 and 2 due to inputs V_{in} (1) and V_{in} (2), respectively. The outputs at the two terminals are amplified but inverted voltage signals of the respective input signals.

Let us discuss what happens when two signals of the same phase (i.e. polarity), amplitude (i.e. magnitude) and frequency are applied to the two input terminals. The outputs due to each input and the total output have been shown in Fig. 9.9.

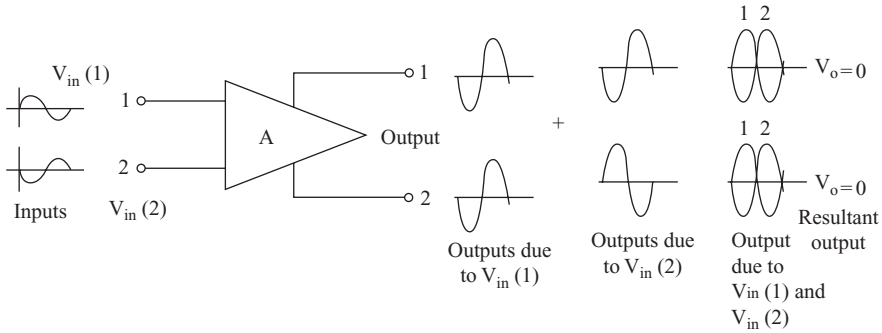


Figure 9.9 *Common mode of operation of differential amplifiers*

It is observed from Fig. 9.9 that when the outputs due to the inputs are superimposed, they cancel each other giving a resultant output, which is zero at both the output terminals. This type of operation is called common mode of operation and the action of the amplifier is called common mode rejection.

Common mode rejection is used in the elimination of unwanted signals such as noise signals appearing at the input terminals. These unwanted signals appearing at the input terminals get eliminated at the output so that the desirable signal does not get distorted. Thus, we can say that the unwanted signals of same polarity appearing at the inputs do not appear at the output of the amplifier.

The ability of an amplifier to reject common mode signals is one of the parameters of a differential amplifier and is expressed as common mode rejection ratio (CMRR). The ratio of open-loop gain, A(OL) to the common mode gain, A(CM) is called common mode rejection ratio.

$$\text{CMRR} = \frac{\text{A}(\text{OL})}{\text{A}(\text{CM})}$$

When expressed in dB,

$$\text{CMRR} = 20 \log_{10} \frac{\text{A}(\text{OL})}{\text{A}(\text{CM})} \tag{9.1}$$

9.3.3 Op-amp Parameters

The parameters of an ideal op-amp were mentioned earlier. Several important parameters of a practical op-amp are described below. Op-amps of different varieties are compared in terms of the following practical parameters.

1. Input offset voltage
2. Input bias current
3. Input impedance
4. Output impedance
5. Open-loop voltage gain
6. Common mode rejection ratio
7. Slew rate

These parameters are discussed below.

Input offset voltage: Ideally an op-amp should produce a zero-volt output for a zero-volt input. However, in practical op-amps a small dc voltage appears at the output with no input voltage. This is due to mismatches between the base-emitter voltages of the differential input stage of the amplifier. The input offset voltage is the differential dc voltage required between the inputs so as to make the differential output zero. This input offset voltage is of the range of 2 mV or even less.

Input bias current: It is the amount of direct current required at the inputs for operation of the first stage of the differential amplifier.

Input impedance: It is the total impedance between the inverting and non-inverting terminals.

Output impedance: The output impedance is the impedance viewed from the output terminal of the op-amp.

Open-loop voltage gain: Open-loop voltage gain is the ratio of output voltage to the input voltage. This ratio depends upon the internal design of the amplifier. Voltage gain (open-loop) is very high. Its value can be as high as hundreds of thousands.

Common mode rejection ratio: It is the ratio of open-loop voltage gain to the common mode voltage gain. CMRR is often expressed in decibels as

$$\text{CMRR} = 20 \log_{10} \frac{\text{A(OL)}}{\text{A(CM)}}$$

Slew rate: Slew rate is defined as the maximum rate of change of output voltage when a step input voltage is applied to an op-amp. Slew rate can be expressed as, $\text{SR} = \frac{dV_o}{dt}$ (maximum) and is expressed in voltage per micro-second. Slew rate indicates how rapidly the output voltage changes in response to changes in the input voltage frequency.

9.3.4 Op-amp Configuration with Negative Feedback

Negative feedback is an important concept in the field of electronics and control engineering. Negative feedback is used in op-amp applications with several advantages.

Negative feedback is a process whereby a portion of the output voltage is fed back i.e. returned to the input with a phase angle that opposes the input signal i.e. the feedback voltage opposing the input voltage.

We know that an op-amp has two input terminals and one output terminal. One of the input terminals is called an inverting input terminal and the other is called a non-inverting input terminal. An op-amp has very high voltage gain. A small difference in input voltages, thus, produces a high output voltage. Suppose a small voltage of 10 mV is applied to the non-inverting input terminal of an op-amp whose open-loop gain is 100,000. The output will be 1 mV × 100,000 = 1000 V which is a very high a voltage and, hence, the amplifier will move towards saturation region. If a fraction of the output is fed back to the inverting terminal, the output voltage will be reduced. This will bring down the output voltage so that the amplifier works in the linear region. Further, any change in the output due to a change in the amplifier gain is immediately compensated by the negative feedback.

An op-amp can be connected in three different ways using negative feedback. These three types of connections make the following configurations, namely

1. Non-inverting amplifier
2. Inverting amplifier
3. Voltage follower

These are described in the following sections.

9.3.5 Non-inverting Amplifiers

The input is applied to the non-inverting input terminal (marked +) of the op-amp. A portion of the output voltage is returned back to the inverting terminal (marked –) as shown is Fig. 9.10.

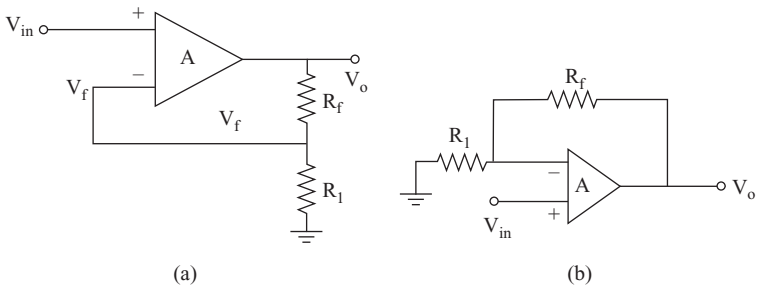


Figure 9.10 *Non-inverting amplifier; (a) is redrawn as in (b)*

The ratio of V_o and V_{in} is the closed-loop voltage gain of the amplifier.
From Fig. 9.10(a), using the voltage-divider rule,

$$V_f = \frac{V_o}{R_f + R_1} \times R_1 = \left(\frac{R_1}{R_f + R_1} \right) V_o \tag{9.2}$$

For the negative feedback amplifier, $(V_1 – V_f)$ is the input. This gets multiplied by the open-loop gain of the amplifier, A, and gives an output voltage V_o. Therefore,

$$\left(V_1 - V_f\right) A = V_o$$

or,

$$V_1 - V_f = \frac{V_o}{A}$$

The open-loop gain of the amplifier is very high. Thus, as $A \to \infty$, $\left(V_1 - V_f\right) \to 0$. This input differential voltage, $(V_1 - V_f)$ becomes very small as a result of the negative feedback and high open-loop gain of the amplifier. If $\left(V_1 - V_f\right)$ is very small, we can take V_f as approximately equal to V_1. Thus, from Eq. (9.2), we write,

$$V_f = V_1 = \frac{R_1}{R_f + R_1} \times V_o = \beta V_o$$

where

$$\beta = \frac{R_1}{R_1 + R_f}$$

or,

$$\frac{V_o}{V_1} = \frac{R_1 + R_f}{R_1}$$

or,

$$V_o = \left(\frac{R_f + R_1}{R_1} \right) \times V_1$$

or,
$$V_o = \left(1 + \frac{R_f}{R_1}\right) V_1$$

$$\frac{V_o}{V_1} = A(CL) = 1 + \frac{R_f}{R_1}$$

The closed-loop gain of the non-inverting feedback amplifier is

$$A(CL) = 1 + \frac{R_f}{R_1} = \frac{1}{\beta} \tag{9.3}$$

This shows that the closed-loop gain of the amplifier is not dependent on the open-loop gain of the amplifier. By properly selecting the values of R_f and R_1 the A(CL) can be set to a desired value.

Example 9.1 Calculate the value of R_f in Fig. 9.11 so that the closed-loop gain of the amplifier can be made equal to 20.

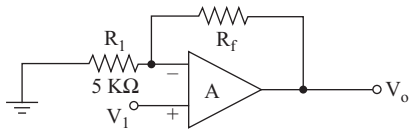


Figure 9.11 *Refers to Example 9.1*

Solution:
The amplifier is connected in inverting mode.

$$A(CL) = 20 = 1 + \frac{R_f}{R_1}$$

or,
$$20 = 1 + \frac{R_f}{5k\Omega} = \frac{(5\ k\Omega)}{5k\Omega} + R_1$$

or,
$$5\ k\Omega \times 20 = 5\ k\Omega + R_1$$

or,
$$R_f = 9.5\ k\Omega\ .$$

9.3.6 Inverting Amplifier

An op-amp connected as an inverting amplifier has been shown in Fig. 9.12. The input signal is applied to the inverting terminal (negative terminal) through a resistor R_1. The output is fed back to the inverting input terminal (negative terminal) through a feedback resistance R_f.

The non-inverting input terminal has been connected to the ground. Let the voltage at point Q is ν. The current through the input resistance, i, is

$$i_1 = \frac{V_{in} - \nu}{R_1}$$

As the input impedance is very high, the current through the amplifier is negligible. The same current i_1 will pass through the feedback resistance R_f. Thus, we can write

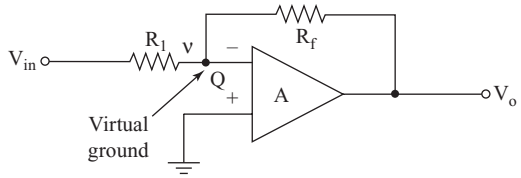


Figure 9.12 *Op-amp as an inverting amplifier*

$$i_1 = \frac{V_{in} - \nu}{R_1} = \frac{\nu - V_o}{R_f}$$

or,
$$\frac{\nu - V_o}{V_{in} - \nu} = \frac{R_f}{R_1} \tag{9.4}$$

The open-loop gain of the amplifier is A so that

$$V_o = A\nu$$

or,
$$\nu = \frac{V_o}{A}$$

or,
$$A = \frac{V_o}{\nu}$$

Assuming amplifier gain to be very high, i.e. if

$$A \to \infty \text{ , } \nu \to 0.$$

Taking $\nu = 0$, Eq. (9.4) would become

$$\frac{0 - V_o}{V_{in}} = \frac{R_f}{R_1}$$

or,
$$\frac{V_o}{V_{in}} = -\frac{R_f}{R_1} \tag{9.5}$$

$\frac{V_o}{V_{in}}$ is the overall gain of the amplifier. This is termed as the closed-loop gain, A(CL). A(CL) of the inverted amplifier is

$$A(CL) = -\frac{R_f}{R_1} \tag{9.6}$$

The negative sign indicates that the output voltage is inverted, i.e. it indicates inversion. The open-loop voltage gain of an amplifier is dependent on its circuit design. However, if there is any variation in the values of circuit components due to ageing, temperature, or replacement, the open-loop gain changes. The closed-loop gain A(CL) is independent of the variation of circuit parameters. The closed-loop gain is independent of the open-loop gain of the op-amps. Thus, we can see that negative feedback has made the gain of the amplifier independent of any change in the amplifier's internal circuit parameters. The voltage gain of the closed-loop op-amp will, therefore, be stable.

9.3.7 Voltage Follower

It is a special case of a non-inverting amplifier where the input voltage is applied to the non-inverting (+) terminal and all the output voltage is fed back to the inverting (−) terminal as shown in Fig. 9.13.

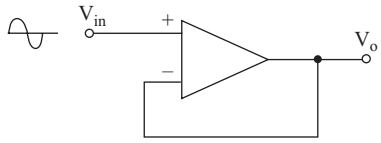


Figure 9.13 *Op-amp voltage follower*

Since the feedback loop is a straight connection of the output voltage to the inverting terminal (–) of the op-amp, $V_f = \beta V_o = V_o$. The value of $\beta = 1$. The closed-loop voltage gain of a non-inverting amplifier is (see Eq. 9.3)

$$A(CL) = \frac{1}{\beta}$$

Here, for a voltage follower, $\beta = 1$. Therefore, the gain of a voltage follower

$$= \frac{V_o}{V_{in}} = A(CL) = 1 \tag{9.7}$$

Thus, for a voltage follower, the output terminal is connected directly to the inverting input (−) terminal. The signal is applied to the non-inverting (+) terminal and the load is connected to the output terminal. The voltage follower has very high input impedance and very low output impedance, and a unity closed-loop voltage gain. These features make a voltage follower suitable as a buffer amplifier for interfacing high-impedance sources and low-impedance loads.

9.3.8 Impedances of Op-amps

(a) Impedances of non-inverting amplifier: Negative feedback in a non-inverting configuration increases the input impedance and reduces the output impedance (Fig. 9.14). If Z_{in} and Z_o are respectively the input and output impedances of the op-amp without feedback, their values with negative feedback Z_{in}' and Z_o' are

$$Z_{in}' = (1 + A\beta) Z_{in} \tag{9.8}$$

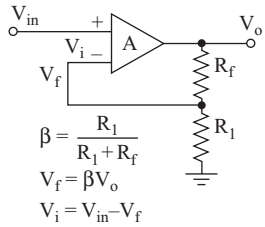


Figure 9.14 *Non-inverting amplifier where A is the open-loop gain and β is the feedback fraction or attenuation of the feedback circuit*

$$Z_o' = \frac{Z_o}{1+A\beta} \tag{9.9}$$

(b) Impedances of inverting amplifier:

$$Z_{in}' = R_1 \tag{9.10}$$

$$Z_o' = \frac{Z_o}{1+A\beta} \tag{9.11}$$

Because of the virtual ground of the input terminal, input impedance Z_{in}' is equal to input resistance R_1 (Fig. 9.15). The output impedance is reduced as in the case of non-inverting amplifier.

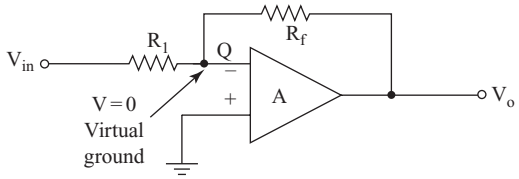


Figure 9.15 *Inverting amplifier; zero voltage at Q is called the virtual ground*

(c) Impedances of a voltage follower: A voltage follower is a special case of non-inverting configuration of an op-amp with unity feedback, i.e. with β=1. Therefore, the input and output impedances of the feedback amplifier are

$$Z_{in}' = (1+A)\, Z_{in} \tag{9.12}$$

$$Z_o' = \frac{Z_o}{(1+A)} \tag{9.13}$$

where A is the open-loop gain of the amplifier.

***Example* 9.2** An op-amp with an open-loop gain of 100,000 and input and output impedances of 10 MΩ and 50 Ω, respectively, is connected in the non-inverting mode of feedback as shown in Fig. 9.16. Calculate the input and output impedance of the feedback amplifier. Also calculate the closed-loop voltage gain.

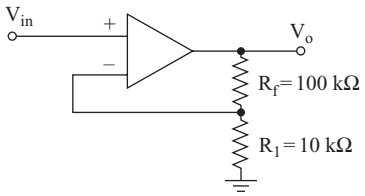


Figure 9.16

Solution:
The feedback fraction of the attenuation β is calculated as

$$\beta = \frac{R_1}{R_1 + R_f} = \frac{10}{10 + 100} = 0.9$$

$$\begin{aligned} Z_{in}' &= (1+A\beta)\, Z_{in} \\ &= (1 + 100,000 \times 0.9)10\ \text{m}\Omega \\ &= 90,001 \times 10\ \text{m}\Omega \\ &= 900,010\ \text{m}\Omega \end{aligned}$$

$$Z_o' = \frac{Z_o}{(1 + 100,000 \times 0.9)} = \frac{50}{90,001}\ \Omega = \frac{50 \times 1,000}{90,001}\ \text{m}\Omega$$

$$= 0.55\ \text{m}\Omega$$

$$\text{A (Closed-loop)} = \frac{1}{\beta} = \frac{1}{0.9} = 1.11.$$

Example 9.3 Calculate the values of input and output impedance for the feedback circuit shown in Fig. 9.17. Also calculate the closed loop voltage gain. The open-loop parameters are A = 100,000, Z_{in} = 5 MΩ and Z_o = 25 Ω.

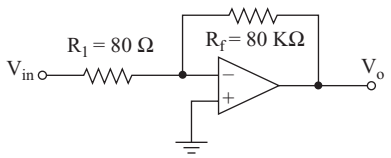


Figure 9.17

Solution:

$$Z_{in}' = R_1 = 80\ \Omega \qquad \beta = \frac{R_1}{R_1 + R_f} = \frac{80}{80 + 80,000} = 0.001$$

$$Z_o' = \frac{Z_o}{1 + 100,000 \times 0.001} = \frac{25}{101} = 0.24\ \Omega$$

$$A(CL) = -\frac{R_f}{R_1} = -\frac{80,000}{80} = -1000.$$

The negative sign indicates that the feedback used is inverting.

Example 9.4 The open-loop data of an op-amp are A = 200,000, Z_{in} = 2 MΩ, Z_o = 75 Ω. Determine the input and output impedance when (a) the op-amp is connected in non-inverting mode with feedback fraction β = 0.043; (b) when the op-amp is connected as a voltage follower.

Solution:

(a) In inverting configuration,

$$Z_{in}' = (1+A\beta)Z_{in}$$
$$= (1 + 200,000 \times 0.043) \times 2\,\text{m}\Omega$$
$$= 17,200\ \text{M}\Omega$$

$$Z_o' = \frac{Z_o}{(1 + 200,000 \times 0.043)} = \frac{75}{8601} = 0.0087\,\Omega.$$

(b) A voltage follower is a special case of non-inverting amplifier with unity feedback, i.e. with $\beta = 1$. Therefore, the impedances are

$$Z_{in}' = (1 + 200,000 \times 1) \times 2\,\text{M}\Omega$$
$$= 400,002\,\text{M}\Omega$$

$$Z_o' = \frac{Z_o}{1 + A\beta} = \frac{75}{1 + 200,000 \times 1} = \frac{75}{200,001}\,\text{ohms}$$
$$= 375 \times 10^{-6}\,\text{ohms}.$$

The result shows that the voltage follower has greater input impedance and lower output impedance than a non-inverting op-amp.

9.3.9 Summary of Op-amp Configurations

Non-inverting amplifier

A non-inverting amplifier circuit is shown in Fig. 9.18.

Feedback fraction, $\beta = \dfrac{R_1}{R_1 + R_f}, V_f = \beta V_o, (V_{in} - V_f)A = V_o$

$$A(CL) = \frac{V_o}{V_{in}} = \frac{R_1 + R_f}{R_1} = \frac{1}{\beta}$$

Input impedance, $Z_{in}' = (1 + A\beta)Z_{in}$

Output impedance, $Z_o' = \dfrac{Z_o}{(1 + A\beta)}$.

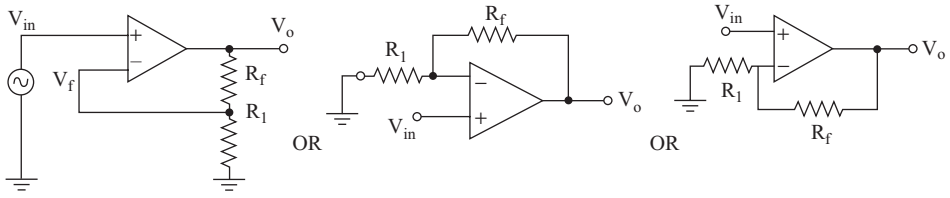


Figure 9.18 *Non-inverting amplifier*

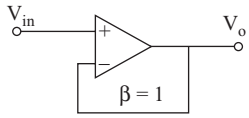


Figure 9.19 *Voltage follower*

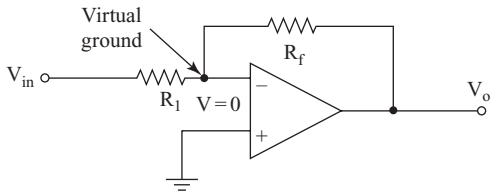


Figure 9.20

Voltage follower

A voltage follower circuit is shown in Fig. 9.19.

$$A(CL) = \frac{V_o}{V_{in}} = \frac{1}{\beta} = 1 \text{ as } \beta = 1$$

$$Z_{in}' = (1 + A) Z_{in}$$

$$Z_o' = \frac{Z_o}{(1 + A)}$$

Inverting amplifier

An inverting amplifier circuit is shown in Fig. 9.20.

Current through the input resistance R_1 is the same as the current through the feedback resistance R_f such that

$$V_0 = A\nu \quad \frac{V_{in} - \nu}{R_1} = \frac{\nu - V_0}{R_f}$$

$$A = \frac{V_o}{\nu} \quad \text{As } \nu = 0, \quad \frac{V_o}{V_{in}} = A(CL) = -\frac{R_f}{R_1}$$

$$A \to \infty$$

$$\nu \to 0 \text{ (Virtually at ground potential)}$$

$$Z_{in}' = Z_{in} = R_1$$

$$Z_o' = \frac{Z_o}{(1 + A\beta)} \text{ where } \beta = \frac{R_1}{R_1 + R_f}$$

9.4 BASIC OP-AMP CIRCUITS

Op-amps are used in a wide variety of applications. In this section, we will discuss the use of an op-amp as an adder and subtractor, i.e. a summing amplifier, constant gain multiplier, voltage follower, comparator, integrator and differentiator.

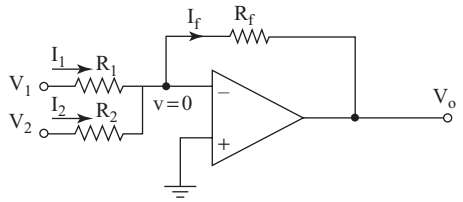


Figure 9.21 *Op-amp as a summing amplifier*

9.4.1 Op-amp as a Summing Amplifier

Figure 9.21 shows a two-input inverting summing amplifier. The sum of input currents is equal to the feedback current. This is due to the concept of infinite input impedance of the amplifier and the virtual ground.

$I_1 + I_2 = I_f$ since no current is flowing through the amplifier because of its infinite input impedance.

$$I_1 = \frac{V_1 - v}{R_1}, \quad I_2 = \frac{V_2 - v}{R_2} \quad \text{and} \quad I_f = \frac{v + V_o}{R_f}$$

As $v = 0$

Thus,

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = -\frac{V_o}{R_f}$$

If $R_1 = R_2 = R$

$$V_o = -\frac{R_f}{R}(V_1 + V_2)$$

If we make $R_f = R$,

$$V_o = -(V_1 + V_2)$$

If there are n numbers of inputs, we will have an output voltage whose sum is the arithmetic sum of all the input voltages but inverted. For a summing amplifier,

$$V_o = -(V_1 + V_2 + V_3 + \cdots\cdots + V_n) \tag{9.14}$$

Example 9.5 The input voltage to a summing amplifier has been shown in Fig. 9.22. Assuming $R_1 = R_2 = R_3 = R_f$, calculate the output voltage.

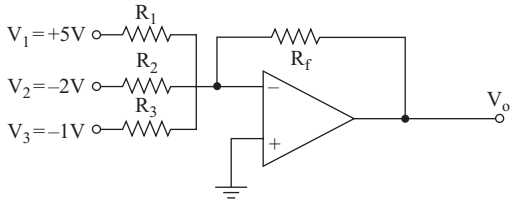


Figure 9.22

Solution:
The output voltage,

$$\begin{aligned} V_o &= -(V_1 + V_2 + V_3) \\ &= -(5 - 2 - 1) \\ &= -2 \text{ V}. \end{aligned}$$

9.4.2 Summing Amplifier with Gain Greater than Unity

For a summing amplifier with two voltage inputs, the output is $V_o = -\dfrac{R_f}{R}(V_1 + V_2)$.

We can have an output voltage, which is a multiple of the sum of the input voltages by choosing the ratio R_f/R.

This way the output will be the sum of all the input voltages multiplied by a constant whose value will be determined by the ratio R_f/R.

Example 9.6 An op-amp is used as a summing amplifier as shown in Fig. 9.23. Calculate the output voltage.

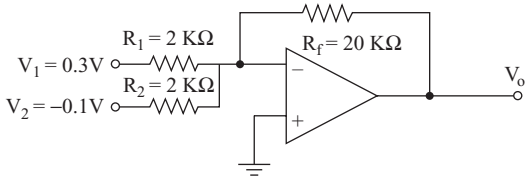


Figure 9.23

Solution:
We have $R_1 = R_2 = R = 2\ \text{K}\Omega$ and $R_f = 20\ \text{K}\Omega$

$$\begin{aligned} V_o &= \frac{R_f}{R}(V_1 + V_2) \\ &= -\frac{20}{2}(0.3 - 0.1) \\ &= -10 \times 0.2 \text{ V} \\ &= -2 \text{ V}. \end{aligned}$$

Here, the sum of all the input voltages has been multiplied by a constant of 10.

9.4.3 Averaging Amplifier

By setting the ratio of R_f/R as the reciprocal of the number of inputs, the output will be a measure of the average of the input voltages. If there are five input voltages, the ratio of R_f/R will be set as 1/5. The output voltage will be the average of the input voltages. Such an amplifier configuration can be termed an **averaging amplifier**. This operation is illustrated with an example.

Example 9.7 An op-amp as a summing amplifier is shown in Fig. 9.24. Show that the output voltage is the average of the input voltages.

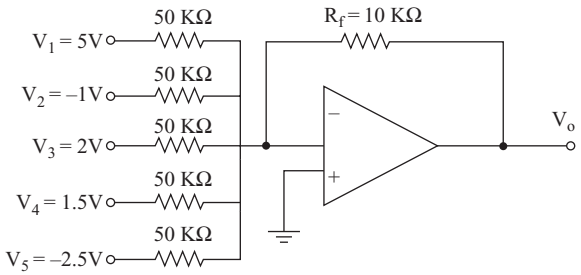


Figure 9.24

Solution:

$$V_o = \frac{R_f}{R}\left(V_1 + V_2 + V_3 + V_4 + V_5\right)$$

$$= \frac{10}{50}\left(+5 - 1 + 2 + 1.5 - 2.5\right)$$

$$= \frac{1 \cdot 5}{5} = 1 \text{ V}.$$

Here, the ratio of R_f/R is the reciprocal of the number of inputs, i.e. 1/5. By simple calculation, we see that the average of the input voltage is 1 V.

9.4.4 Op-amp as a Constant Gain Multiplier

An op-amp as an inverting amplifier has been shown in Fig. 9.25.

For a constant gain multiplier, the output voltage,

$$V_o = \frac{R_f}{R_1} V = -KV_1 \tag{9.15}$$

where

$$K = \frac{R_f}{R_1}$$

The output is obtained by multiplying the inputs with a gain factor, −K. This is called a constant gain multiplier or a scale changer. If we want $V_o = KV_1$, then we will have another inversion of $-KV_1$ with an op-amp where $R_f/R = 1$.

A very low voltage can be amplified and then measured using a scale changer. The output reading has to be divided by the scale factor.

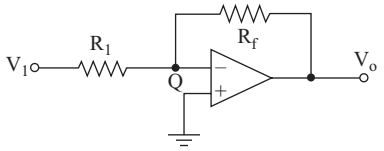


Figure 9.25 *Op-amp as a constant gain multiplier*

Example 9.8 Determine the value of the output voltage in the summing amplifier shown in Fig. 9.26.

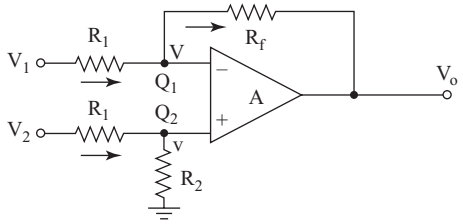


Figure 9.26

Solution:

Here, the weight of a particular input is determined by the ratio of R_f to the resistance of that input. The output voltage is

$$V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)$$

Substituting the given values,

$$\begin{aligned} V_o &= -\left[\frac{5\ \text{k}\Omega}{10\ \text{k}\Omega}\times 5 + \frac{5\ \text{k}\Omega}{5\ \text{k}\Omega}\times(-2) + \frac{5\ \text{k}\Omega}{15\ \text{k}\Omega}\times 3\right] \\ &= -(2.5 - 2 + 1) \\ &= -1.5\ \text{V}. \end{aligned}$$

Example 9.9 Find the output voltage of the three-input summing amplifier circuit shown in Fig. 9.27.

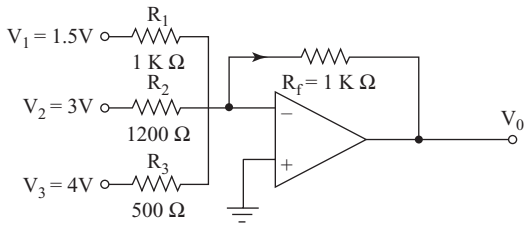


Figure 9.27

Solution:

Considering the ratios of R_f/R for each of the inputs,

$$\begin{aligned} V_o &= -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right) \\ &= -\left(\frac{1000}{1000}\times 1.5 + \frac{1000}{1200}\times 3 - \frac{1000}{1000}\times 4\right) \\ &= -(1.5 + 2.5 - 8) \\ &= 4\ \text{V}. \end{aligned}$$

9.4.5 Op-amp as a Subtractor

Figure 9.28 shows an op-amp used as a differential amplifier. Here, the difference between the two input voltages V_1 and V_2 can be calculated or can be amplified. We assume that the amplifier has infinite gain and infinite input impedance. No current will flow through the amplifier. As $\nu A = V_o$, if A $\to\infty$, $\nu \to 0$. Thus, the points Q_1 and Q_2 will have zero potential, i.e. $\int \nu = 0$. These points although not actually grounded are considered as virtually (as it) grounded.

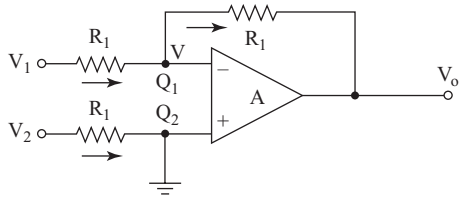


Figure 9.28 *Op-amp used as a subtractor*

Looking at the inverted input terminal and feedback path, and equating the currents,

$$\frac{V_1 - \nu}{R_1} = \frac{\nu - V_o}{R_f} \tag{9.16}$$

From the non-inverting input terminal,

$$\frac{V_2 - \nu}{R_1} = \frac{\nu}{R_2} \tag{9.17}$$

Substituting Eq. (9.16) from Eq. (9.17)

$$\frac{V_2}{R_1} - \frac{\nu}{R_2} - \frac{V_1}{R_1} - \frac{\nu}{R_1} = \frac{\nu}{R_2} - \frac{\nu}{R_2} + \frac{V_o}{R_2}$$

or,
$$\frac{V_2 - V_1}{R_1} = \frac{V_o}{R_2}$$

The output voltage for op-amp subtractor,

$$V_o = \frac{R_2}{R_1}(V_2 - V_1) \tag{9.18}$$

Thus, the output voltage V_0 is the difference of the two input voltages multiplied by the factor R_2/R_1. Taking R_2/R_1, $V_0 = V_2 - V_1$.

9.4.6 The Op-amp Integrator

An ideal integrator has been shown in Fig. 9.29. The input resistor is R and a capacitor C is connected in the feedback path.

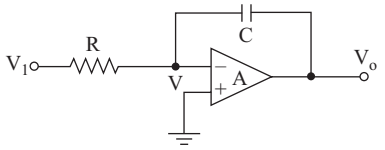


Figure 9.29 *An ideal op-amp integrator*

In an integrator, the output voltage should be the integral of the input voltage.
From Fig. 9.29,

$$\frac{V_1 - v}{R} = i$$

$$v - V_o = \frac{q}{C}$$

and $$Av = V_o$$

or, $$v = \frac{V_o}{A}. \qquad \text{As} \quad A \to 0, v \to 0.$$

Assuming infinite gain of the amplifier and virtual ground as the input,

$$\frac{V_1}{R} = i$$

and $$-V_o = \frac{q}{C} = \frac{\int i dt}{C}$$

or, $$V_o = -\frac{\int \frac{V_1}{R} dt}{C} = \left(-\frac{1}{CR}\right)\int V_1 dt.$$

$$V_o = -\frac{1}{CR}\int V_1 dt. \tag{9.19}$$

Thus, the output of the amplifier is the integral of the input voltage multiplied by a constant $\left(-\frac{1}{CR}\right)$. Integrators are used in ramp generators, filters, and analog computers.

9.4.7 The Op-amp Differentiator

An ideal differentiator is shown in Figure 9.30. The placement of capacitor and the resistor are different in a differentiator when compared to an integrator. The capacitor is now at the input stage.

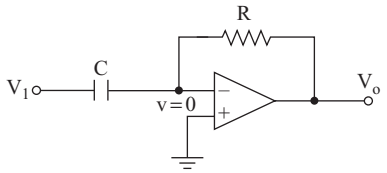


Figure 9.30 *An ideal op-amp differentiator*

Assuming that the op-amp has high input impedance and infinite gain, the voltage at the virtual ground, v, is taken as zero. Current through the feedback resistor R is

$$i = \frac{v - V_o}{R} = -\frac{V_o}{R} \tag{9.20}$$

The same current will flow through the capacitor. We can write,

$$q = \int i dt$$

$$Cv_1 = \int i dt$$

or,
$$i = C\frac{dV_1}{dt} \tag{9.21}$$

Equating Eqs. (9.20) and (9.21),

$$-\frac{V_o}{R} = C\frac{dV_1}{dt}$$

or,
$$V_o = (-CR)\frac{dV_1}{dt} \tag{9.22}$$

The output voltage is the derivative of the input voltage V_1 multiplied by a factor (−CR).

9.4.8 Working of a Differentiator

A differentiator produces an inverted output voltage that is proportional to the rate of change of input voltage. Thus, if the input is a ramp with a constant slope the output will be a constant voltage with an inverted, i.e. negative value. To examine how a differentiator works, lets us apply a positive going ramp voltage to the input of the differentiator as shown in Fig. 9.31. Let V_c be the voltage across the capacitor.

Because of the virtual ground with v = 0, V_{in} is equal to V_c. Since no current flows through the amplifier, I_{in} and I_c are equal.

From the two basic expression, $Q = CV_c$ and $Q = \int i_c dt$, we get $I_c = \left(\frac{V_c}{t}\right)C$

As the slope of capacitor voltage $\left(\frac{V_c}{t}\right)$ is constant,

I_c is constant. Since I_c is the same as I_f, the output voltage can be calculated as

$$V_o = I_f R_f = I_c R_f$$

or,
$$V_o = -\left(\frac{V_c}{t}\right)C\ R_f$$

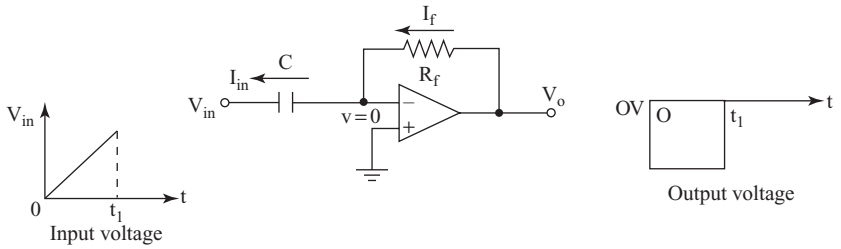


Figure 9.31 *A differentiator with a ramp input*

For the inverting amplifier, the output is negative when the input is a positive ramp and the output is positive when the input is a negative ramp. Thus, for an input voltage, the output voltage of a differentiator is a square wave as shown in Fig. 9.32.

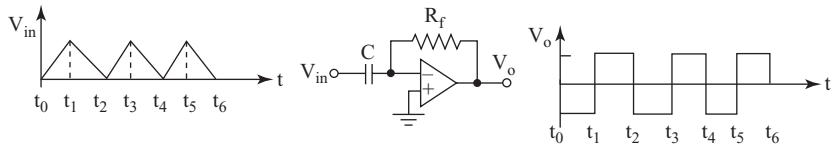


Figure 9.32 *Input–output voltage wave shapes of a differentiator*

9.4.9 Working of an Integrator

Notice that in the op-amp integrator shown in Fig. 9.33, the feedback element is a capacitor that forms an RC circuit with the input resistor R. Using the concept of virtual ground, the current through the input resistor is considered equal to the current through the feedback capacitor. This is because current will not flow through the amplifier.

Thus,

$$I_c = I_{in} = \frac{V_{in}}{R}$$

Since V_{in} is a constant, I_{in} and I_c are constants. A constant current will charge the capacitor. We can write,

Charge, $$Q = I_c\, t$$

and $$Q = C\, V_c$$

Therefore, $$V_c = \left(\frac{I_c}{c}\right) t\,.$$

This is of the form y = mx, a straight line passing through the origin in x–y plane. The constant charging current charges the capacitor linearly and will produce a linear (straight line) voltage across C. The voltage at the positive side of the capacitor is held at 0 volt by the virtual ground. The voltage at the negative side will grow linearly but negatively until the op-amp gets saturated. The output voltage will be the same as the voltage at the negative terminal of the capacitor. Thus, a negative ramp is available at the output. The rate of the change of output is calculated as

$$\frac{\Delta V_o}{\Delta t} = \frac{\Delta V_c}{\Delta t} = \frac{I_c}{C} = -\frac{V_{in}}{RC}.$$

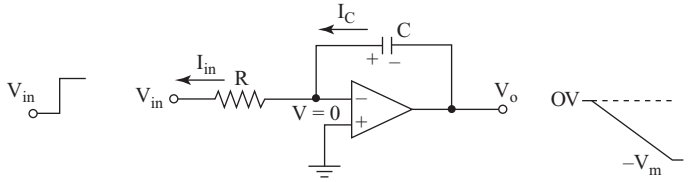


Figure 9.33 *A constant input voltage produces a ramp on the output*

9.4.10 Op-amp as a Comparator

Operational amplifiers are often used to compare the amplitudes of one voltage with the other. Op-amps are used in open-loop configurations with one voltage on one of the input terminals and a reference voltage on the other. The output of the comparator will indicate which one of the two inputs is greater.

As shown in Fig. 9.34 (a), an input voltage V_i is applied to the positive input terminal and a known voltage is connected to the other input terminal, which is called the reference voltage V_{ref}. V_o is the output voltage.

In the open-loop configuration the output voltage is expressed as

$$V_o = A(V_i - V_{ref})$$

where A is the open-loop gain of the amplifier, which is assumed to be equal to infinity. Since the open-loop voltage gain is very high, a minute voltage difference between the two inputs will drive the amplifier into saturation as shown in Fig. 9.34 (b) For example, let us assume that a practical op-amp has an open-loop voltage gain of 200,000. A voltage difference, $V_i - V_{ref} = 0.5$ mV will produce an output voltage of

$$V_o = (V_i - V_{ref}) \times A = 0.5 \times 10^{-3} \times 200{,}00$$

$$= 100 \text{ V}.$$

This is too high a voltage for an op-amp to be in the linear region of operation. Most op-amps have an output voltage limitation of ± 15 V beyond which the device will be driven into saturation. If the voltage of the non-inverting terminals, i.e. v_i exceeds the voltage of the inverting terminal, i.e. V_{ref}, the op-amp output voltage reaches saturation stage and the output voltage becomes positive. On the other hand, if V_i is slightly less than V_{ref}, the op-amp output voltage reaches saturation stage at –V volts. The output voltage of the comparator is expressed as

$$\left.\begin{aligned} V_o &= -V_{sat} \text{ if } V_i < V_{ref} \\ V_o &= +V_{sat} \text{ if } V_i < V_{ref} \end{aligned}\right\} \tag{9.23}$$

Thus, the comparator will detect voltages other than zero by selecting the reference voltage suitably. This is also called non-zero-level voltage detection. A Schmitt trigger circuit is a fast operating voltage level detector. When the input voltage reaches a particular level, the output voltage switches rapidly between maximum positive and negative values.

It is possible to use the comparator as a zero-crossing detector. In this case one of the input terminals is grounded as shown in Fig. 9.35.

As shown in Fig. 9.35 (b) a sinusoidal voltage is applied to the non-inverting input of the amplifier. When the sine wave is negative, the output is at maximum negative level, i.e. at $-V_{sat}$ level. When the

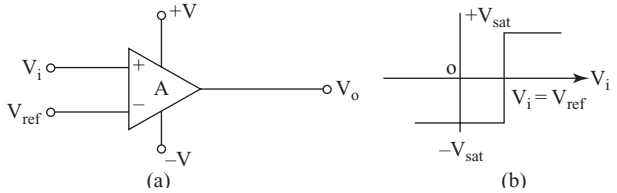


Figure 9.34 *(a) An op-amp used as a comparator (b) transfer characteristic of a comparator*

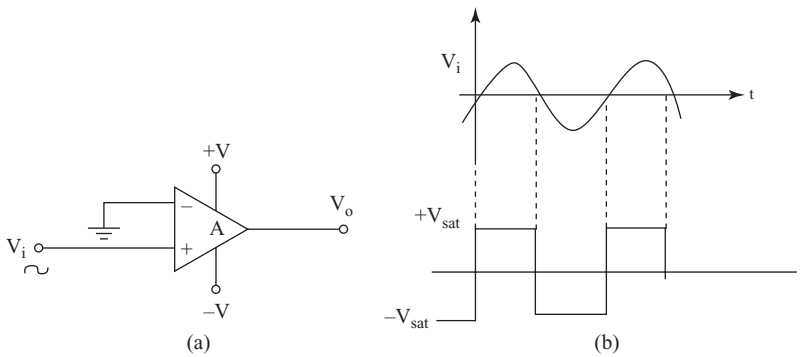


Figure 9.35 *(a) An op-amp as a zero-crossing detector; (b) input and output voltages of the comparator*

sine wave crosses zero, the amplifier is driven to maximum positive saturation level. Thus, an op-amp can be used as a zero-crossing detector. Such a zero-crossing detector can be used to produce a square wave from a sine wave.

9.4.11 The Buffer Amplifier or Voltage Follower

In a voltage-follower as shown in Fig. 9.36, the output voltage is the same as the input voltage both in magnitude and in phase, i.e. the output follows the input voltage. It is a unity gain non-inverting type feedback amplifier. The closed-loop gain for a non-inverting op-amp is

$$\frac{V_o}{V_i} = A(CL) = 1 + \frac{R_f}{R_1}$$

with $R_f = 0$

$$V_o = \left(1 + \frac{0}{R_1}\right) V_1$$

or, $V_o = V_i$

The gain $\frac{V_o}{V_i} = 1.$

This circuit is used in isolating a high-impedance source from a low-impedance load, i.e. to use as a buffer.

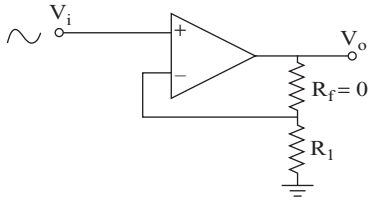


Figure 9.36 *Op-amp voltage follower with $R_f = 0$*

9.4.12 More Applications of Op-amps

Op-amps have plenty of applications in the field of electronics and control engineering. We find applications of op-amps in:

Active filters: Filter is a frequency selective circuit. It passes a specified band of frequencies and rejects signals in the rest of the frequency band.

Oscillators: Op-amps are capable of generating repetitive waveforms of fixed amplitude and frequency without any external input signal.

Analog computers: Op-amps are used in analog computers that can be used to solve differential equations with even non-linear coefficients. Physical systems and processes are simulated using mathematical equations and they are solved using analog computers.

Special purpose op-amp circuits: IC 741 is a general purpose op-amp and finds applications in a wide variety of places. A basic instrumentation amplifier, which is made up of three op-amps and a number of resistors, is used in the area of medical electronics.

Example 9.10 Two op-amps are connected in cascade (series) as shown in Fig. 9.37. Calculate the value of R where $V_1 = 3$ V and the output voltage is 15 volts.

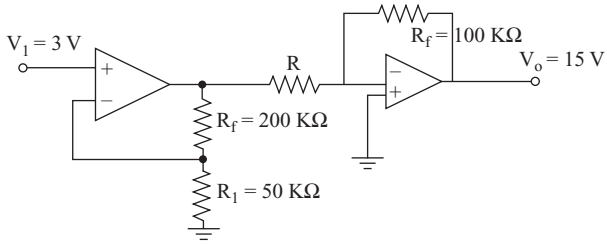


Figure 9.37

Solution:

The first op-amp is connected in non-inverting mode and the second is in inverting mode. Let the closed loop voltage gain of the first op-amp be $A(CL)_1$ and that of the second op-amp be $A(CL)_2$.

$$\frac{V_o}{V_i} = -A(CL)_1 \times A(CL)_2$$

$$A(CL)_1 = 1 + \frac{R_f}{R_1} = 1 + \frac{200}{50} = 5$$

$$A(CL)_2 = -\frac{R_f}{R} = -\frac{100}{R(k\Omega)}$$

$$\frac{V_o}{V_1} = \frac{15}{3} = 5$$

Therefore,

$$5 = -5 \times \left(-\frac{100}{R}\right) = +\frac{500}{R}$$

or,

$$R = +\frac{500}{R}$$

or,

$$R = 100\,k\Omega.$$

Example 9.11 Find the output voltage and current through the resistance R_f in the circuit shown in Fig. 9.38.

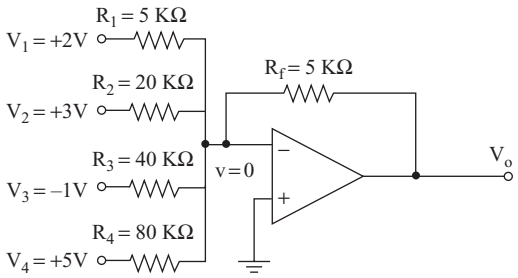


Figure 9.38

Solution:

The output voltage, V_o is calculated as

$$V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}(V_3) + \frac{R_f}{R_4}V_4\right)$$

$$= -\left[\frac{5}{5}\times 2 + \frac{5}{20}\times 3 + \frac{5}{40}\times(-1) + \frac{5}{80}\times 5\right]$$

$$= -[2 + 0.75 - 0.125 + 0.3125]$$

$$= -2.9375 \text{ V}$$

Assuming virtual ground concept with v = 0, current through the feedback resistor R_f is

$$R_f = \frac{v - V_o}{R_f} = \frac{2.9375}{5\times 10^3} = 0.58 \text{ mA}\ .$$

Example 9.12 Draw the output voltage wave form for the op-amp integrator shown in Fig. 9.39.

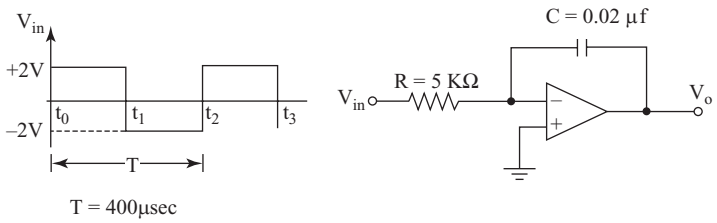


Figure 9.39

Solution:

The rate of change of output voltage is

$$\frac{\Delta V_o}{\Delta t} = -\frac{V_{in}}{RC} = -\frac{2}{5\times 10^3 \times 0.02\times 10^{-6}}$$

$$= -20\times 10^3 \text{ V/sec}$$

$$= -20 \times 10^{3} \times 10^{-6}\ \text{V}/\mu\text{sec}$$
$$= -20 \times 10^{-3}\ \text{V}/\mu\text{sec}$$
$$= -20\text{mV}/\mu\text{sec}.$$

For the time the input is positive i.e. for $t = T/2 = 400/2 = 200 \propto \text{sec}$. The output is a negative-going ramp and when the input is negative, the output is a positive-going ramp.

$$\Delta V_o = -20\ \text{mV} \times 200\ \mu\text{sec} = -4\ \text{V}$$

The input and output voltage waveforms are shown in Fig. 9.40.

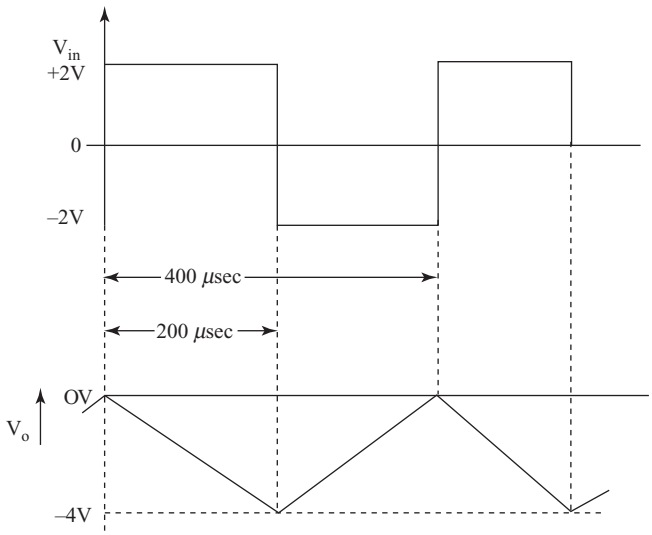


Figure 9.40 *Input–output voltage waveforms of an op-amp integrator circuit*

Example 9.13 Calculate the voltage gain for the amplifier circuit shown in Fig. 9.41.

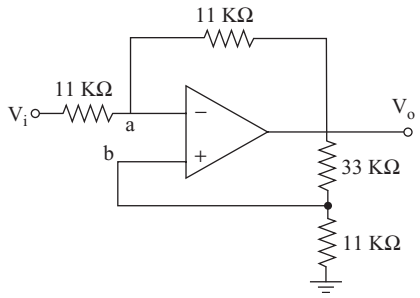


Figure 9.41

Solution:
The voltage at inputs a and b are equal. Assuming infinite voltage gain of the amplifier,

$$\nu_b = \frac{11}{11+33} \times V_o = \frac{V_o}{4} = V_a$$

As no current flows through the op-amp, current with the input resistance is equal to the current with the feedback resistance. Accordingly,

$$\frac{V_i - V_a}{11\,k\Omega} = \frac{V_a - V_o}{11\,k\Omega}$$

or,
$$V_o = 2V_a - V_i = \frac{2V_o}{4} - V_i = \frac{V_o}{2} - V_i$$

or,
$$\frac{V_o}{2} = -V_i$$

or,
$$V_o = -2V_i$$

Notice that 33 kΩ resistor is three times the 11 kΩ resistor. The multiplying factor is 3. If n is the multiplying factor, then it can be shown that $V_o = -\frac{n+1}{n-1} V_i$.

Example 9.14 Calculate the output voltage for the amplifier circuit shown in Fig. 9.42.

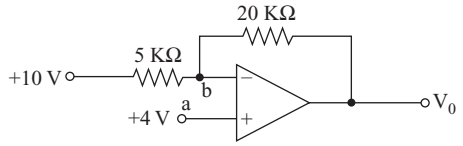


Figure 9.42

Solution:
Assuming an infinite gain of the amplifier, voltage at a is taken as equal to voltage at b, i.e. equal to 4 V. The current through the 5 kΩ resistor is taken as the same as that through the 20 kΩ resistor.

Therefore,

$$\frac{10 - V_b}{5\,k\Omega} = \frac{V_b - V_o}{20\,k\Omega}$$

$$\frac{10-4}{5\,k\Omega} = \frac{4 - V_o}{20\,k\Omega}$$

or,
$$4 - V_o = 4(10-4) = 24$$

or,
$$V_o = -20\,V.$$

Example 9.15 Calculate the value of the output voltage for the amplifier circuit shown in Fig. 9.43.

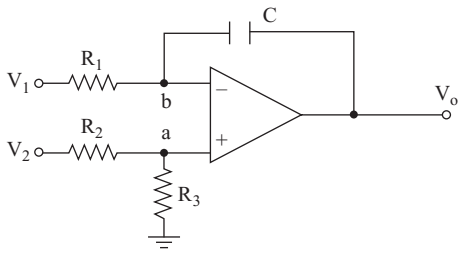


Figure 9.43

Solution:

Assuming infinite amplifier gain, potential at point a is taken as equal to potential at point b.

$$V_a = \frac{R_3}{R_2 + R_3} \cdot V_2$$

$$V_b = \frac{R_3}{R_2 + R_3} \cdot V_2$$

$$V_a - V_o = V_c$$

$$\frac{R_3}{R_2 + R_3} V_2 - V_o = \frac{1}{c}\int_o^t idt$$

$$q = \int idt$$

$$q = CV_c$$

or,
$$V_c = \frac{1}{C}\int idt$$

$$V_o = \frac{R_3}{R_2 + R_3}\frac{V_2}{R_3} - \frac{1}{c}\int_o^t idt \tag{9.24}$$

Again,
$$i = \frac{V_1 - V_b}{R_1} = \frac{1}{R_1}\left(V_1 - \frac{R_3 V_2}{R_2 + R_3}\right) \tag{9.25}$$

From Eqs. (9.24) and (9.25),

$$V_o = \left(\frac{R_3}{R_2 + R_3}\right)V_2 - \frac{1}{CR_1}\int_0^t \left(V_1 - \frac{R_3 V_2}{R_2 + R_3}\right) dt.$$

Example 9.16 In the amplifier circuit shown in Fig. 9.44, show that $V_o = V_2 - V_1$ if

$$R_1 = R_2.$$

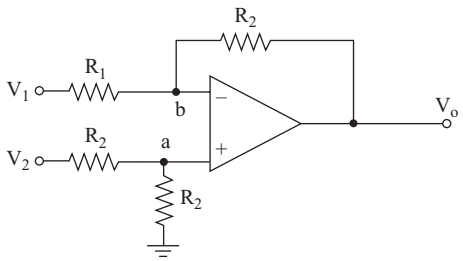


Figure 9.44

Solution

$$V_a = \frac{R_2}{R_1 + R_3} \cdot V_2$$

$$V_b = V_a = \frac{R_2}{R_1 + R_2} \cdot V_2 \tag{9.26}$$

Again,
$$\frac{V_1 - V_b}{R_1} = \frac{V_b - V_o}{R_2} \tag{9.27}$$

From Eqs. (9.26) and (9.27)

$$\frac{V_1}{R_1} - \frac{V_b}{R_1} = \frac{V_b}{R_2} - \frac{V_0}{R_2}$$

when $R_1 = R_2$

$$\frac{V_1}{R_1} - \frac{V_b}{R_1} = \frac{V_b}{R_1} - \frac{V_0}{R_1}$$

or,
$$V_o = V_1 - 2V_b. \tag{9.28}$$

Again,
$$V_b = \frac{R_2}{R_1 + R_2} \cdot V_2 = \frac{R_2}{2R_2} V_2 = \frac{V_2}{2} \tag{9.29}$$

From Eqs. (9.28) and (9.29),

$$V_o = V_1 - 2 \times \frac{V_2}{2}$$

or,
$$V_0 = V_1 - V_2$$

Example 9.17 Calculate the value of output voltage in terms of the input voltages in the amplifier circuit shown in Fig. 9.45.

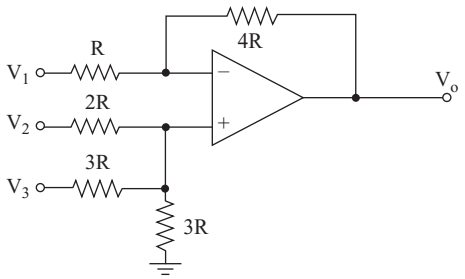


Figure 9.45

Solution:

Applying the superposition theorem, we calculate the output voltage for each of the input voltages and then determine the combined effect.

(i) When V_1 is applied and $V_2 = V_3 = 0$.

$$V_{o1} = -\frac{4R}{R} V_1 = -4V_1$$

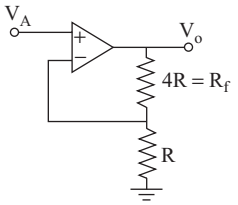


Figure 9.46

(ii) When V_2 is applied and $V_1 = V_3 = 0$

Considering the two 3R resistances in parallel (as V_3 is also at zero potential, i.e. earthed), the equivalent resistance is 1.5R.

$$V_A = \frac{V_2 \cdot 1.5R}{2R + 10.5R} = \frac{V_2 \cdot 1.5R}{3.5R} = \frac{3}{7} V_2 .$$

Let the output voltage is $V_{o2.}$

$$V_{o2} = \left(1 + \frac{R_f}{R}\right) V_A = \left(1 + \frac{4R}{R}\right) V_A$$

$$= 5 \times \frac{3}{7} V_2 = \frac{15}{7} V_2$$

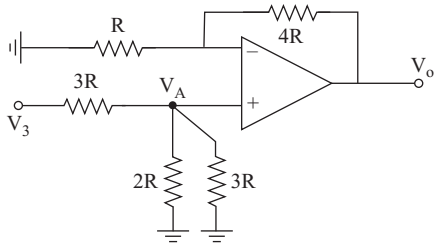


Figure 9.47

(iii) V_3 is applied, $V_1 = V_2 = 0$ (at ground potential. The connections are show)

$$= \frac{6/5\,R}{21/5\,R} V_3 = \frac{2}{7} V_3$$

$$V_{o3} = \left(1 + \frac{4R}{R}\right) VA$$

$$= 5 \cdot \frac{2}{7} V_3 = \frac{10}{7} V_3$$

The total output $= V_{01} + V_{02} + V_{03}$

$$V_0 = \left[-4V_1 + \frac{15}{7} V_2 + \frac{2}{7} V_3\right]$$

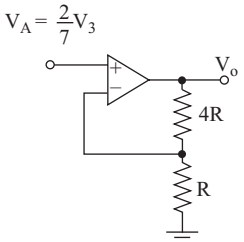


Figure 9.48

9.5 REVIEW QUESTIONS

A. Short Answer Type Questions

1. Explain why negative feedback is most useful in op-amp applications.
2. Explain the concept of feedback is op-amp with an example.
3. Explain the process of fabrication of monolithic integrated circuits in brief.
4. What is an integrated circuit? What are the advantages of ICs?
5. Illustrate the inverting and non-inverting configurations of op-amp circuits.
6. State the characteristics of an ideal op-amp.

7. Calculate the closed-loop voltage gains of inverting and non-inverting operational amplifiers.
8. Derive expressions for voltage gain, input and output impedance of an inverting amplifier using an op-amp.
9. What is a voltage follower? Discuss the applications of a voltage follower?
10. Show how an op-amp can be used as a differentiator.
11. Prove that the closed-loop voltage gain of a non-inverting amplifier is approximately inversely proportional to the feedback fraction.
12. Show the effect of negative feedback on the input and output impedances of a non-inverting amplifier.
13. Show the effect of negative feedback on the input and output impedances of a voltage follower.
14. Show that for an inverting amplifier input and output impedances are

$$Z_{in}\,(\text{inverted}) = R_i\,(\text{external input resistance})$$

$$Z_o\,(\text{inverted}) = \frac{Z_o\,(\text{open-loop})}{1 + A\beta}$$

where A is the open-loop gain and β is the feedback fraction of the op-amp.

15. Show the effect of negative feedback on the input and output impedances of an op-amp in (i) non-inverting configuration; (ii) inverting configuration; and (iii) as a voltage follower in a tabular form.
16. Show that a voltage follower has the highest input impedance and the lowest output impedance of the three configurations of an op-amp.
17. Explain the three basic configurations of an op-amp—(a) inverting; (b) non-inverting; and (iii) voltage follower.
18. Prove that negative feedback reduces the voltage gain but increases the stability with the help of a suitable example.
19. State and briefly explain the important parameters of an op-amp.
20. Prove that the output voltage of a summing amplifier is proportional to the sum of the input voltages.
21. What is an averaging amplifier. Show that it is a summing amplifier with closed-loop gain equal to the reciprocal of the number of inputs.
22. Explain how an op-amp can be used as a (i) differentiator (ii) integrator.
23. Draw the circuit for an op-amp integrator and explain its working. Also show the input and output voltage waveforms as an example.
24. Write short notes for an op-amp on the following.
    (a) Slew rate; (b) CMRR; (c) Closed-loop voltage gain.

B. Numerical Problems

25. A differential amplifier has a voltage gain of 2000 and a common-mode gain of 0.2. Determine the CMRR and express it in decibels.

    (Ans. CMRR = 10,000, CMRR = 80 dB)

26. Determine the closed-loop voltage gain of the amplifier circuits shown in Fig. 9.49.

    (Ans. (a) 22.3; (b) −100)

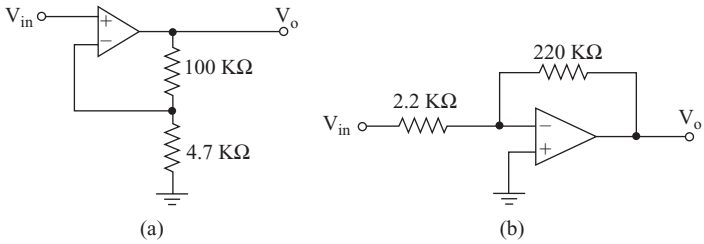


Figure 9.49

27. Determine the closed-loop voltage gain, input and output impedances of the op-amp shown in Fig. 9.50.

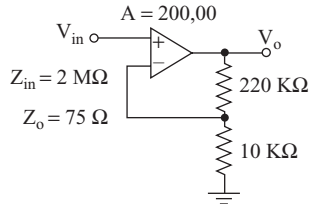


Figure 9.50

(Ans. $A(CL) = 23, Z_{in}' = 17.4 \times 10^{9}\,\Omega, Z_o' = 8.6 \times 10^{-3}\,\Omega$)

28. An inverting amplifier has a resistance of $R_1 = 1$ kΩ in the input voltage terminal and feedback resistance f = 3 kΩ. Calculate the output voltage, input impedance, and input current when the input voltage in 2 volts.

(Ans. $V_o = -6V, Z_{in}' = 1\,k\Omega, i_1 = 2$ mA)

29. Calculate the output voltage for a summing amplifier shown in Fig. 9.51,

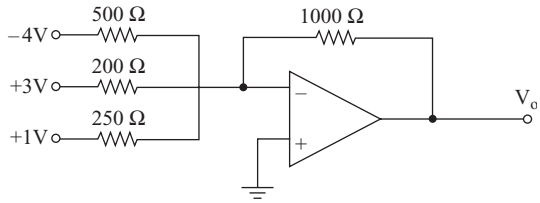


Figure 9.51

(Ans. –11 V)

30. Determine the closed-loop voltage gain, input and output impedances of the op-amp shown in Fig. 9.52.

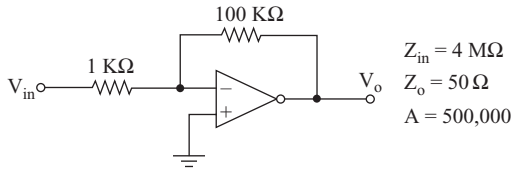


Figure 9.52

(Ans. A (CL)= -100; $Z_{in}' = 1\,k\Omega$, $Z_o' = 0.101\,\Omega$)

31. Calculate the output voltage, V_o for the amplifier circuit shown in Fig. 9.53.

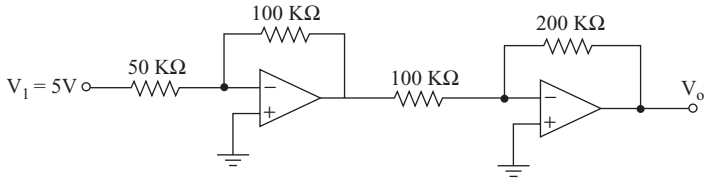


Figure 9.53

(Ans. 20 V)

32. An input signal of V_{in} = 5 sinwt is applied to the input terminal of the comparator shown in Fig. 9.54. The maximum output level of the op-amp is ± 12 V. Draw the output voltage wave form with reference to the input voltage wave form.

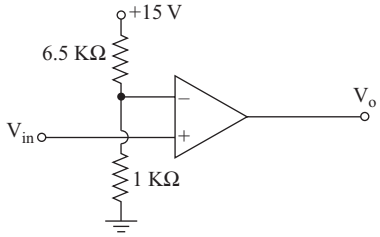


Figure 9.54

(Ans. Vr_{ef} = 2 V, the output voltage will be ±12 V rectangular starting at a time when V_{in} = 2 V)

33. Show that the amplifier shown in Fig. 9.56 gives an output voltage whose magnitude is the average of the input voltages.

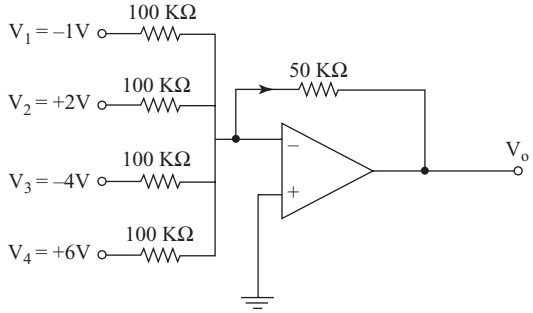


Figure 9.55

(Ans. −1.5 V)

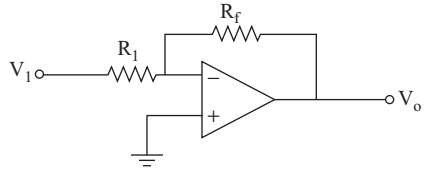
C. Multiple Choice Questions

1. If you know only the open-loop gain of an amplifier, you can determine closed-loop gain of
   (a) an inverting amplifier
   (b) a non-inverting amplifier
   (c) a voltage follower;
   (d) none of the configurations.
2. The highest possible input impedance is possible to achieve with
   (a) inverting amplifier
   (b) non-inverting amplifier
   (c) differential amplifier
   (d) voltage follower.
3. An inverting amplifier has the following circuit value: $R_f = 100\ k\Omega$, $R_i = 5\ k\Omega$ and open-loop gain of 20,000. The closed-loop gain will be
   (a) −1000 (b) −40
   (c) −200 (d) −20.
4. The feedback path of an integrator consist of
   (a) a resistor
   (b) a capacitor
   (c) a resistor and a capacitor in series
   (d) a resistor and a capacitor in parallel.
5. The inputs to a two-input summing amplifier are +1 V and +1.8 V with input resistance of 22 kΩ in each case. The value of feedback resistance to produce an output that is five times the sum of the input voltages will be
   (a) 110 kΩ (b) 11 kΩ
   (c) 66 kΩ (d) 44 kΩ.
6. An op-amp comparator circuit employs
   (a) negative feedback
   (b) positive feedback
   (c) no feedback
   (d) regenerating feedback.
7. An op-amp with negative feedback provides
   (a) higher closed-loop gain and lower input impedance
   (b) higher closed-loop gain and higher input impedance
   (c) lower closed-loop gain and lower input impedance
   (d) lower closed-loop gain and higher input impedance
8. The voltage gain of a non-inverting amplifier is expressed as

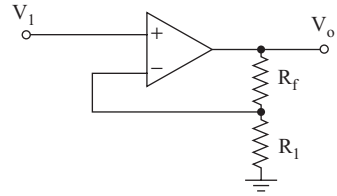
$$\text{(a) } \frac{V_o}{V_1} = 1 - \frac{R_f}{R_1};\ \text{(b) } \frac{V_o}{V_1} = 1 + \frac{R_f}{R_1}$$

$$\text{(c) } \frac{V_o}{V_1} = 1 + \frac{R_1}{R_f} \quad \text{(d) } \frac{V_o}{V_1} = 1 - \frac{R_1}{R_f}$$

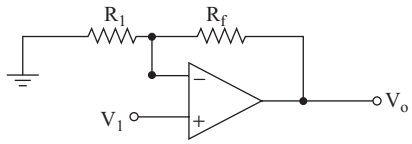
9. Which of the following statements are not correct?
   (a) In a non-inverting amplifier the input voltage is applied to the positive terminal
   (b) In an inverting amplifier the positive terminal is earthed
   (c) In an inverting amplifier input voltage is applied to the negative terminal through an input resistance
   (d) In a non-inverting amplifier the positive terminal is connected to earth.
10. Which of the following circuits is not correct?
    (a) Inverting amplifier:



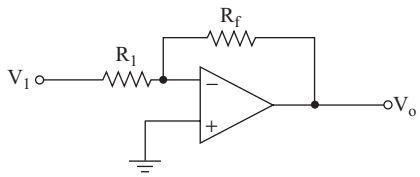
    (b) Non inverting amplifier:



(c) Non-inverting amplifier:

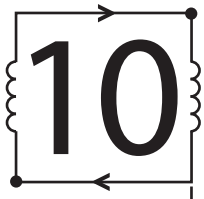


(d) Inverting amplifier:



Answers to Multiple Choice Questions

1. (c) 2. (d) 3. (d) 4. (b) 5. (a) 6. (c) 7. (d) 8. (b) 9. (d) 10. (d)



Digital Electronics

TOPICS DISCUSSED

- Number systems
- Octal number system
- Hexadecimal number system
- Logic gates
- Universal gates
- Boolean algebra
- Combinational logic circuits and function realization using logic gates
- Converting logic statements into Boolean expressions and realization of the network
- Flip-flops

10.1 INTRODUCTION

Digital electronics find applications in computers, calculators, integrated circuits, and many digital devices and circuits. Digital circuits are being used in many applications like in display devices, digital clocks, digital counters, digital instruments, etc. A small integrated circuit (chip) can perform tasks of large number of transistors, diodes, resistors, etc. Digital electronics is an interesting subject of study.

Digital electronics includes electronic systems that use digital signals. Digital electronics or any digital circuit are usually made from large assemblies of logic gates. Digital electronics, digital circuits, and digital systems are often used interchangeably. Digital circuits and systems used in computers and electronic controls are *logic gates* and *multivibrators*. Logic gates are combinational (i.e., with no memory) and sequential (i.e., with memory). Combinational gates are AND, NOT, OR, NAND and NOR gates. Sequential gates include flip-flops (bistable multivibrators) which are available as registers and counters.

In this chapter brief descriptions of the above mentioned digital circuits and systems will be provided. Electrical signals that are continuous and can have any value over a given range are called analog signals. Electronic circuits that are used to process (i.e., amplify or modify) analog signals are called analog circuits. In a digital circuit, a signal is represented in one of two states i.e., low or high (on or off). These two voltage levels are called logic levels. Thus, a digital circuit functions in two states, i.e., in a binary manner.

Signals of two discrete values or levels, high and low, are called digital signals. The circuits which process digital signals are called digital circuits. Digital electronics, or any digital circuit, are made from large assemblies of logic gates which are simple electronic representation of boolean logic functions.

Digital circuits have a number of advantages over analog circuits. For example, signals represented digitally can be transmitted without degradation of quality. That is why analog signals are converted to digital signals through A to D converters and the digital signals are transmitted. These are again converted through D to A converters and the actual analog signal is recovered. For example, a continuous audio signal can be transmitted as a sequence of 1s and 0s and can be reconstructed without much error. An hour of music can be stored in a compact disc using digital signals which may be a few billion binary digits (1s and 0s). All systems comprising light, temperature, conductivity, pressure, magnetic fields, etc., are analog systems. The continuous signals of these are converted into discrete digital signals.

Before discussing logic gates and some digital circuits we will discuss data representation using number systems. The number system used to represent data in a computer is known as the binary number system.

10.2 NUMBER SYSTEMS

A number system is a set of rules of symbols used to represent numbers. There are number systems like decimal, binary, octal, and hexadecimal. The knowledge of number systems is very important for the design of digital circuits and systems.

10.2.1 Decimal Number System

We are familiar with the decimal number system which uses 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, i.e., ten symbols. It is a base-10 counting because the total number of symbols used is ten. When we represent a number, say 752 and 907, the positional values of the numbers and their decimal equivalent would be as shown.

Number	Value	Position	Positional value	Decimal equivalent
7 5 2				
	2	0	10°	$2 \times 10^{\circ} = 2$
	5	1	10^{1}	$5 \times 10^{1} = 50$
	7	2	10^{2}	$7 \times 10^{2} = 700$
				752
9 0 7				
	7	0	10°	$7 \times 10^{\circ} = 7$
	0	1	10^{1}	$0 \times 10^{1} = 0$
	9	2	10^{2}	$9 \times 10^{2} = 900$
				907

A decimal number of, say, 692 is represented as

$$\text{Decimal } 692 = 6 \cdot 10^{2} + 9 \cdot 10^{1} + 2 \cdot 10^{0}$$

A decimal number of say, 824.48 is represented as

$$\text{Decimal } 824.48 = 8 \times 10^{2} + 2 \times 10^{1} + 4 \times 10^{6} \bullet \; 4 \times 10^{-1} + 8 \times 10^{-2}$$

The multiples of 10 used are called weighted values. The decimal number 824.48 is represented as $(824.48)_{10}$ as 10 is the base.

10.2.2 Binary Number System

In the binary system a number is expressed by two symbols or digits, 0 and 1. Since the total number of symbols or digits used is two, it has a base 2. The binary symbols 0 and 1 are called binary digits or bits. A group of eight bits is called a byte. In this system the positional value is used in the same way as the decimal system. For example, two binary numbers, 11010 and 110011 are represented as their decimal equivalent as

Number	Value	Position	Positional value	Decimal equivalent
1 1 0 1 0				
	0	0	2^0	$0 \times 2^0 = 0$
	1	1	2^1	$1 \times 2^1 = 2$
	0	2	2^2	$0 \times 2^2 = 0$
	1	3	2^3	$1 \times 2^3 = 8$
	1	4	2^4	$1 \times 2^4 = 16$
				26
1 1 0 0 1 1				
	1	0	2^0	$1 \times 2^0 = 1$
	1	1	2^1	$1 \times 2^1 = 2$
	0	2	2^2	$0 \times 2^2 = 0$
	0	3	2^3	$1 \times 2^3 = 0$
	1	4	2^4	$1 \times 2^4 = 16$
	1	5	2^5	$1 \times 2^5 = 32$
				51

10.2.3 Conversion of Binary to Decimal

A binary number can be converted into its decimal equivalent number by noting that successive digits from the extreme right of a binary number are the coefficients of ascending power of 2 beginning with zeroeth power of 2. A few examples will illustrate this conversion.

Example 10.1 Binary number is 1100101. Convert this number into its decimal equivalent.

Solution:

1	1	0	0	1	0	1
1×2^6	1×2^5	0×2^4	0×2^3	1×2^2	0×2^1	1×2^0

$64 + 32 + 0 + 0 + 4 + 0 + 1 = 101$

Decimal number is 101. Therefore, $(1100101)_2 = (101)_{10}$

Example 10.2 Binary number is 11010. Convert this number into its decimal equivalent.

Solution:

1	1	0	1	0
1×2^4	1×2^3	0×2^2	1×2^1	0×2^0

$16 + 8 + 0 + 2 + 0 = 26$

Decimal number is 26. Therefore, $(11010)_2 = (26)_{10}$

Example 10.3 Binary number is 101101. Convert this number into its decimal equivalent.

Solution:

1	0	1	1	0	1
1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0

$32 + 0 + 8 + 4 + 0 + 1 = 45$

Therefore, $(101101)_2 = (45)_{10}$

Fractional binary number to fractional decimal number

The decimal equivalent of a binary fraction is determined by multiplying each digit in the fraction by 2^{-1}, 2^{-2} 2^{-3}, 2^{-4}, . . . etc., beginning with the first digit after the binary point. The products are then added to get the decimal fraction. A few examples will illustrate this procedure

Example 10.4 Convert the fractional binary number 0.1011 into decimal fraction.

Solution:

The number 0.1011 is converted into decimal number as

$1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}$

$= 0.5 + 0 + 0.125 + 0.0625$

$= 0.6875$

Example 10.5 Convert the fractional binary number 0.1101 into decimal fraction

Solution:

The number 0.1101 is converted into decimal number as

$1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$

$= 0.5 + 0.25 + 0 + 0.625$

$= 0.\ 8125$

10.2.4 Conversion of Decimal to Binary

To convert a decimal number into a binary equivalent number, the decimal number is expressed as a sum of ascending power of 2. The positional values of the binary number system are

…	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	128	64	32	16	8	4	2	1

Suppose we are to find the binary equivalent of the decimal number 45. The number 45 is arrived at by adding 32 + 8 + 4 + 1. The 1S and 0S are placed as shown

2^5		2^4		2^3		2^2		2^1		2^0
32	+	0	+	8	+	4	+	0	+	1 = 45

The binary number is 101101

The binary equivalent of 19 is arrived as

16 + 2 + 1

2^4		2^3		2^2		2^1		2^0
16	+	0 +		0	+	2	+	1 = 19

The binary number is 10011

The binary equivalent of decimal numbers 7, 9, 11, respectively are

Example 1

$7 = 4 + 2 + 1$

$2^2 + 2^1 + 2^0$

The binary number of 7 is 111

Example 2

$9 = 8 + 0 + 0 + 1$

$2^3 + 0 + 0 + 2^0$

The binary number of 9 is 1001

Example 3

$$11 = 8 + 0 + 2 + 1$$
$$2^3 + 0 + 2^1 + 2^0$$

The binary number of 11 is 1011.

We can use an alternative method of converting a decimal number into a binary equivalent by dividing the decimal number progressively by 2 till the quotient is zero. The remainders of the successive divisions, expressed in the reverse order, gives the binary equivalent number. A few examples will illustrate the procedure.

Example 10.6 Convert decimal 23 into its binary equivalent number.

			Remainder
Divide	23 by 2	$2\overline{)23}$ $\quad 11$	1
Divide	11 by 2	$2\overline{)11}$ $\quad 5$	1
Divide	5 by 2	$2\overline{)5}$ $\quad 2$	1
Divide	2 by 2	$2\overline{)2}$ $\quad 1$	0
Divide	1 by 2	$2\overline{)1}$ $\quad 0$	1

Now we will arrange the remainders in the reverse order, i.e., from bottom upwards. The binary equivalent number is 10111

Example 10.7 Determine the binary equivalent of decimal number 17.

Solution:

			Remainder
Divide	17 by 2	$2\overline{)17}$ $\quad 8$	1
Divide	8 by 2	$2\overline{)8}$ $\quad 4$	0
Divide	4 by 2	$2\overline{)4}$ $\quad 2$	0
Divide	2 by 2	$2\overline{)2}$ $\quad 1$	0
Divide	1 by 2	$2\overline{)1}$ $\quad 0$	1

The remainder when put in reverse order give the binary equivalent a 10001.

Example 10.8 Convert the following decimal numbers into their binary equivalent.

(a) 16.55 (b) 85.0625

Cross-check the conversion by reverse operation.

Solution:

(a) $(16.55)_{10} = (?)_2$

First we will convert the integer part.

$16 \div 2 = 8$ Remainder 0
$8 \div 2 = 4$ Remainder 0
$4 \div 2 = 2$ Remainder 0
$2 \div 2 = 1$ Remainder 0
$1 \div 2 = 0$ Remainder 1

↑ read this way

For the fractional part,

Product	
$0.55 \times 2 = 1.1$	Integer part $\rightarrow 1$
$0.1 \times 2 = 0.2$	Integer part $\rightarrow 0$
$0.2 \times 2 = 0.4$	Integer part $\rightarrow 0$
$0.4 \times 2 = 0.8$	Integer part $\rightarrow 0$
$0.8 \times 2 = 1.6$	Integer part $\rightarrow 1$
$0.6 \times 2 = 1.2$	Integer part $\rightarrow 1$

↓ read this way

We close this process here as the fractional part of the product is not becoming 0.

Thus, $(16.55)_{10} = (10000 \cdot 100011)_2$

We will now cross-check the result by reverse conversion.

$(10000 \cdot 10001)_2 = (?)_{10} = \text{N,say}$

$$N = 1\times2^4 + 0\times2^3 + 0\times2^2 + 0\times2^1 + 0\times2^0 \cdot 1\times2^{-1} + 0\times2^{-2} + 0\times2^{-3} + 0\times2^{-4} + 0\times2^{-5} + 0\times2^{-6}$$

$$= (16+0+0+0+0)\cdot\left(0.5+.0+0+0+\frac{1}{32}+\frac{1}{64}\right)$$

$$= 16.55$$

(b) $(85 \cdot 0625)_{10} = (?)_2$

First consider the integer part,

$85 \div 2 = 42$ Remainder 1
$42 \div 2 = 21$ Remainder 0
$21 \div 2 = 10$ Remainder 1
$10 \div 2 = 5$ Remainder 0
$5 \div 2 = 2$ Remainder 1
$2 \div 2 = 1$ Remainder 0
$1 \div 2 = 0$ Remainder 1

↑ read this way

$(85)_{10} = (1010101)_2$

For the fractional part,

	Product	Integer part
0.0625×2	$= 0.125$	$\rightarrow 0$
0.125×2	$= 0.25$	$\rightarrow 0$
0.25×2	$= 0.5$	$\rightarrow 0$
0.5×2	$= 1.0$	$\rightarrow 1$

↑ read this way

$(0.0625)_{10} = (0001)_2$

Thus, $(85\cdot0625)_{10} = (1010101\cdot0001)_2$

To cross-check by reverse conversion,

$1010101\cdot0001 = (?)_{10}$

$$N = 1\times2^6 + 0\times2^5 + 1\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 \cdot 0\times2^{-1} + 0\times2^{-2} + 0\times2^{-3} + 1\times2^{-4}$$
$$= (64+16+4+1)\cdot(1\times2^{-4})$$
$$= 85\cdot0625.$$

Example 10.9 Determine the binary equivalent of decimal number 10.

Solution:

			Remainder
Divide	10 by 2	$2\overline{)\,10}$ 5	0
Divide	5 by 2	$2\overline{)\,5}$ 2	1
Divide	2 by 2	$2\overline{)\,2}$ 1	0
Divide	1 by 2	$2\overline{)\,1}$ 0	1

The remainders are arranged in reverse order to get the binary equivalent as 1010.

Conversion of fractional decimal number to fractional binary number

When the decimal number is a fraction, the conversion can be carried out through the following steps.

First, multiply the decimal fraction by 2. The result contains an integer part and a fractional part. Separate the integer part and write it in a column. The fractional part becomes the new fraction. Repeat these steps till the fractional part becomes zero or the desired number of binary places are obtained. The 1s and 0s separated placed together gives the required fractional binary number. A few examples will illustrate the procedure.

Example 10.10 Convert decimal 0.8125 into its equivalent binary number.

Solution:

		Fractional part	Integer part
0.8125×2	$= 1.6250$	0.6250	1
0.6250×2	$= 1.2500$	0.2500	1
0.2500×2	$= 0.5000$	0.5000	0
0.5000×2	$= 1.0000$	0	1

The binary decimal number is 0.1101.

Example 10.11 Convert 0.65 into its binary fraction.

Solution:

		Fractional part	Integer part
0.65×2	$= 1.3$	0.3	1
0.3×2	$= 0.6$	0.6	0
0.6×2	$= 1.2$	0.2	1
0.2×2	$= 0.4$	0.4	0
0.4×2	$= 0.8$	0.8	0
0.8×2	$= 1.6$	0.6	1
0.6×2	$= 1.2$	0.2	1

In the last step we get back 0.2 as the fractional part. To get the approximate result we can terminate the process at this stage. Thus, the approximate binary fraction is 0.1010011.

10.2.5 Binary Addition

The binary addition is performed by using the following rules.

(i) $0 + 0 = 0$
(ii) $1 + 0 = 1$
(iii) $0 + 1 = 1$
(iv) $1 + 1 = 10$ ($\because$ decimal equivalent of 10 is 2)

Example 10.12 Carryout binary addition for the following: (i) 10 + 11; (ii) 111 + 111.

Solution:

(i) Add

$$\begin{array}{r} 10 \\ +11 \\ \hline 101(=5) \end{array} \qquad \begin{array}{c} 10+11=101 \\ 2+3=5 \end{array}$$

(ii) Add

$$\begin{array}{r} 111(=7) \\ +111(=7) \\ \hline 1110(=14) \end{array}$$

Here the first column gives 1 + 1 = 10. We put 0 below this column and add the carry 1 to the second column. In the second column 1 + 1 = 10. We add the carry 1 with 0 to get 1 and carry 1 to the third column. In the third column 1 + 1 = 10. We add the carry 1 to 0 to get 1 and the carry 1 is placed in the fourth column.

(iii) Add

$$\begin{array}{r} 10001(=17) \\ +111(=\ 7) \\ \hline 11000(=24) \end{array}$$

First column 1 + 1 = 10, place 0 and carry 1
Second column, 1 + 0 = 1, 1 + 1 = 10, place 0 and carry 1
Third column, 1 + 0 = 1, 1 + 1 = 10, place 0 and carry 1
Fourth column, 1 + 0 = 1, place 1 and carry 0
Fifth column, 1 + 0 = 1, place 1

10.2.6 Binary Subtraction

The following rules are used for binary subtraction

(i) $0 - 0 = 0$
(ii) $1 - 0 = 1$
(iii) $1 - 1 = 0$
(iv) $10 - 1 = 1$ (decimal equivalent of 10 is binary 2)

Example 10.13 Carryout binary subtraction using rules for the following: (i) 1011 + 1000; (ii) 110 − 11.

Solution:

(i) Subtract

$$\begin{array}{r} 1011(=11) \\ -1000(=\ 8) \\ \hline 0011(=3) \end{array}$$

(ii) Subtract

$$\begin{array}{r} 110 (= 6) \\ -11 (= 3) \\ \hline 011 (= 3) \end{array}$$

In the second example, the first column is 10 – 1 = 1 (after borrowing 1 from the second column). The borrow is carried and added to the lower number of second column to get 1 + 1 = 10. Therefore, the second column gives 1 – 0 = 1 and the third column gives 1 – 1 = 0.

10.2.7 Binary Multiplication

Binary multiplication is carried out using the following rules.

(i) $0 \times 0 = 0$
(ii) $1 \times 0 = 0$
(iii) $0 \times 1 = 0$
(iv) $1 \times 1 = 1$

Example 10.14 Carryout the binary multiplication: 111×101.

Solution:

			1	1	1	(= 7)
			1	0	1	(= 5)
			1	1	1	
		0	0	0	×	
	1	1	1	×	×	
1	0	0	0	1	1	(= 35)
1 + 1 = 10		1 + 1 = 10 carry 1	1 + 1 = 10 carry 1	1 + 0 = 1	1 + 0 = 1	

10.3 OCTAL NUMBER SYSTEM

Octal system has a base of 8 counting from 0 to 7. To convert a decimal number into octal equivalent of decimal number we divide the integer repeatedly by 8 till the quotient is zero. The remainders written in reverse order gives the octal equivalent number.

For example let us find the octal equivalent of decimal number 756.

756 ÷ 8 = 94	Remainder = 4
94 ÷ 8 = 11	Remainder = 6
11 ÷ 8 = 1	Remainder = 3
1 ÷ 8 = 0	Remainder = 1

The octal equivalent of 756 is 1364.

To convert an octal to binary each octal symbol is replaced by a 3-bit binary equivalent as shown

Octal	Binary
0	000
1	001
2	010
3	011

Octal	Binary
4	100
5	101
6	110
7	111

Conversion of binary number into an equivalent octal number can be done in the following way.

Split the binary digits into groups of three digits, each beginning from the right end for integers and from the left end for fractions. Replace each group by its equivalent octal number. A few examples are given to illustrate the procedure.

Example (i)

Convert binary 11001 into octal equivalent
The binary number is grouped as 011 001
Replacing each group by its octal equivalent we get the number 31

Example (ii)

Convert binary 0.111 001110 into its equivalent octal
The binary number is grouped into 3-bit form as
0.111 001 110
The equivalent is 0.716

To convert an octal number into its decimal equivalent the following steps be followed.

Consider the octal number from the extreme right as the coefficient of the ascending power of 8, the starting power of 8 at the extreme right has to be taken as zero. If the octal number has a fraction, the fractional numbers have to be considered as the coefficient of ascending power of 8^{-1}.

Examples

(i) $415.2 = 4 \times 8^2 + 1 \times 8^1 + 5 \times 8^0 + 2 \times 8^{-1}$
$= 256 + 8 + 5 + 0.25$
$= 269.25$

(ii) $732.14 = 7 \times 8^2 + 3 \times 8^1 + 2 \times 8^0 + 1 \times 8^{-1} + 4 \times 8^{-2}$
$= 448 + 24 + 2 + 0.125 + 0.0625$
$= 474.1875$

10.4 HEXADECIMAL NUMBER SYSTEM

In hexadecimal system we use 16 symbols, viz 0 to 9 and the capital letters A B C D E F for 10 to 15. Hence, the hexadecimal number system uses a base of 16.

To convert a binary number into a hexadecimal number (Hex) the binary digits are grouped into 4 binary bits, counting from the right end for integral numbers and from the left end for fractions. Each group is replaced by its equivalent.

Binary	Hex	Binary	Hex	Binary	Hex	Binary	Hex
0000	0	0100	4	1000	8	1100	C
0001	1	0101	5	1001	9	1101	D
0010	2	0110	6	1010	A	1110	E
0011	3	0111	7	1011	B	1111	F

Example 10.15

(i) Convert 1111 1110 0011 into hexadecimal equivalent
Group the binary as (from right to left) 1111 1110 0011
Replace the group by their equivalent as FE3

(ii) Convert 1111 10001.100 1100 1101 into hexadecimal equivalent
Group the binary as
0001 1111 0001 . 1001 1001 1010
Replace the groups by their equivalent to get
1F1.99A

(iii) Convert Hex number 6D5 into its binary equivalent.
Starting from right we replace binary equivalent of 5, D, and 4 as

(6)	(D = 13)	(5)
0110	1101	0101

It is seen from the above examples that conversion between Hex and binary is very quick.

For conversion of decimal number into binary equivalent, or useful system, a binary coded decimal system, or BCD system is often used.

It is a code in which individual decimal digits are replaced by a group of 4 binary bits in a row. The weights of the 4 bits from left to right of the binary string are 8,4,2, and 1, respectively. The BCD code is also called eight four two one code.

Example 10.16 Express 2534 in BCD form.

Decimal form	2	5	3	4
BCD form	0010	0101	0011	0100

Thus, $(2534)_{10} = (0010010100110100)_{BCD}$.

It is to be noted that this system is totally different from the method of conversion of decimal numbers into binary numbers.

The binary equivalent of 2534 is calculated as

2^{11}	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
2048	1024	512	256	128	64	32	16	8	4	2	1
1	0	0	1	1	1	1	0	0	1	1	0

2048 + 0 + 0 + 256 + 128 + 64 + 32 + 0 + 0 + 4 + 2 + 0 = 2534

Therefore, $(2534)_{10} = (100111100110)_2$

10.4.1 Application of Binary Numbers in Computers

The computer works with binary numbers, and hence all numbers, letters, symbols, etc. are to be converted into binary equivalents. Computers are made using very-large-scale integrated digital circuits. The main circuit element being transistors which work in two modes, i.e., on or off, i.e., conducting or non conducting. These transistors, in fact, work like a switch. The term binary or bistable means two states of operation either conducting or non-conducting giving rise to voltage level high or voltage level low, i.e., either 1 or 0. The computer works on the basis of binary logic circuits made of logic gates. Various logic gates are discussed in the following section.

10.5 LOGIC GATES

A logic gate is a simple device used to make digital integrated circuits. Diodes and transistors are used to perform switching functions in logic gates. Logic gates basically have one or more inputs and only one output. The output of the gate depends upon the way the inputs are applied and they are named as

AND gate, OR gate, NOT gate, etc. Logic gates may be classified into two categories; combinational type (with no memory) and sequential type (with memory).

In digital circuits or systems there are two possible states or conditions. A switch may be closed or open, i.e., on or off. These two conditions may be represented by 1 and 0, respectively. The two logic levels, i.e., 1 and 0 represent, respectively, voltage high or voltage low. Logic systems can be positive or negative. In positive logic voltage high is represented by 1 and voltage low is represented by 0. In negative logic, voltage high is 0 and voltage low is 1. Binary number system may be used to represent the states. In a digital system, analog quantities are first converted to digital forms using the binary code. After the signal is processed, it is again converted into analog form for display.

The possible states or conditions in digital systems are expressed as 1 and 0. The closed switch or the existence of a signal may be called 1 and open switch or non existence of a signal may be called 0. 1 and 0 are referred to as binary numbers. As mentioned earlier, in decimal system we have 0 to 9 as ten numbers. Using these numbers we can represent any number, e.g., year 2009 is a representation of

$$2 \times 10^3 + 0 \times 10^2 + 0 \times 10^1 + 9 \times 10^0$$

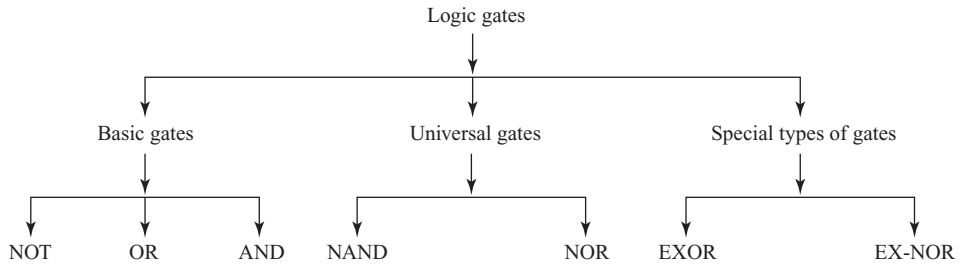
Here, the base is 10 as we use 10 digits. In the binary system we only use 1 and 0 with 2 as the base. The number 82 is represented as

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

$$1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

The number 85 is represented as 1010101.

Logic circuits or logic gates provide an output of logical 1 or logical 0 with a certain logical input. The logic gates are classified into two categories viz three logic gates and two universal gates. There are also two special types of gates. These are shown below.



10.5.1 NOT Gate

A NOT gate works as an inverter. It has a single input and a single output. The output signal is the negative of the input signal. The output is not the same as input, the output negates the input. If input is A, the output is $\overline{A}$ (A-bar).

Figure 10.1 shows a transistor circuit. When input A is 0 V, the transistor is off because $V_{BE} = 0$. The output voltage is the same as V_{CC}, i.e., 5 V which is referred to as 1. If input A is 1, i.e., 5 V, the transistor goes into saturation and the output will be low which is referred to as 0. The logic symbol and truth table for a NOT gate have also been shown in Fig. 10.1 (b) and (c), respectively.

Thus, the NOT gate is a logic gate that gives an output that is opposite to the state of its input.

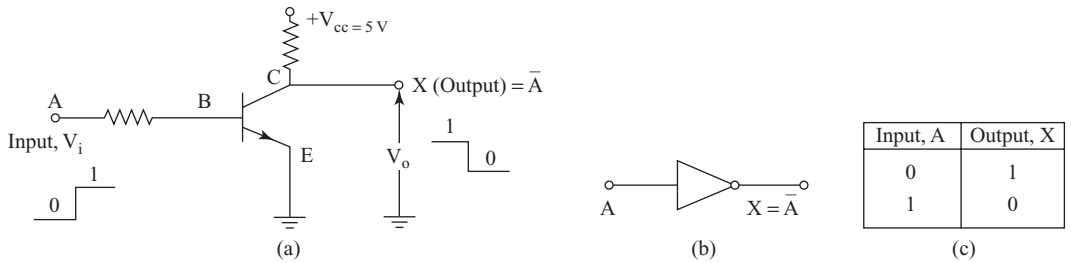


Figure 10.1 *(a) Circuit for a postive logic NOT gate; (b) symbol of a NOT gate; (c) truth table of a NOT circuit*

10.5.2 OR Gate

The OR gate provides an output 1 when one of its inputs is in 1 state. That is, its output is 1 whenever at least one of its inputs is 1. The output of an OR gate is 0 only when all its inputs are 0. The circuit using diodes for such a logic has been shown in Fig. 10.2 (a). The standard symbol for an OR gate and its truth table have also been shown along with the figure.

Output will occur when there is input in any one of the inputs A or B or at both. If A = B = 0, then the two diodes D_1 and D_2 will be reverse biased, and hence output, X will be zero. If A = 1 and B = 0 or if A = 0 and B = 1, i.e., when any one of the inputs is high, the output, X will be 1. If A = 1 and B = 1, i.e., when both the diodes conducting output X will be 1. If A = 1 and B = 0, i.e., when diode A will be in forward biased condition, the output X will be 1.

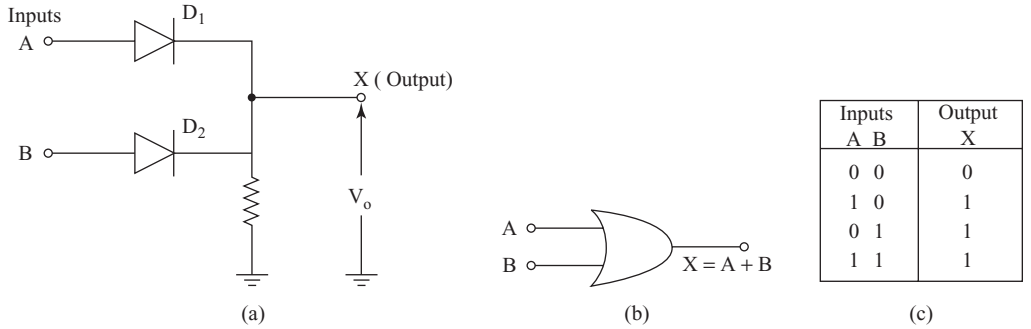
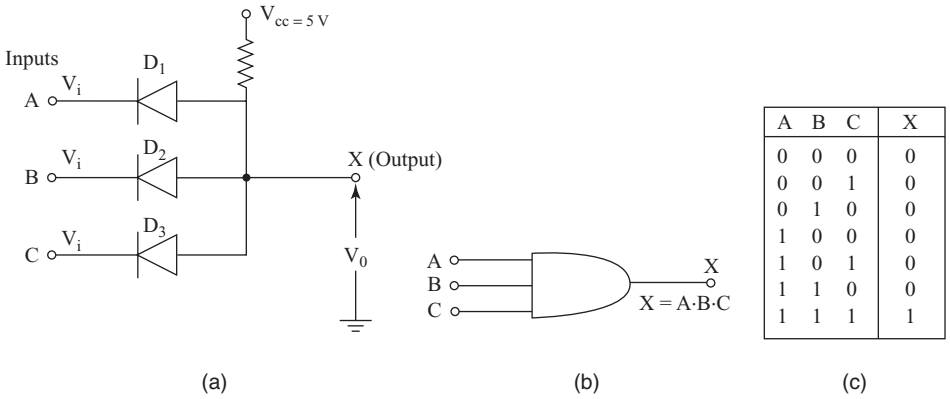


Figure 10.2 *(a) Logic OR gate circuit using diodes; (b) symbol of an OR gate; (c) truth table*

Although two diodes have been used to show an OR gate, in actual practice OR function is obtained using a transistor–transistor logic (TTL). The 2-input OR gates is available in the TTL 74 xx series. 7432 is a 2-input TTL OR gate integrated circuit. The IC chip has four OR gates.

10.5.3 AND Gate

This gate has two or more inputs but only one output. The AND gate is a logic gate that gives an output of 1 only when all of its inputs are 1. Thus, its output will be 0 when at least one of its inputs is 0. Figure 10.3 shows a 3-input AND gate circuit using three diodes. The AND gate may be considered equivalent to a circuit in which a number of switches are connected in series. Only when all the switches are on, will there be an output.



A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Figure 10.3 *(a) Logic AND gate circuit; (b) symbol; (c) truth table*

As in Fig. 10.3 (a), when inputs A = B = C = 0, all the diodes will be forward biased and will conduct, the output will be zero.

When A = B = C = 1, all the diodes will be reverse biased and will remain off. The output will be high, i.e., equal to 1.

When any of the inputs is high, the corresponding diode will conduct, and thus output will be zero.

As shown in figure, X = 1 only when A = B = C = 1. The 2-input AND gate is available in IC 7408. There are such four AND gates in the IC chip.

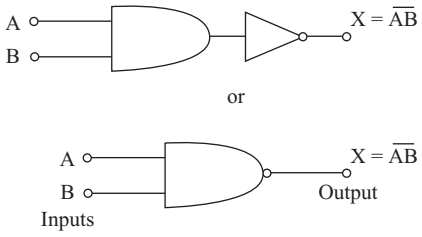
10.5.4 NAND Gate

The NAND gate is an AND gate with a NOT gate at its end. For the same combination of inputs, the output of a NAND gate will be opposite to that of an AND gate. The combination of AND and NOT gate can be made using a diode transistor logic circuit with minor modification. The symbol of a 2-input NAND gate along with its truth table have been shown in Fig. 10.4.

10.5.5 NOR Gate

The NOR gate is an OR gate with a NOT gate at its end. For the same combination of inputs, the output of a NOR gate will be opposite to that of the output of a NOR gate. The symbolic representation and truth table of a NOR gate have been shown in Fig. 10.5.

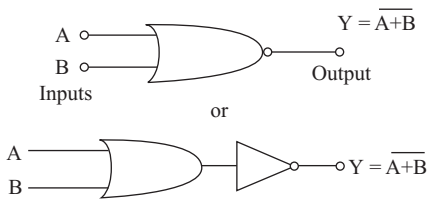
A truth table describes the behaviour of a logic gate. It shows the value of the output for every possible combination of inputs.



Truth table

Inputs		Output
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Figure 10.4 *Standard symbol and truth table of a NAND gate*



Truth table

Inputs A	Inputs B	Output Y
0	0	1
0	1	0
1	0	0
1	1	0

Figure 10.5 *Standard symbol and truth table of a NOR gate*

10.5.6 The EXOR Gate

The EXOR gate stands for Exclusive OR gate. The EXOR gate is a logic gate that gives an output 1 when only one of its inputs is 1. The symbol and truth table of an EXOR gate have been shown in Fig. 10.6 .

As can be seen from the truth table, the output will be low (0) only when all the inputs are similar.

Symbol

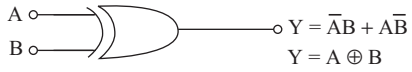


Figure 10.6 *Symbol of EXOR gate*

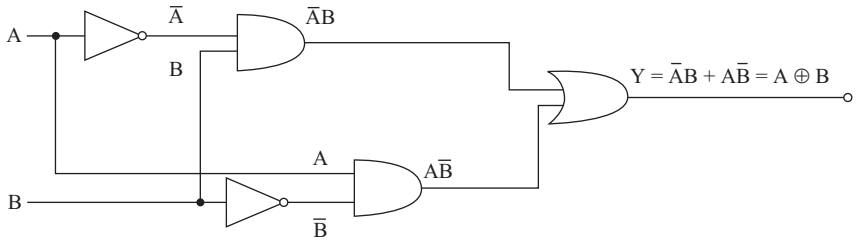


Figure 10.7 *EXOR gate from simple gates*

Truth table

Inputs A	Inputs B	Output Y
0	0	0
0	1	1
1	0	1
1	1	0

Figure 10.8 *Standard symbol and truth table of an EXOR gate*

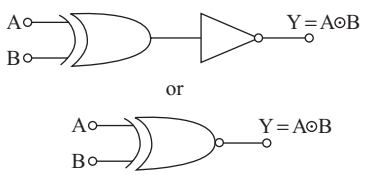
Note the difference between an OR gate and EXOR gate. In an OR gate, the output is 1 when any one of the inputs is 1 or both the inputs are 1. In an EXOR gate the output is 1 when any one of its inputs is 1. If both the inputs are 1 the output is 0. That is why an OR gate as such is also called an inclusive OR gate whereas EXOR gate is an exclusive OR gate.

10.5.7 EX-NOR Gate

The operation of an EX-NOR gate is just opposite to the operation of an EX OR gate. Inputs are two or more and output is one. The output will be high for similar types of inputs only. The symbol

and truth table have been shown. If A and B are two inputs then the output expression will be $Y = \overline{A}\,\overline{B} + AB$ which is expressed as

$$Y = \overline{A}\,\overline{B} + AB$$
$$= A \cdot B$$



Inputs		Output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

EX-NOR gate can be made from simple gates as shown in Fig. 10.9

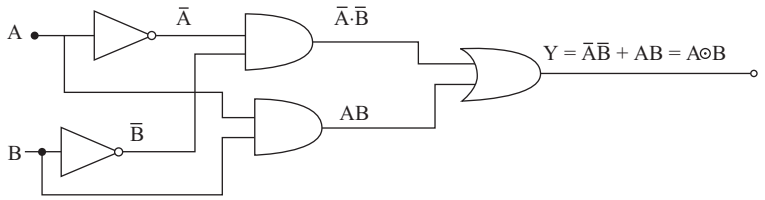


Figure 10.9

Thus, we have seen seven types of gates. Three basic gates are NOT, OR, and AND gates and the combinational gates are NAND, NOR, EXOR and EX-NOR gates.

NAND and NOR gates are called the universal gates as all other gates can be created from a suitable network of just NAND or just NOR gates.

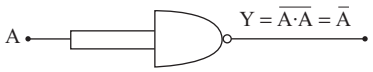
A logic gate performs a logical operation on one or more logic inputs and produces a single logic output. Logic gates are primarily implemented using resistors, diodes, and transistors.

10.6 UNIVERSAL GATES

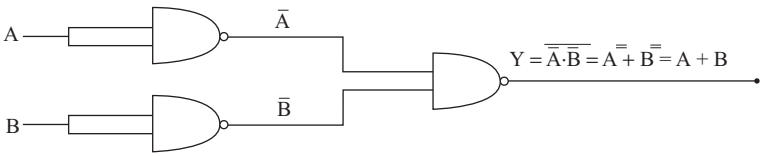
NAND and NOR gates are called universal gates. The basic gates i.e, AND, OR, NOT gates can be realized using NAND or NOR gates. These are presented below.

10.6.1 Realization of Logic Gates Using NAND and NOR Gates

(a) Realize NOT gate using NAND gate:

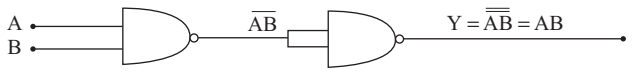


(b) Realize OR gate using NAND gate:

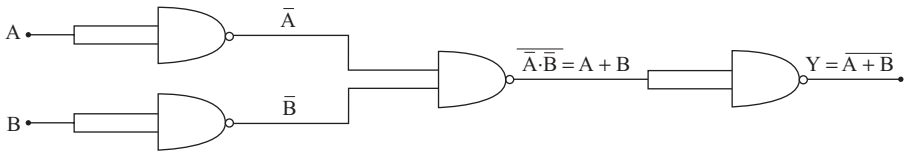


We have applied De Morgan's theorem to simplify the output Y. De Morgan's theorem is dealt with in the next section.

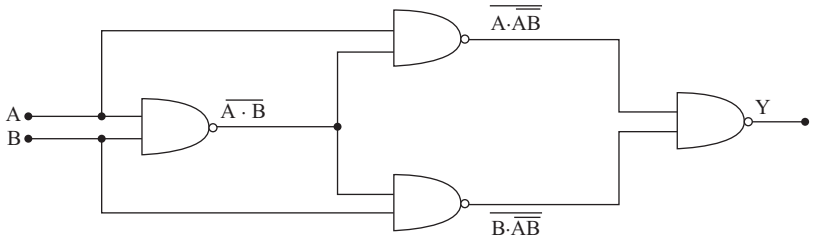
(c) Realize AND gate using NAND gate:



(d) Realize NOR gate using NAND gate:



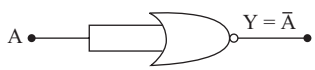
(e) Realize EXOR gate using NAND gate:



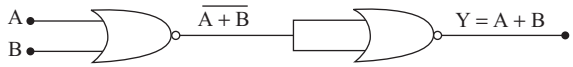
$$
\begin{aligned}
Y &= \overline{(A \cdot \overline{AB})} \cdot \overline{(B \cdot \overline{AB})} \\
&= \overline{\overline{A \cdot \overline{AB}}} + \overline{\overline{B \cdot \overline{AB}}} \\
&= A \cdot \overline{AB} + B \cdot \overline{AB} \\
&= A \cdot \left(\overline{A} + \overline{B}\right) + B \cdot \left(\overline{A} + \overline{B}\right) \\
&= A \cdot \overline{A} + A \cdot \overline{B} + B \cdot \overline{A} + B \cdot \overline{B} \\
&= A\overline{B} + B\overline{A} \\
&= A \oplus B
\end{aligned}
$$

10.6.2 Realization of Basic Logic Gates Using NOR Gates

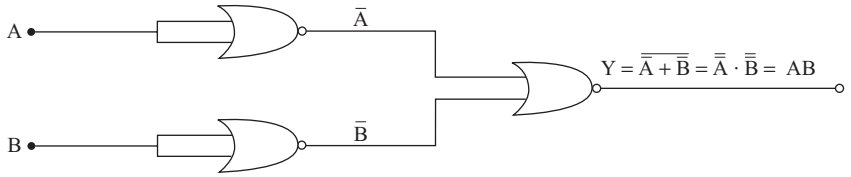
(a) Realize NOT gate using NOR gate:



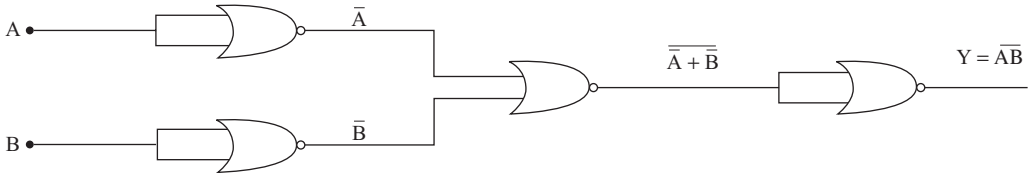
(b) Realize OR gate using NOR gate:



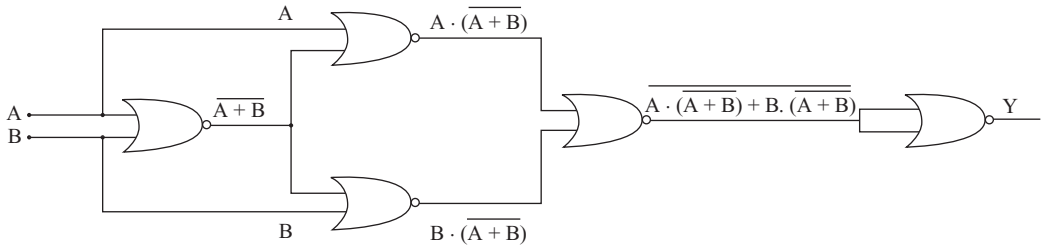
(c) Realize AND gate using NOR gate:



(d) Realize a NAND gate using NOR gate:



(e) Realize EXOR gate using NOR gate:



$$Y = A \cdot (A + B) + B \cdot (A + B) = A \cdot \overline{(A + B)} + B \cdot \overline{(A + B)}$$
$$= A \cdot (\overline{A} + \overline{B}) + B \cdot (\overline{A} + \overline{B})$$

$$A\overline{B} + B\overline{A} = A \oplus B$$

10.7 BOOLEAN ALGEBRA

In 1854, George Boole developed a new form of algebra which has come to be known as Boolean algebra. Boolean algebra is being applied for the solution of logical problems.

The operation of logic gates in relation to one another may be represented and analysed using a branch of mathematics called Boolean algebra. Expressions used in Boolean algebra are called Boolean expressions. In Boolean algebra only capital letters like A, B, C, D, etc. are used to represent the variables.

The variables in Boolean algebra are used in basic operations, namely (i) the OR addition indicated by plus (+) sign, (ii) the AND multiplication indicated by a cross (×) or a dot (•) sign, (iii) the NOT operation indicated by a bar ($\bar{}$) over the variable.

In Boolean algebra, the AND operation is represented by multiplication, since the result of multiplication of a combination of binary numbers 1 and 0 will be equal to 1 only if all its inputs are equal to 1. The Boolean expression for A and B is A.B.

The OR operation is represented by addition because the only way to obtain a result of an addition operation equal to 0 is to make all inputs equal to 0, which basically describes an OR operation. The Boolean expression for A or B is A + B.

NOT operation means inversion. The Boolean expression for NOT operation is A = NOT(A) =$\overline{A}$. NAND operation is an AND operation followed by a NOT operation. The NOR operation is an OR operation followed by a NOT operation.

10.7.1 Boolean Laws and Boolean Theorems

Three Boolean operations are illustrated below:

(1) OR addition (OR laws)

If we add 0 to anything, it does not affect the value. This is true in ordinary algebra and in OR addition. If the variable is OR'ed with 1, the resultant will be 1. If the variable is OR'ed with itself, the resultant is the variable itself. A variable OR'ed with its complement equals 1. The OR laws are presented as:

A + 0 = A
A + 1 = 1
A + A = A
A + $\overline{A}$ = 1
Truth Table for OR Operation

A	B	Y
0	0	0
0	1	1
1	1	1
1	0	1

From the truth table for OR operation,
If A = 0, we get 0 + 0 = 0 (OFF)
0 + 1 = 1 (ON)
If A = 1, we get 1 + 1 = 1 (ON)
1 + 0 = 1 (ON)

(2) AND multiplication (AND laws)

The laws are expressed as:
A × 0 = 0 (the variable AND'ed with 0 equals 0)
A × 1 = A (the variable AND'ed with 1 equals the variable)
A × $\underline{A}$ = A (the variable AND'ed with itself equals the variable)
A · $\overline{A}$ = 0 (the variable when AND'ed with its complement equals to 0)

Truth Table for AND Operation

A	B	Y
0	0	0
1	0	0
0	1	1
1	1	1

If A = 0, we get:
$0 \cdot 0 = 0$
$0 \cdot 1 = 0$
If A = 1, we get
$1 \times 0 = 0$
$1 \cdot 1 = 1$

(3) NOT laws (laws of complements)
A bar over a letter indicates NOT operation.

$\text{If } A = 0, \overline{A} = 1$ (Complement of 0 is 1)

$\text{If } A = 1, \overline{A} = 0$ (Complement of 1 is 0)

$\overline{\overline{A}} = A$ (Double complement of A is A)

(4) Commutative laws
The commutative laws are as follows:

- The two inputs of an OR gate can be interchanged (transposed) without effecting the output.
- The two inputs of an AND gate can be interchanged (transposed) without effecting the output.

$A + B = B + A$
$A \cdot B = B \cdot A$

(5) Associative laws
This law states that the manner in which variables are grouped in OR and AND operations has no effect on the output. For example,
$A + (B + C) = (A + B) + C$
$A (B \cdot C) = (A \cdot B) C$
The two equations given above show that the inputs of OR and AND gates have been grouped in two different ways. However, they remain the same as shown in Fig. 10.10.

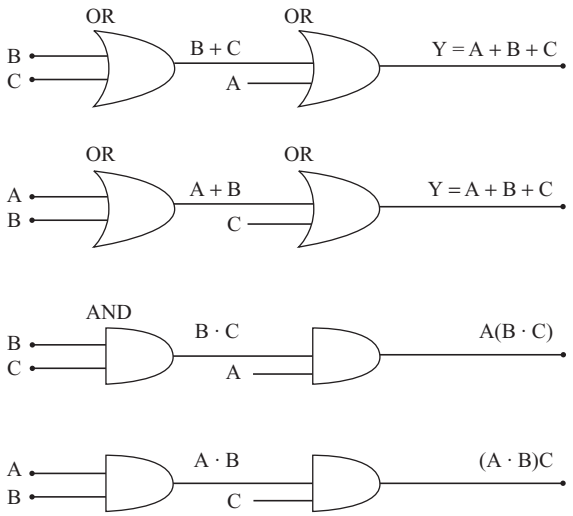


Figure 10.10 *Illustration of associative laws*

(6) Distributive laws

The distributive laws are as follows:

$A + (B \cdot C) = (A + B)(A + C)$

$A \times (B + C) = A \times B + A \times C$

We can see from the above two equations that we can perform OR operation followed by AND operation or perform AND operation followed by OR operation. In both the cases the result will be the same as shown in Fig. 10.11.

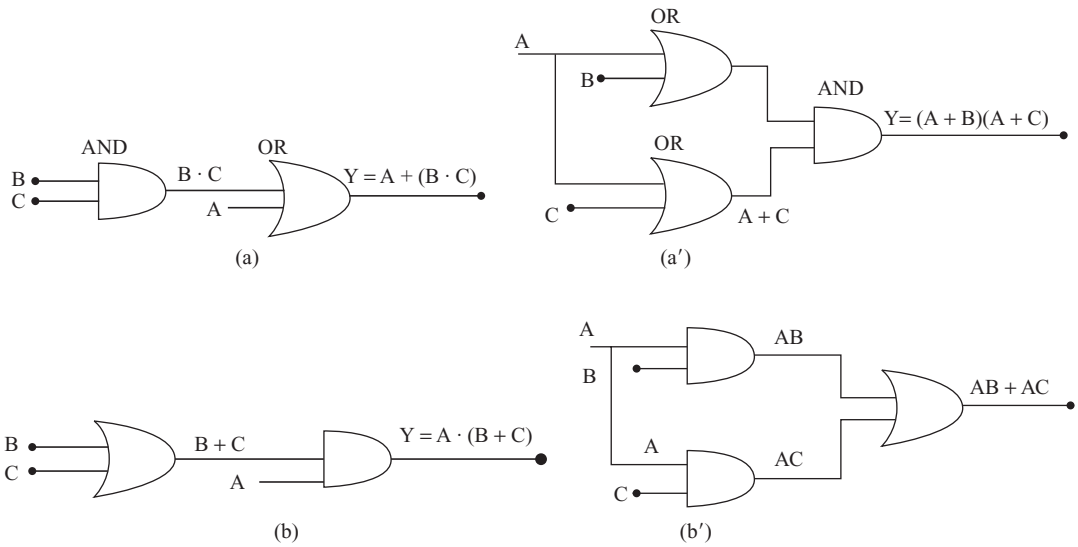


Figure 10.11 *Distributive laws: (a) is equivalent (a′); (b) is equivalent (b′)*

(7) Double inversion

The double inversion of a variable results in the variable itself. For example, $\overline{\overline{A}}=A$. This is shown in Fig. 10.12.

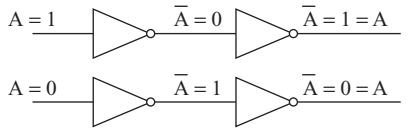


Figure 10.12 *Double inversion of a variable*

(8) Duality theorem

Another Boolean relation can be derived from a Boolean relation by (i) changing each OR sign (+) by an AND sign (+), (ii) by changing each AND sign (·) by an OR sign (+), and (iii) by complementing any 0 or 1 appearing in the given relation. Such relations are called dual relation.

For example, $A + 0 = A$ is one relation. Its dual relation is $A \cdot 1 = A$.

This has been obtained by changing (+) into (·) and complementing 0 by 1 to get the same output.

Similarly, let us consider another expression:

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

Its dual relation can be obtained by changing (.) into (+) and vice versa as

$$A + B \cdot C = (A + B) \cdot (A + C)$$

Two more examples of dual forms are shown below. These are known as absorption laws:

$$A + A\,B = A$$

$$A \cdot (A + B) = A$$

Proof: $A + A\,B = A\,(1 + B) = A$ since $1 + B = 1$

$$A \cdot (A + B) = A \cdot A + A \cdot B = A + A\,B = A$$

Also, $$AB+\overline{A}C+BC=AB+\overline{A}C$$

Dual form: $(A+B)(\overline{A}+C)(B+C)=(A+B)(\overline{A}+C)$

10.7.2 De Morgan's Theorems

De Morgan's theorems are extremely useful in simplifying Boolean expressions.
De Morgan's first theorem states,

$$\overline{A+B}=\overline{A}.\overline{B}$$

This means that the inverse of the OR operation of two variables A and B is the same as the AND operation of the inverses of individual variables.

De Morgan's second theorem states that the inverse of the AND operation of variables A and B is the same as the OR operation of the inverses of A and B. The logic representation of the second theorem is

$$\overline{A.B}=\overline{A}+\overline{B}$$

The representation of the logic expressions of De Morgan's Theorem is shown in Figure 10.13.

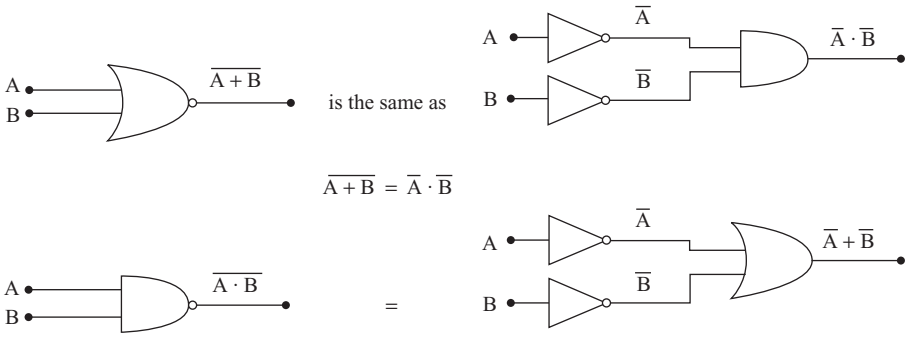


Figure 10.13 *(a) De Morgan's first theorem, (b) De Morgan's second theorem*

10.7.3 Proof of De Morgan's Theorem

$\overline{A+B}=\overline{A}.\overline{B}$

A and B are two variables, which can take a value of 0 or 1. The following are the possibilities.

	A	B
i)	0	0
ii)	0	1
iii)	1	0
iv)	1	1

We will prove De Morgan's theorem by considering the different values of A and B.

(i) For A = 0, B = 0

$\overline{A+B} = \overline{0+0} = \overline{0} = 1$

$\overline{A} \cdot \overline{B} = \overline{0} \cdot \overline{0} = 1.1 = 1$

Therefore, $\overline{A+B} = \overline{A} \cdot \overline{B}$

(ii) For A = 0, B = 1

$\overline{A+B} = \overline{0+1} = 0$, and

$\overline{A} \cdot \overline{B} = \overline{0} \cdot \overline{1} = 1.0 = 0$

Therefore, $\overline{A+B} = \overline{A} \cdot \overline{B}$

(iii) For A = 1, B = 0

$\overline{A+B} = \overline{0+1} = \overline{1} = 0$

$\overline{A} \cdot \overline{B} = \overline{1} \cdot \overline{0} = 0.1 = 0$

Therefore, $\overline{A+B} = \overline{A} \cdot \overline{B}$

(iv) For A = 1, B = 1

$\overline{A+B} = \overline{1+1} = 1.0 = 0$

$\overline{A} \cdot \overline{B} = \overline{1} \cdot \overline{1} = 0.0 = 0$

Therefore, $\overline{A+B} = \overline{A} \cdot \overline{B}$

The theorem has been proved for all possible value of the variables A and B.
Similarly, for the second theorem the relation is

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

Taking values of A = 0, B = 0; A = 0, B = 1; A = 1, B = 0; and A = 1, B = 1

(i) LHS: $\overline{A \cdot B} = \overline{0.0} = \overline{0} = 1$

RHS: $\overline{A} + \overline{B} = \overline{0} + \overline{0} = 1+1 = 1$

This shows that LHS = RHS or $\overline{A \cdot B} = \overline{A} + \overline{B}$

(ii) LHS: $\overline{A \cdot B} = \overline{0} \cdot \overline{1} = \overline{0} = 1$

RHS: $\overline{A} + \overline{B} = \overline{0} + 1 = 1 + 0 = 1$

Thus, $\overline{A \cdot B} = \overline{A} + \overline{B}$

(iii) $\overline{A \cdot B} = \overline{1.0} = \overline{0} = 1$

$\overline{A} + \overline{B} = \overline{1} + \overline{0} = 0 + 1 = 1$

Thus, $\overline{A \cdot B} = \overline{A} + \overline{B}$

(iv) $\overline{A \cdot B} = \overline{1.1} = \overline{1} = 0$

$\overline{A} + \overline{B} = \overline{1} + \overline{1} = 0 + 0 = 0$

Thus, $\overline{A \cdot B} = \overline{A} + \overline{B}$

We have taken two variables A and B and proved that both the theorems hold good. They will hold good for more than two variables also.

It is easy to remember De Morgan's theorem this way: Break the line on top and replace (+) with (·) or vise-versa while equating the variables. For example,

$\overline{A+B+C} = \overline{A}.\overline{B}.\overline{C}$ (as per first theorem)

$\overline{A.B.C} = \overline{A}+\overline{B}+\overline{C}$ (as per second theorem)

As mentioned earlier, George Boole developed rules for manipulation of binary variables. These rules or theorems form part of Boolean algebra. Logical expressions and their realization form the basis for digital systems like computers, calculators, etc. Table 10.1 provides the theorems relating to Boolean theorem.

Table 10.1 *Theorems Related to Boolean Algebra*

Theorem Number	Fundamental Rule
1	$A + 0 = A$
2	$A \cdot 1 = A$
3	$A + 1 = 1$
4	$A \cdot 0 = 0$
5	$A + A = A$
6	$A \cdot A = A$
7	$A+\overline{A}=1$
8	$A.\overline{A}=0$
9	$A.(B + C) = AB + AC$
10	$A + BC = (A + B)(A + C)$
11	$A + AB = A$
12	$A(A + B) = A$
13	$A+\overline{A}B = A+B$
13a	$\overline{A}+AB = \overline{A}+B$
14	$A(\overline{A}+B) = AB$
15	$AB + A\overline{B} = A$
16	$(A + B) \cdot (A + B) = A$
17	$AB + \overline{A}C = (A + C)(\overline{A} + B)$
18	$(A+B)(\overline{A}+C) = AC+\overline{A}B$
19	$AB+\overline{A}C+BC = AB+\overline{A}C$
20	$(A+B)(\overline{A}+C)(B+C) = (A+B)(\overline{A}+C)$
21 De Morgan's theorem:	$\overline{A.B.C} = \overline{A}+\overline{B}+\overline{C}$
22 De Morgan's theorem:	$\overline{A+B+C} = \overline{A}.\overline{B}.\overline{C}$

10.7.4 Simplification of Boolean Expressions

It is possible to simplify Boolean expressions so as to reduce the number of logic gates required to realize the logic expression. In the design of a circuit using gates, the traditional method is to simply the logical expression and realize it using logic gates.

A number of methods such as algebraic method, Karnaugh-map technique, Quine–McCluskey method, etc. are used to simply the Boolean function. Here we will only discuss the algebraic method. Any logic function can conveniently be expressed in the following forms:

(i) Sum-of-products (SOP) or sum of minterms
(ii) Product-of-sum (POS) or product of maxterms

An example will clarify the concept. Let us express the logic function $Y=(A+BC)(B+\overline{C}A)$ in SOP and POS form.

In the sum-of-products form, this expression is written as

$$Y = (A+BC)(B+\overline{C}A) \quad \text{(i)}$$

or, $$Y = AB+A\overline{C}A+BCB+BC\overline{C}A$$

or, $$Y = AB+A\overline{C}+BC \quad \text{(ii)}$$

(Since $A.A = A, B.B = B$ and $C\overline{C} = 0$)

This representation is in the sum-of-products form.
The expression is represented in the product-of-sum form as

$$\begin{aligned} Y &= (A+BC)(B+\overline{C}A) \\ &= (A+B)(A+C)(B+\overline{C})(B+A) \\ &= (A+B)(A+C)(B+\overline{C}) \end{aligned}$$

Since, $A+BC = (A+B)(B+C)$

$B+\overline{C}A = (B+\overline{C})(B+A)$

$(A+B)(A+B) = A+B.$

$Y = (A+BC)(B+CA)$ Can be represented

or $$Y = (A+B)(A+C)(B+C) \quad \text{(iii)}$$

This representation is in the product-of-sums (POS) form. We can now realize the simplified logical functions using logic gates.

Example 10.17 Convert the following expression in the sum-of-products (SOP) form.

$$Y = AB(C+\overline{C})+A\overline{C}(B+\overline{B})+BC(A+\overline{A})$$

Solution:

$$\begin{aligned} Y &= ABC+AB\overline{C}+A\overline{C}B+A\overline{CB}+BCA+BC\overline{A} \\ &= ABC+ABC+AB\overline{C}+AB\overline{C}+A\overline{CB}+BC\overline{A} \\ &= ABC+AB\overline{C}+A\overline{BC}+\overline{A}\,BC \end{aligned}$$

Example 10.18 Simplify the expression $Y = AB + \overline{A}C + BC$ to and draw the logic circuit.

Solution:

$$\begin{aligned} Y &= AB + \overline{A}C + BC \\ &= AB + \overline{A}C + (A + \overline{A})BC \qquad (\because A + \overline{A} = 1) \\ &= AB + ABC + \overline{A}C + \overline{A}BC \\ &= AB(1 + C) + \overline{A}C(1 + B) \\ &= AB + \overline{A}C \qquad (\because 1 + C = 1 \\ &\qquad\qquad\qquad\qquad\quad 1 + B = 1) \end{aligned}$$

The simplified logic circuit for $Y=AB+\overline{A}C$ is shown in Fig. 10.14.

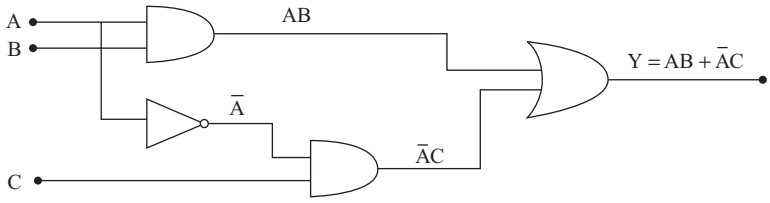


Fig 10.14 *Logic Circuit for* $Y=AB+\overline{A}C$.

Students are advised to draw the logic circuit using the original expression $Y=AB+\overline{A}C+BC$ and observe the requirement of number of logic gates.

Example 10.19 Simplify the following logic expressions:

(a) $A\overline{(B + \overline{C})}D$

(b) $\overline{\overline{A}\,\overline{B}C\,D}$

Solution:

(a) $A\ \overline{(B + \overline{C})}\ D$

$= \overline{A} + \overline{\overline{(B + \overline{C})}} + \overline{D}$

$= \overline{A} + B + \overline{C} + \overline{D}$

(We have broken the bar and changed the dots into plus signs.)

(Since $\overline{\overline{A}} = A$)

(b) $\overline{\overline{A}\,\overline{B}\,C\,D}$

$= \overline{\overline{\overline{A}\,\overline{B}\,C}} + \overline{D}$

$= A\,B\,C + \overline{D}$

$= (\overline{A} + \overline{B})C + \overline{D}$

$= \overline{A}C + \overline{B}C + \overline{D}$

Example 10.20 Apply Boolean's leaves to prove the following equivalent relations:

(i) $AB + \overline{A}C + BC = AB + \overline{A}C = AB + \overline{A}C$

(ii) $ABC + A\overline{B}C + AB\overline{C} = A(B + C)$

(iii) $(\overline{AC} + B)(\overline{\overline{A} + \overline{C}}) = ABC$

Solution:

(a) $AB + \overline{A}C + BC$

$= AB + \overline{A}C + BC(A + \overline{A})$ $\quad (\because A + \overline{A} = 1)$

$= AB + ABC + \overline{A}C + B\overline{A}C$

$= AB(1 + C) + \overline{A}C(1 + B)$

$= AB + \overline{A}C$ $\quad (\because 1 + C = 1 + B = 1)$

(b) $ABC + A\overline{B}C + AB\overline{C}$

$AC(B + \overline{B}) + AB\overline{C}$

$= AC + AB\overline{C}$

$= A(C + B\overline{C})$

$= A(C + \overline{C}B)$

$= A(C + B)$

$= A(\mathbf{B} + C)$ $\quad (\because A + \overline{A}B = A + B)$

(C) $(\overline{AC} + B)(\overline{\overline{A} + \overline{C}})$

$= (\overline{AC} + B)(\overline{\overline{A}}.\overline{\overline{C}})$

$= (\overline{AC} + B)(AC)$

$= \overline{AC}.AC + ABC$

$= AB$ $\quad (\because A.\overline{A} = 0)$

Example 10.21 Simplify the logic functions (i) $Y = \overline{\overline{AB} + \overline{A} + AB}$ (ii) $Y = (A + B)(\overline{A} + \overline{C})(B + C)$

Solution:

(i) $\overline{Y = AB + \overline{A} + AB}$ (break the bar and change the dot to plus sign)

$= \overline{\overline{A} + \overline{B} + \overline{A} + AB}$ $(\because A + A = A, \overline{A} + \overline{A} = \overline{A})$

$= \overline{\overline{A} + \overline{B} + AB}$ $(\because \overline{A} + AB + \overline{A} + \overline{B})$

$= \overline{\overline{A} + B + \overline{B}}$ $(\because B + \overline{B} = 1)$

$= \overline{A} + 1$

$= \overline{\overline{A} + 1}$

$= A.1$

$= 0.$

(ii) $Y = (A + B)(\overline{A} + C)(B + C)$

$= (A\overline{A} + AC + B\overline{A} + BC)(B + C)$ $(\because A\overline{A} = 0)$

$= (AC + B\overline{A} + BC)(B + C)$

$= ABC + BB\overline{A} + BBC + ACC + B\overline{A}C + BCC$ $(\because AA = A)$

$= ABC + B\overline{A} + BC + AC + B\overline{A}C + BC$ $BB = BCC = C$

$= AC(B + 1) + B\overline{A} + BC(\overline{A} + 1)$ $(\because BC + BC + BC)$

$= AC + B\overline{A} + BC$ $(\because B + 1 = 1, \overline{A} + 1 = 1)$

$= AC + B\overline{A} + BC(A + \overline{A})$ $(\because A + A = 1)$

$= AC + B\overline{A} + ABC + \overline{A}BC$

$= AC(1 + B) + B\overline{A}(1 + C)$

$= AC + B\overline{A}$

Example 10.22 With the help of a truth table establish the station:

$AB + AC + B\overline{C} = AC + B\overline{C}$

Solution:

We assign all possible values i.e. 0 and 1 for the variable A, B, C and then calculate the products and their sums in the truth table.

A	B	C	$\overline{C}$	AB	AC	$B\overline{C}$	$AB + AC + BC$	$AC + B\overline{C}$
0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	1	0	0	1	1	1

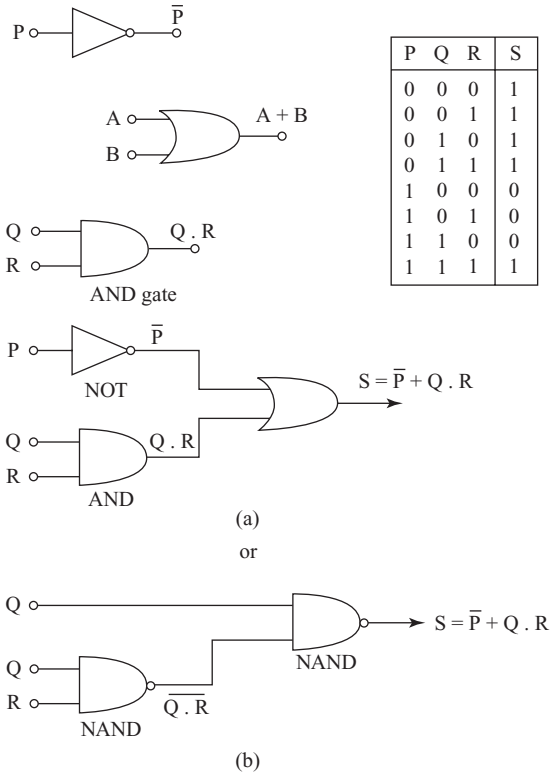
A	B	C	$\overline{C}$	AB	AC	$B\overline{C}$	AB + AC + BC	AC + $B\overline{C}$
0	1	1	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	0	1	0	1	1
1	1	0	1	1	0	1	1	1
1	1	1	0	1	1	0	1	1

It is seen that $AB+AC+B\overline{C}=AC+B\overline{C}$ for all possible values of A, B, and C.

10.8 COMBINATIONAL LOGIC CIRCUITS AND FUNCTION REALIZATION USING LOGIC GATES

Combinational logic circuits are constructed by interconnecting different logic gates. A combinational circuit with its truth table has been shown in Fig. 10.15. The same circuit can also be implemented using NAND gates only as shown in Fig. 10.15 (b).

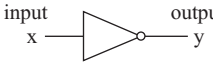
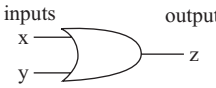
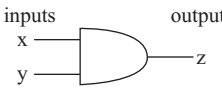
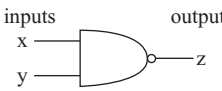
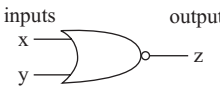
The logic gates and related Boolean equations are shown in tabular form in Table 10.2.



P	Q	R	S
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Figure 10.15 *(a) Truth table for a combinational circuit; (b) NAND gate implementation of the same logic*

Table 10.2 *Logic Gates and Their Truth Tables*

Gate name	Gate symbol and Boolean equation	Truth table
NOT	 Boolean equation: $y = \overline{x}$	input x | output y 0 | 1 1 | 0
OR	 Boolean equation: $z = x + y$	input x y | output z 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
AND	 Boolean equation: $z = x.y$	input x y | output z 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
NAND	 Boolean equation: $z = \overline{x.y}$	input x y | output z 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR	 Boolean equation: $z = \overline{x + y}$	input x y | output z 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
EXOR	Boolean equation: $z = x \oplus y$ $= \overline{x}y + x\overline{y}$	input x y | output z 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0

Example 10.23 Write the Boolean expression and truth table for the logic gate circuit shown in Fig. 10.16.

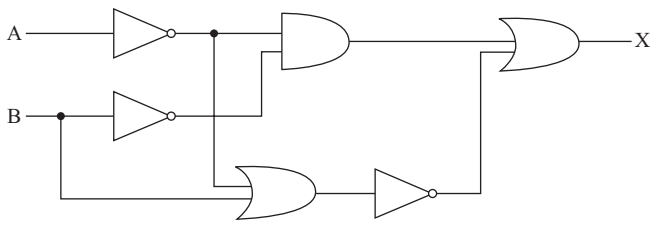


Figure 10.16

Solution:
The Boolean expression is developed as shown below.

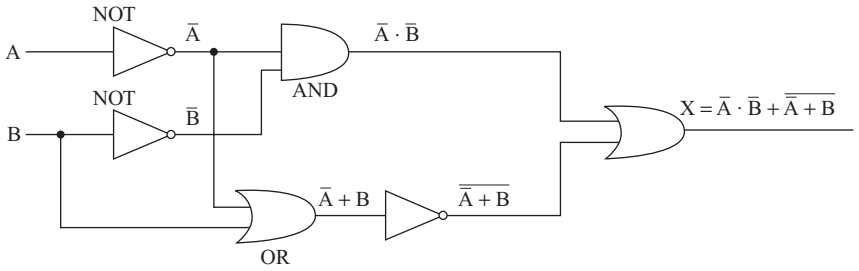


Figure 10.17

The Boolean expression is

$$X = \bar{A}.\bar{B} + \overline{\bar{A} + B}$$
$$= Y + Z$$

where $Y = \bar{A}.\bar{B}$

and $Z = \overline{\bar{A} + B}$

Now let us write the truth table. We first write the various combinations of 0 and 1 for the two inputs, A and B. There are $2^N = 2^2 = 4$ combinations. For each row of inputs, we will calculate Y and Z, respectively. Then we would add Y and Z, to get X.

Truth table

A	B	Y	Z	X
0	0	1	0	1
0	1	0	0	0
1	0	0	1	1
1	1	0	0	0

Example 10.24 For the logic circuit shown in Fig. 10.18, write the Boolean expression and construct the truth table.

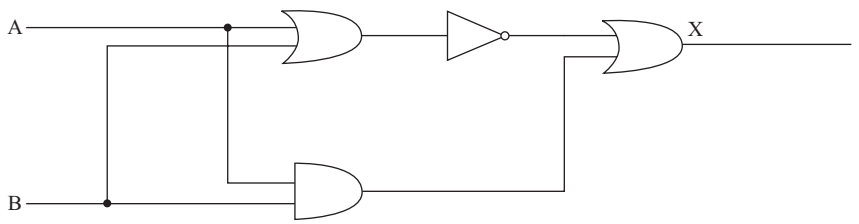


Figure 10.18

Solution:

The Boolean expression is developed from Fig. 10.19 as

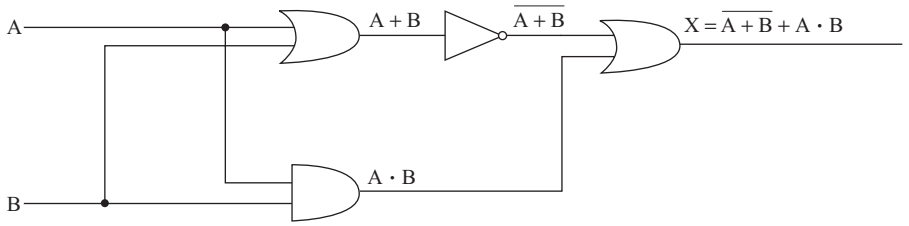


Figure 10.19

The Boolean expression is

$$X = \overline{A + B} + A.B$$

$$= Y + Z$$

where $Y = \overline{A + B}$ and $Z = A \cdot B$

For constructing the truth table, we will first write the various combinations of the inputs at A and B in the form of binary numbers (the total number of possible combinations being Z^N where N is the number of inputs). We then calculate Y and Z, and then X is calculated.

Truth table

A	B	Y	Z	X = Y + Z
0	0	1	0	1
0	1	0	0	0
1	0	0	0	0
1	1	0	1	1

Boolean expression for any logic function can be obtained from the truth table. This is done by taking the sum of all terms which correspond to all those combinations of inputs for which the output attains a high value. Let us consider the truth table of the above example.

A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

The first row and fourth row have to be considered where output is high, i.e., 1. Inputs A and B are to be considered 1. The complement $\overline{A}$ and $\overline{B}$ therefore 0. For row 1

$$Y = \overline{A + B}$$

For row 4

$$Z = A \cdot B$$

The Boolean expression for the output is

$$X = Y + Z = \overline{A + B} + A \cdot B$$

Example 10.25 The truth table for three variable inputs and single output logic circuit is shown below. Note that for three inputs the combinations are $2^3 = 8$. Construct the logic circuit using AND and OR gates.

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Solution:

From observing the truth table we find that output is high, i.e., 1 for the inputs at rows 1, 3, and 6. Inputs at A, B, and C is 1 and complement $\bar{A}$, $\bar{B}$ $\bar{C}$ is 0.

By writing the sum of all terms for which the output attains a high

$$X = \overline{ABC} + \bar{A}B\bar{C} + A\bar{B}C$$

Using the above Boolean expression, the logic circuit is drawn as in Fig. 10.20.

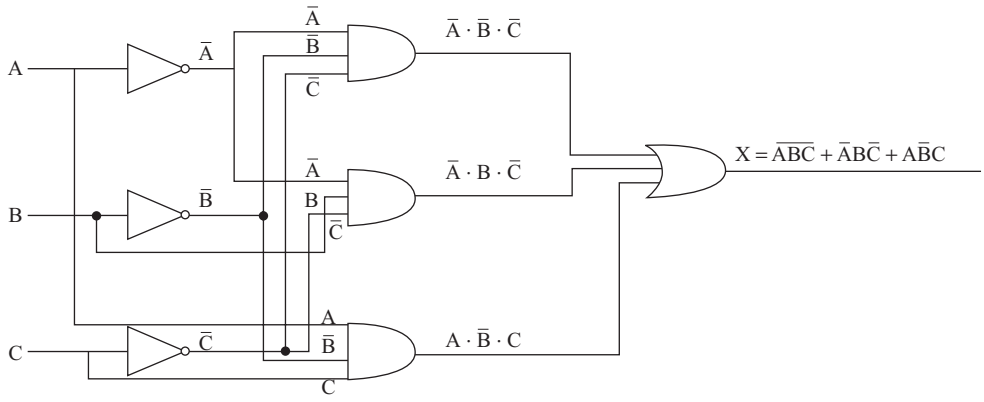


Figure 10.20

Example 10.26 Draw the logic circuit for the Boolean expression made in two forms. Also write the truth table

$$Y = \overline{A}\,\overline{C} + B\,\overline{C} \quad \text{(i)}$$

and

$$Y = \overline{C}\,(\overline{A} + B) \quad \text{(ii)}$$

Solution:
for form (i)

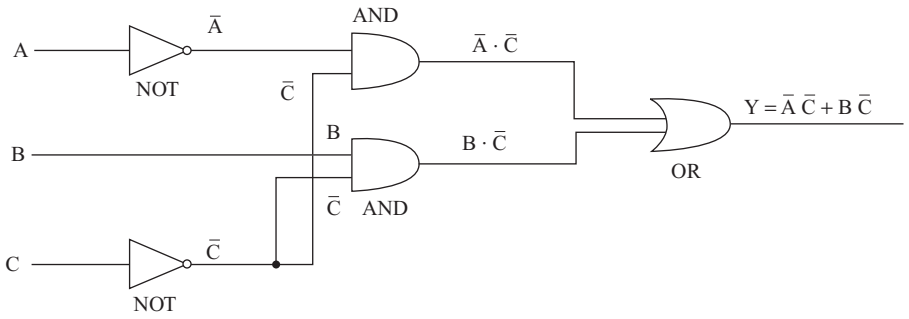


Figure 10.21

for form (ii)

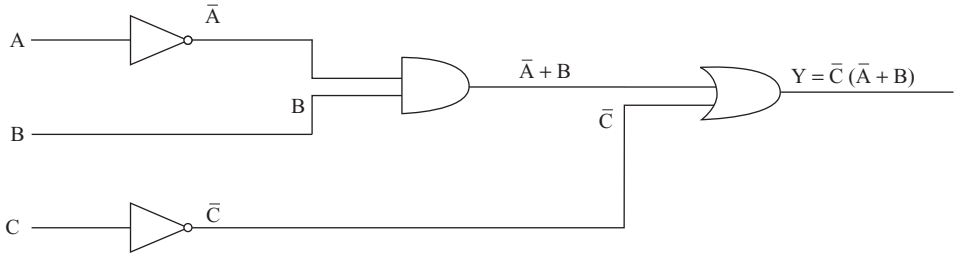


Figure 10.22

The truth table for

$$Y = \overline{A}\,\overline{C} + B\overline{C}$$
$$= P + Q$$

is shown as

A	B	C	P	Q	Y
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	0	1	1

1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	0	1	1
0	1	1	0	0	0
1	1	1	0	0	0

10.9 CONVERTING LOGIC STATEMENTS INTO BOOLEAN EXPRESSIONS AND REALIZATION OF THE NETWORK

Define a logic problem in words, write the logic statements and realize the network. Alternately, express the logic problem through input and output combination with a truth table. Derive the Boolean expression from the truth table either by methods of SOP and POS. Then implement the logic using AND and OR gates and invertors. The logic expression could be simplified before implementation so as to reduce the number of gates required.

Example 10.27 Design a logic circuit for a three-input and one-output system where the output will be high only when any of the two inputs. The truth table for the above problem is shown below.

A B C Y	Product term
0 0 0 0	
0 0 1 0	
0 1 0 0	
0 1 1 1	
1 0 0 0	$\overline{A}$ B C
1 0 1 1	A $\overline{B}$ C
1 1 0 1	A B $\overline{C}$
1 1 1 1	A B C

The product term in the last column in the above table shows the product of the three inputs when the output in high. The Boolean expression in the sum-of-product form is

$$Y=\overline{A}BC+A\overline{B}C+AB\overline{C}+ABC$$

We will now simplify the above expression using Boolean algebraic laws.

$$Y=\overline{A}BC+A\overline{B}C+AB\overline{C}+ABC+ABC+ABC$$

(We know, A + A = A So, ABC+ABC+ABC = ABC+ABC = ABC.)

$$=(\overline{A}+A)BC+(\overline{B}+B)AC+(\overline{C}+C)AB$$

$$Y=BC+AC+AB \quad (\because \overline{A}+A=1)$$

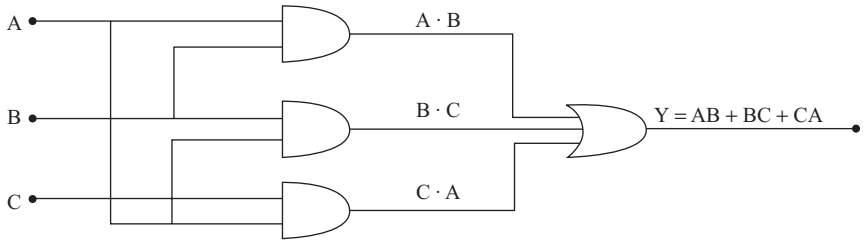


Figure 10.23 *Logic circuit for Y = AB + BC + CA*

The logic circuit for Y is shown in Fig. 10.23.

Example 10.28 Prove that the Boolean expression $\overline{\overline{AB}+\overline{A}+AB}$ is equal to zero. Establish the same with the help of a truth table by assuming all possible values of the variable A and B.

Solution:

$$Y=\overline{\overline{AB}+\overline{A}+AB}$$

$$= \overline{1+\overline{A}} \qquad \text{Since } \overline{A-B}+AB=1$$

$$= \bar{1} \qquad \text{Since } 1+\overline{A}=1$$

$$= .0$$

Truth table

A	B	AB	$\overline{AB}$	$\overline{A}$	$\overline{AB}+\overline{A}+AB$	$\overline{AB}+\overline{A}+A$
0	0	0	1	1	1	0
0	1	0	1	1	1	0
1	0	0	1	0	1	0
1	1	1	0	0	1	0

It can be seen from the last column of the truth table that the output is always zero.

Example 10.29 Obtain the Boolean expression for the output Y in the logic circuit shown in Fig. 10.24. Simplify the expression and show that the circuit is equivalent to an AND gate with two inputs.

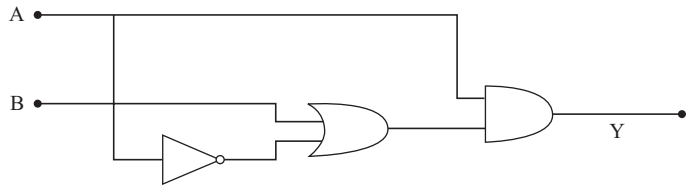


Figure 10.24

Solution:
We take A and B as the two variables and draw the logic diagram to obtain the output Y.

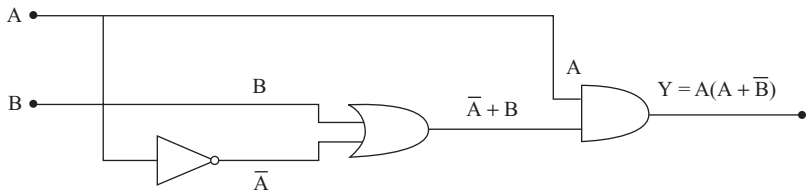


Figure 10.25

The expression is $Y=A(\overline{A}+B)$

or $Y=A\overline{A}+AB$

or $Y=AB$ since $A\overline{A} = 0$.

The simplified expression is $Y = AB$ which is simply an AND gate as shown.

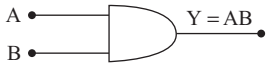


Figure 10.26

Example 10.30 Realize the logic function $Y = AB + CD$ using two inputs NAND gates only.

Solution:
We Write, $Y = AB + CD$

or $Y=\overline{AB+CD}$

or $Y=\overline{\overline{AB}\cdot\overline{CD}}$

The above expression is implemented using two-input NAND gates as shown.

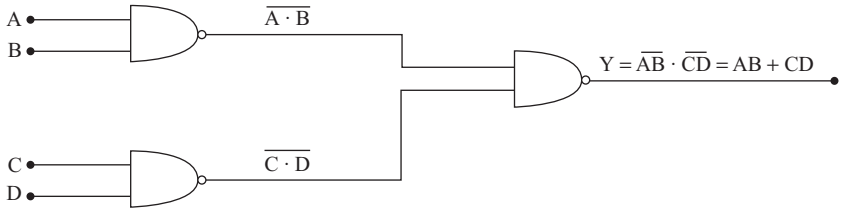


Figure 10.27

Example 10.31 Write the Boolean equation from the truth table given below and implement the logic equation through logic gates.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Solution:

From the truth table it is observed that the output Y is high, i.e 1 at five places. The sum-of-products equation is framed using the terms at the five input states as

$$Y=\overline{A}\,\overline{B}C+A\,\overline{B}\,\overline{C}+A\,\overline{B}C+AB\overline{C}+ABC$$

Now we will simplify the above expression using Boolean laws.

$$\begin{aligned}Y&=\overline{A}\,\overline{B}C+A\,\overline{B}\,\overline{C}+A\,\overline{B}C+AB\overline{C}+ABC\\&=\overline{A}\,\overline{B}C+A\,\overline{B}(\overline{C}+C)+AB(\overline{C}+C)\\&=\overline{A}\,\overline{B}C+A\,\overline{B}+AB\ \ (\text{since } \overline{A}+A=1)\\&=\overline{A}\,\overline{B}C+A(\overline{B}+B)\\&=\overline{A}\,\overline{B}C+A\end{aligned}$$

or $$Y=A+\overline{B}\,\overline{C}\ \ (\text{since } A+\overline{A}B=A+B)$$

The logic gate implementation of the simplified expression $Y=A+\overline{B}C$ is shown in Fig. 10.28.

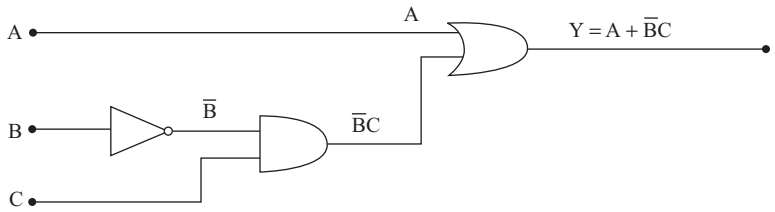


Figure 10.28

Example 10.32 Develop the truth table for the logic diagram shown in Fig. 10.29.

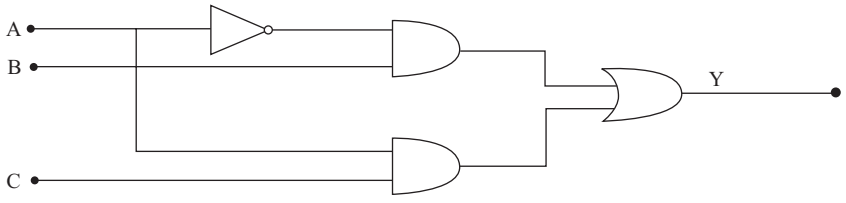


Figure 10.29

Solution:
The Boolean expression for the logic diagram will be as shown.

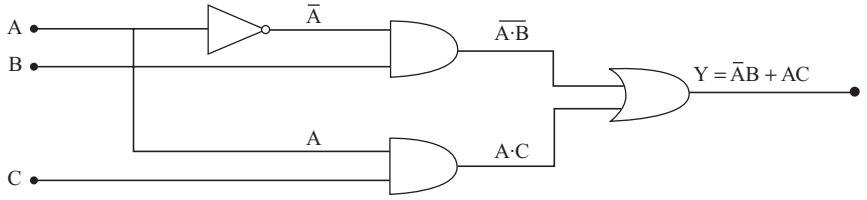


Figure 10.30

$Y=\overline{A}B+AC.$

The truth table is shown below.

A	B	C	$\overline{A}B$	AC	$Y=\overline{A}B+AC$
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	1	0	1
0	1	1	1	0	1
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	0	0	0
1	1	1	0	1	1

Example 10.33 A logic circuit has three inputs and one output. The output is 1 if any two of the three inputs are 1. The output is 0 for any other combination of the inputs. Draw the block diagram of this logic circuit.

Solution:
The truth table for the requirement stated is as shown.

A	B	C	Y	
0	0	0	0	
0	0	1	0	
0	1	1	1	$\overline{A}BC$
0	1	0	0	
1	1	0	1	$AB\overline{C}$
1	1	1	0	
1	0	1	1	$A\overline{B}C$
1	0	0	0	

The output as sum-of-products is

$$Y=\overline{A}BC+AB\overline{C}+A\overline{B}C$$

$$Y=B(\overline{A}C+A\overline{C})+A\overline{B}C.$$

The diagram for the logic circuit for this expression is shown in Fig. 10.31.

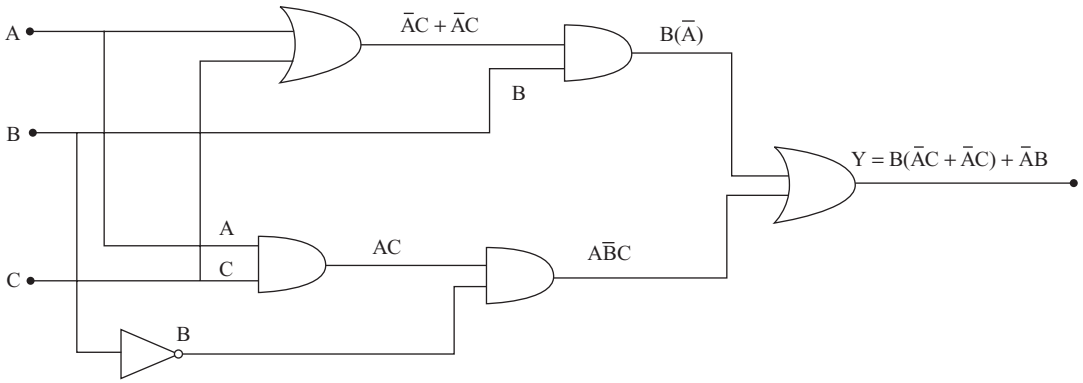


Figure 10.31

Example 10.34 Show how an OR gate can be implemented with AND and NOT gates.

Solution:

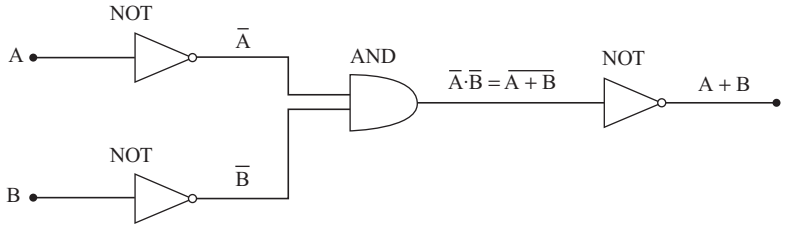


Figure 10.32

According to De Morgan's first theorem, $\overline{A}\cdot\overline{B} = \overline{A+B}$.
The above circuit gives the OR operation for the variables A and B.

Example 10.35 Four NOR gates are connected as shown with two inputs A and B. What will be the simplified expression for the output Y?

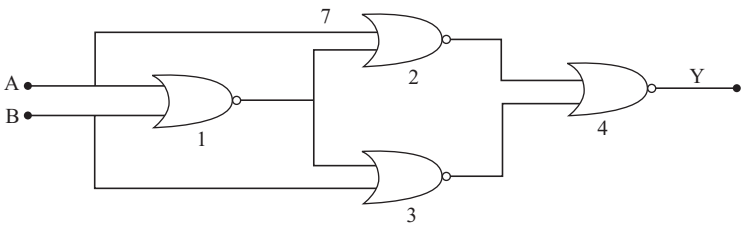


Figure 10.33

Solution:

Output of gate 1 is $\overline{A+B}$. Output of gate 2 is $\overline{A+\overline{A+B}}$. Output of gate 3 is $\overline{B+\overline{A+B}}$. Output of gate 4 is $Y=\overline{\overline{A+\overline{A+B}}+\overline{B+\overline{A+B}}}$

$$Y=\overline{A+\overline{A+B}}+\overline{B+\overline{A+B}}$$

$$Y=\overline{\overline{A}(A+B)+\overline{B}(A+B)}$$

$$Y=\overline{A\overline{A}+\overline{A}B+\overline{B}A+B\overline{B}} \quad (\therefore B\overline{B}=0\ A\overline{A}=0)$$

$$Y=\overline{\overline{A}B+\overline{B}A}$$

The output is the inverse of an EXOR gate, which is called an EXNOR or XNOR gate.
This shows that an XNOR gate can be realized using NOR gates

10.10 FLIP-FLOPS

In the previous section we have seen logic gates of combinational type where the output at a given time depends upon the inputs at that time. The previous records of inputs does not effect the output at all.

The output of a flip-flop is either a low voltage (0) or a high voltage (1). The output remains in one of these states indefinitely unless an external trigger is applied to change that state. Thus a flip-flop will retain or remember a state indefinitely after the removal of the input trigger. A flip-flop is considered a storing device of 1-bit memory. Since a flip-flop will have two stable states (0 or 1), it is also referred to as *bistable multivibrator*.

In most flip-flop circuits the change of state of the flip-flop is of a definite rate, and hence they are of clocked type.

10.10.1 RS Flip-flop

The RS flip-flop is used to temporarily hold or store information until it is required. A single RS circuit will store one binary digit, i.e., either 1 or 0. For storing a six digit binary number, six RS flip-flops will be required. As shown in Fig. 10.34, a RS flip flop which is a one-bit memory device has two inputs $\overline{R}$ and S. R stands for 'RESET' and S stands for 'SET'. The two output terminals are marked Q and $\overline{Q}$ One input will set the device and another will reset the device back to its original state. The output will either be at logic level 1 or 0 depending upon the set/reset condition. The simplest way to make a 1-bit set/reset RS flip-flop is to connect together a pair of NAND gates which are cross-coupled as shown in Fig. 10.17. The output $\overline{Q}$ is the inverse or complement of Q.

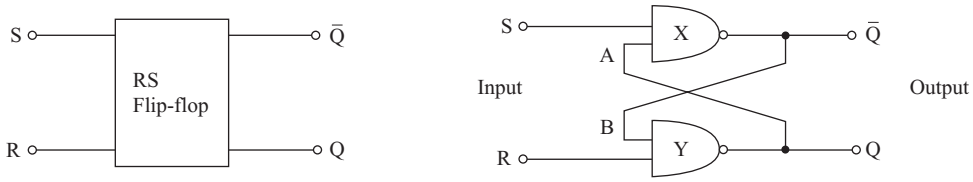


Figure 10.34 *RS flip-flop using NAND gates*

The truth table of a RS flip-flop has been shown

R	S	Q	$\overline{Q}$
0	0	Last value * *	(No change)
0	1	1	0 (Set)
1	0	0	1 (Reset)
1	1	invalid	

Application of a positive trigger at the input S will be setting the flip-flop. The output at Q will be 1 and at $\overline{Q}$ will be 0. The application of positive trigger at R will be resetting the flip-flop. The output at $\overline{Q}$ will be 1 and at Q will be 0.

For the set state, the input at R is at logic level 0 and input at S is at logic level 1, the output of NAND gate Y has atleast one of its inputs at logic level 0, and therefore the output Q must be at logic level 1. The output Q is fed back to input A so that both the inputs to the NAND gate X are at logic level 1. The output $\overline{Q}$ must be at 0 level.

For reset state, R = 1 and S = 0. The NAND gate X will have one of its inputs at level 0, and hence output at $\overline{Q}$ is 1. The flip-flop will function as follows.

(a) For R = 0, S = 0, i.e., with no input at the input terminals, the output will retain its previous state i.e, last value retained, i.e., output at Q may be 0 or 1.

(b) For R = 0, S = 1, i.e., trigger input is provided in the set input terminal, the output at Q will be 1.

(c) For R = 1, S = 0, Q = 0

(d) For R = 1, S = 1 the output is forced to become 0 and 1 simultaneously. Such input condition is called illegal or invalid forbidden input condition.

An RS flip-flop can also be created by two single input NAND gates N_1 and N_2 and two 2-input NAND gates N_3 and N_4 connected in a fashion as shown in Fig. 10.35.

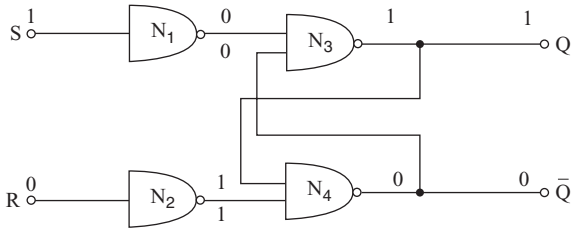
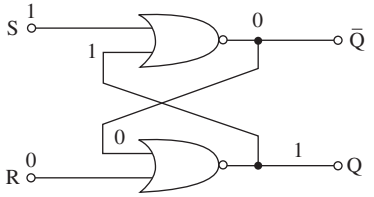


Figure 10.35 *RS flip-flop using combination of 1-input and 2-input NAND gates*

Flip-flop can also be formed by using 2-input NOR gates as shown in Fig. 10.36.



S	R	Q	$\overline{Q}$
0	0	No change	
0	1	0	1 Reset
1	0	1	0 Set
1	1	0 In	0 valid

Figure 10.36 *Flip-flop using NOR gates*

The circuit works in a similar way as in the case of NAND gate circuit. When both the inputs are at logic level 1, an invalid condition is created.

10.10.2 Gated or Clocked RS Flip-flop

Sequential circuits like an RS flip-flop are commonly synchronized with a train of pulses, called clock pulses. Clock pulses are generated by a master oscillator.

An RS flip-flop that has a clock input as shown in Fig. 10.37 is called a clocked RS flip-flop. The clock input is a square wave. When the clock is at low, the output at Q and $\overline{Q}$ will remain unchanged.

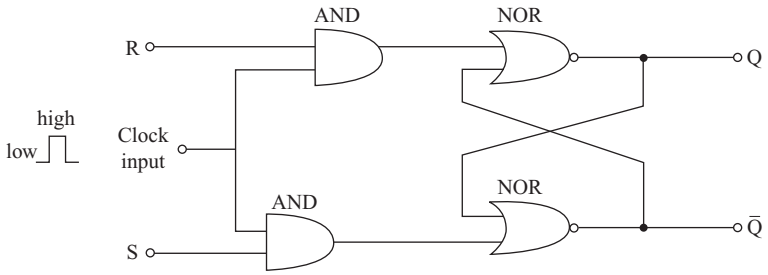


Figure 10.37 *Clocked RS flip-flop*

When the clock input is high (i.e., 1) the flop-flip will be set for R = 0 and S = 1 and will be reset for R = 1 and S = 0 as has been shown in the truth table.

Truth table for a clocked RS flip-flop

CLK	R	S	Q
0	0	0	No change
0	0	1	No change
0	1	0	No change
0	1	1	No change
1	0	0	No change
1	0	1	1 Set
1	1	0	0 Reset
1	1	1	Invalid *

Clocked RS flip-flop can also be made using 2-input gates as shown in Fig. 10.38.

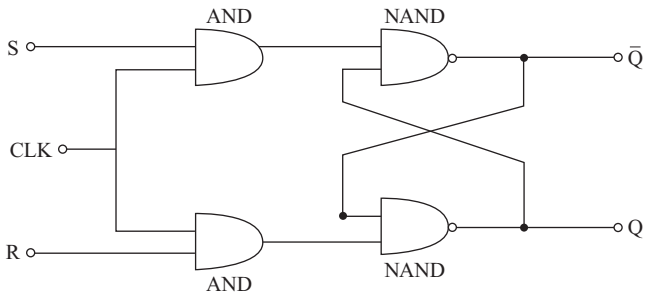


Figure 10.38 *Clocked SR flip-flop*

When the CLK input is at 0 level, the outputs of the two AND gates are also at logic level 0. The output of the Flip-flop at Q and $\overline{Q}$ will be uneffected and will be latching at the last known state. With a high clock input (i.e., 1) the flip-flop is set and reset as has been shown in the truth table. Synchronization of the clock timing signal to the flip-flop creates what is sometimes called a clocked SR flip-flop.

10.10.3 JK Flip-flop

JK flip-flop is the modified version of RS flip-flop with no invalid or illegal output state (see state marked invalid * in the truth table of RS flip-flop). The two inputs of RS flip-flop of Fig. 10.34 is now replaced by 3-input AND gates. The third input of each gate is the feedback from the output Q and $\overline{Q}$ as shown in Fig. 10.39. In RS flip-flop the invalid state occurred due to the activation of both the inputs. Here, the two inputs are interlocked so that they cannot be activated simultaneously.

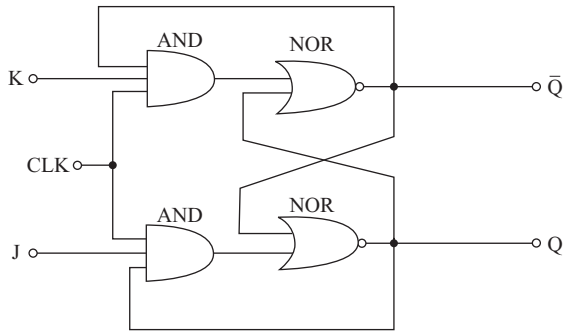


Figure 10.39 *JK flip-flop*

10.10.4 D Flip-flops

The output Q and $\overline{Q}$ of a D flip-flop change only on the positive going edge of the incoming clock pulse as shown in Fig. 10.40. If D = 1 and the positive going clock edge appears, then output Q = 1 and $\overline{Q} = 0$. When D = 0 and positive going pulse appears, then output Q = 0 and $\overline{Q} = 1$ For negative going edge of the incoming clock pulse, the flip-flop is inactive don't care.

For edge-triggered flip-flops, the clock signal is applied in the form of sharp positive and negative spikes instead of in the form of square or rectangular pulse train. Such sharp spikes are obtained from the rectangular pulse with the help of a passive differentiator circuit as has been shown in Fig. 10.40. Edge-trigger flip-flops can be of two types, viz positive edge triggered and negative edge triggered.

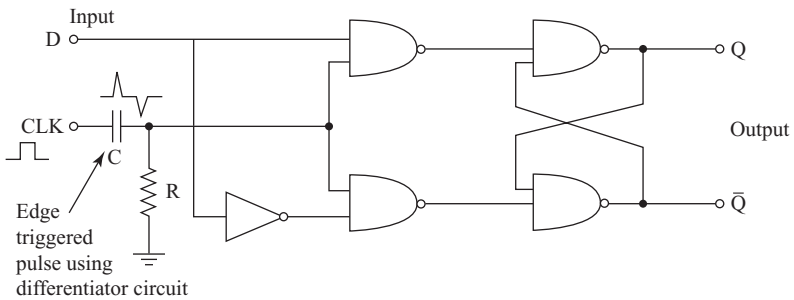


Figure 10.40 *D flip-flop using edge-triggered NAND gates*

10.10.5 T Flip-flops (Toggle Flip-flop)

These are basically JK flip-flops with both J and K terminals connected together permanently. Thus, such a flip-flop will have only one input terminal, T as has been shown in Fig. 10.41.

If T = 0, J = 0 and K = 0, the output Q and $\overline{Q}$ will remain unchanged. When T = 1, J = 1 and K = 1, the output will toggle corresponding to each and every edge of the clock signal.

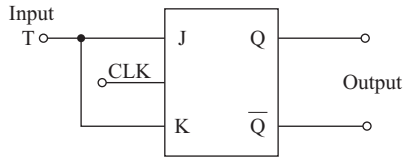


Figure 10.41 *A JK flip-flop converted into T flip-flop*

10.10.6 Master–Slave JK Flip-flop

Master-slave JK flip-flop as shown in Fig. 10.42 is a cascade connection of two flip-flops. The master is a clocked JK flip-flop and the slave is a RS flip-flop. The output of the second flip-flop is fed back to the input of the first. Positive clock pulses are applied to the first, called the master and same clock pulses after inversion are applied to the slave Flip-flop. When the clock pulse is at 1 (positive level) the master is active and the slave is inactive. When clock is at 0 (low level) the slave gets active and the master stays inactive

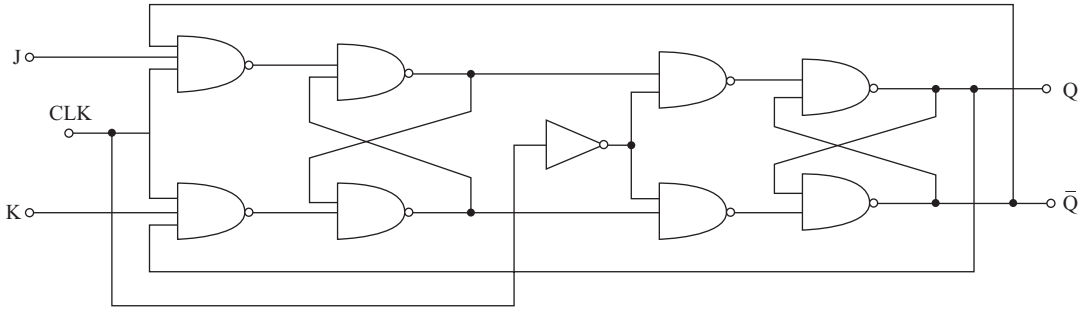


Figure 10.42 *Master–slaveJK flip-flop*

10.10.7 Counters and Shift Registers

Flip-flops are wired together that form circuits to do counting functions. Manufacturers make self-contained counters in IC form.

A register is a group of flip-flops used to store binary numbers. The output on each flip-flop is connected to the input of the adjacent input to form a register. The connection of the flip-flops are made in such a way that each data bits get transferred or shifts one flip-flop to the left or to the right. Flip-flops are classified the way data are entered and retrieved. For example, in serial-in serial-out registers, input data are applied one bit at a time to the first flip-flop in the form of a chain and is retrieved or read out from the last flip-flop in the form of a chain, one bit at a time. You must have seen the use of shift registers in your calculators. As we enter any digit on the key board, the numbers shift to the left on the display.

Taken, for example, what happens when we enter 65. We press 6 on the key board. 6 appears on the extreme right on the display. Next we press 5 on the key board. The number 6 gets shifted to the left creating space for number 5. This way the shift register carries out two functions, viz it holds or remembers the number 6 even after the pressure on the key board is released as it has a temporary memory, then it shifts the number to the left on the display each time we press a new digit or a character on the key board. The short time memory and shifting characteristics have made shift registers extremely useful in digital electronic systems.

10.10.8 Arithmetic Circuits

Combinational logic circuits are used to make adders and subtractors. In the central processing unit (CPU) of a computer, arithmetic functions are performed in a section called arithmetic logic unit (ALU). This section can perform functions like add, subtract, multiply, dvide, compare, complement, shift, and perform logic functions like AND, OR, and XOR.

10.10.9 Memory Function or Data Storage

You must have seen floppy disks and CD-ROMs. CD-ROM (compact disk read-only memory) is an optical storage device which can store many thousands of typed pages of information. Such optical disks can store much more data than floppy disks. Flip-flops form the basic memory cell.

Some semiconductor memory devices are RAM (random-axis memory), ROM (read-only memory), PROM (programmable read-only memory), EPROM (erasable PROM), etc. The hard disk drive is currently the most important large capacity storage (bulk storage) memory device used in computers these days.

10.10.10 Digital Systems

Like any other system a digital system consists of a number of subsystems assembled together to perform some specific task. In a digital system, the sub-system may be adders, subtractors, counters, shift registers, RAMs or ROMs. Subsystems are manufactured on a single chip of IC. Even the whole system is made available on a single IC chip. Such digital integrated systems are classified as SSI (small-scale integration), MSI (medium-scale integration), LSI (large-scale integration), VLSI (very-large- scale integration), and ULSI (utra-large-scale integration). A VLSI contains logic gates ranging from 10,000 to 99,999. Your digital wrist watch, for example, is an assembly of a number of digital subsytems on a single chip. The calculator we use, if we open it, we will notice, contains a small pencil cell, a mini read out display, key board from which few wires are attached to an IC chip. The IC is an LSI chip which is intended to perform thousands of logic functions consisting of storage, processing, and control that are required in a calculating system.

10.11 REVIEW QUESTIONS

A. Short Answer Type Questions

1. State the advantages of digital signals over analog signals.
2. Differentiate between analog and digital signals. What are the disadvantages of analog systems?
3. What are the different types of number systems? Explain each of them.

4. What is a binary number system. Mention its applications
5. Explain with example how binary numbers can be converted into decimal numbers.
6. Illustrate how decimal numbers can be converted into binary numbers.
7. Convert the following decimal numbers into binary numbers:
   (a) 49; (b) 756; (c) 0.578; (d) 110.58; (e) 109; (f) 18.33; (g) 212.
8. Convert the following fractional decimal numbers into binary numbers:
   (a) 0.782; (b) 0.359; (c) 0.568; (d) 0.075.
9. Convert the following binary numbers into decimal numbers:
   (a) 1101; (b) 1111; (c) 1001; (d) 1100101.
10. Convert the following fractional binary numbers to fractional decimal numbers:
    (a) 0.1111; (b) 0.1001; (c) 0.0101.
11. Add the following binary numbers:
    (a) 1011+1110; (b) 101010+111011.
12. Explain the difference between hexadecimal and octal systems.
13. Perform the following binary multiplication:
    (i) $(111)_2 \times (1.2)_2$; (ii) $(10101)_2 \times (1.1)_2$; (iii) $(10100)_2 \times (0.1)_2$.
14. State De Morgan's theorem
15. Write down truth tables for the following gates:
    (a) NAND gate; (b) AND gate; (c) NOR gate.
16. Explain a logic OR gate using diodes. Also write the truth table.
17. What are universal gates? Explain with an example.
18. What are flip-flops? Explain one of them.
19. Explain RS flip-flop. What is D flip-flop?
20. Explain JK flip-flop. What is T flip flop?
21. Draw and explain the circuit for master–slave JK flip-flop.
22. What is a digital system? Give an example of a digital system and name its subsystems.

B. Numerical Problems

23. Convert the binary number 1001 into decimal number, and the decimal number 19 into binary number.

    [Ans. 9,10011]

24. Convert decimal numbers 7547 and 100.625 into hexadecimal numbers.

    [Ans. $(7547)_{10} = (107\ B)_{16}$; $(100.625)_{10} = (64.A)_{16}$]

25. Convert the following hexadecimal numbers to binary.
    (a) AEO; (b) 75 F; (c) E 25.

    [Ans. (a) 101011100000; (b) 011101101111 (c) 111000100101]

26. Convert the following octal numbers into binary.
    (a) 71; (b) 560; (c) 173.

    [Ans. (a) 111001; (b) 101110000; (c) 001111011]

27. Perform the following binary operations.
    (a) $(1011)_2 \times (1001)_2$; (b) $(111)_2 \times (190)_2$.

    [Ans. (a) 1100011; (b) 101010]

28. Express the decimal number 9 into (a) binary; (b) BCD; (c) octal; (d) hexadecimal.

    [Ans. (a) 1001; (b) 1001; (c) 11; (d) 9]

29. Add the hexadecimal numbers 3E91 and 2F93.

    [Ans. 6E 24]

30. Solve the following conversions.
    (a) $(110001)_2 = (?)_{10}$; (b) $(23.6)_{10} = (?)_2$; (c) $(\text{BCA } 3.\text{AD})_{16} = (?)_2$.

    [Ans. (a) 49; (b) 10111.1001100; (c) 10111100101000011.10101101]

31. Establish the following identities of Boolean algebra.
    (i) $A = (A + B) = A$ (ii) $\overline{A}B + \overline{A} + AB = 1$
    (iii) $A + AB = A$ (iv) $\overline{\overline{A + B}} + \overline{\overline{A}\,\overline{B}} = A$

32. Simplify the function $Y = \overline{A}\,\overline{B}C + A\overline{B}C + \overline{A}B\overline{C}$ and draw the logic circuit.

    (Ans. $Y = \overline{B}C + \overline{A}B\overline{C}$)

33. Implement the Boolean expression $Y = (A + B)\left(\overline{A} + \overline{B}\right)$ in the form of a logic diagram. Frame the truth table and show that the logic diagram is equivalent to an EXOR gate.

34. Verify the following Boolean identities:
    (i) $ABC + A\overline{B}C + AB\overline{C} = AC + AB\overline{C}$
    (ii) $AB + A\overline{B} = A$
    (iii) $A\left(\overline{A} + B\right) = AB$

35. Draw the logic circuit for the Boolean expression, $V = A(\overline{A} + B)$ after simplification.

    [Ans. Y = A B]

36. Simplify the logic function $Y = \overline{A}BC + A\overline{B}C + \overline{A}B\overline{C}$ and draw the logic circuit.

    $\left(\text{Ans. } Y = \overline{B}C + \overline{A}B\overline{C}\right)$

C. Multiple Choice Questions

1. A NOR gate is equivalent to
   (a) inverters connected to the output of an AND gate
   (b) an inverter connected to the inputs of a NAND gate
   (c) inverters connected to the inputs of an OR gate
   (d) an inverter connected to the output of an OR gate.

2. The sum of binary 10011 and 0111 is
   (a) 01011 (b) 11010
   (c) 11101 (d) 10111.

3. Which of the following statements is true according to De Morgan's theorems?
   (a) $\overline{A + B} = \overline{A} \cdot \overline{B}$ (b) $\overline{A} + \overline{B} = \overline{A} \cdot \overline{B}$
   (c) $A + B = \overline{A} \cdot \overline{B}$ (d) $\overline{A \cdot B} = \overline{A} \cdot \overline{B}$.

4. Connecting of inverters at all the inputs of an AND gate produces a
   (a) NAND gate (b) OR gate
   (c) NOR gate (d) XOR gate.

5. Which of the following gates is represented by the Boolean expression: A + B + C + D = Y
   (a) 4-input AND gate (b) 4-input OR gate
   (c) 4-input NAND gate (d) 4-input NOR gate.
6. Which of the following is the Boolean expression for a 3-input NAND gate?
   (a) $\overline{A \cdot B \cdot C} = Y$ (b) $\overline{A + B + C} = Y$
   (c) $\overline{A} \cdot \overline{B} \cdot \overline{C} = Y$ (d) $A \cdot B \cdot C = Y$.
7. Which flip-flop has only one data input?
   (a) RS flip-flop (b) D flip-flop
   (c) JK flip-flop (d) Triggering flip-flop.
8. Which of the following can be called an universal flip flop?
   (a) JK flip-flop (b) D flip-flop
   (c) RS flip-flop (d) Clocked RS flip-flop.
9. In a JK flip-flop, repeated clock pulses causes the output to turn on-off-on-off—when
   (a) both inputs J and K are at 0 (b) J is at 0 and K is at 1
   (c) K is at 0 and J at 1 (d) both inputs J and K are at 1.
10. The master–slave JK flip-flop is an example of
    (a) level-triggered device (b) positive edge-triggered device
    (c) negative edge-triggered device (d) pulse-triggered device.
11. An astable multivibrator
    (a) generates a continuous flow of pulses (b) is a flip-flop
    (c) generates a single short pulse (d) always is in one of the two stable states.
12. A monstable multivibrator is also called a
    (a) flip-flop (b) clock
    (c) one shot multivibrator (d) free running multivibrator.

Answers to Multiple Choice Questions

1. (d)	2. (b)	3. (a)	4. (c)	5. (b)	6. (a)
7. (b)	8. (a)	9. (d)	10. (d)	11. (a)	12. (c)

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Solved Question Paper

West Bengal University of Technology
Basic Electrical and Electronics Engineering-II
CS/B.Tech. New/Semester 2
Subject Code: ES-201/2011

Time: 3 hours **Full marks: 70**

Part I (EE)

Group A

(Multiple Choice Questions)

1. Choose the correct alternatives for any five of the following: **5 × 1 = 5**

(i) If a balanced load is connected across a balanced supply, the sum of the three-phase current is
  (a) zero
  (b) $I_A - I_B + I_C$
  (c) $-I_A - I_B - I_C$
  (d) $I_C - I_A - I_B$.

(ii) If the field of a dc shunt motor is opened:
  (a) it will continue to run at its rated speed
  (b) the speed of the motor will become very high
  (c) the motor will stop
  (d) the speed of the motor will decrease.

(iii) In a transformer connected to constant voltage source, an increase in secondary load current will ultimately
  (a) reduce the mutual flux in the core
  (b) increase the mutual flux in the core
  (c) not cause any change in the magnitude of mutual flux
  (d) decrease the mutual flux when the secondary current is below 0.8 power factor lag.

(iv) The power drawn by a three-phase induction motor when first run in star connection and then delta is
  (a) same
  (b) $\sqrt{3}$ times that of star
  (c) 3 times that of star
  (d) $\frac{1}{\sqrt{3}}$ times that of star.

(v) Frequency of the induced EMF in the rotor circuit is
  (a) maximum at standstill
  (b) maximum at synchronous speed
  (c) zero at standstill
  (d) none of these.

(vi) In the application of Gauss's law, the correct statement is/are
  (a) Gauss's law depends on symmetry. If symmetry does not exist, we cannot use Gauss's law
  (b) for closed surfaces $\vec{D}$ is either normal or tangential
  (c) $\oint_s \vec{D}.d\vec{s} = \int_\upsilon \nabla.\vec{D}\, d\upsilon$
  (d) (a), (b) and (c).

Group B

(Answer any two of the following) **2 × 5 = 10**

2. A positive charge Q is uniformly distributed over an imaginary, thin spherical shell of radius a in air. Using Gauss's law, calculate the field intensity at any point inside and outside the shell. What shall be the field intensity at the surface of the shell?
3. Derive speed–torque characteristics of a dc series motor. Discuss its applications.
4. Explain why the rotor of an induction motor rotates in the same direction as the stator magnetic field. Why is the speed of the motor less than the synchronous speed?
5. Draw and explain the phasor diagram of a single-phase transformer under loaded condition.

Group C

(Answer any two of the following) **2 × 10 = 20**

6. (a) Explain the conditions required for the self-excitation of a dc generator.
   (b) A shunt generator delivers 200 A at 250 V. The armature resistance and shunt field resistance are 0.02 Ω and 50 Ω, respectively. The iron and friction losses are equal to 950 W. Determine:
     (i) EMF generated
     (ii) Copper losses
     (iii) Output of the prime mover
     (iv) Efficiency of the generator **2 + 8**
7. (a) A single-phase 50 Hz core-type transformer has a core of cross-sectional area 400 sq. cms. The permissible maximum flux density is 1 Wb/m^2. Calculate the number of turns on the high and low voltage sides for a 3,000/220 V ratio.
   (b) Determine the full-load efficiency at unity power factor for a 4 kVA 200/400 V, 50 Hz single-phase transformer of which the following are test figures.

   OC test: 200 V, 0.8 A, 70 W
   SC test: 17.5 V, 9 A, 50 W **4 + 6**
8. (a) A six-pole, 3-ϕ, 50 Hz induction motor develops a full-load torque of 150 Nm when the rotor current makes 120 complete cycles per minute. Determine the shaft power output.
   (b) A three-phase, 50 Hz, four-pole, 400 volt induction motor has the following data referred to stator side:

$$R_1 = 0.2\ \Omega,\ R_2 = 0.15\ \Omega,\ X_1 = j0.5\ \Omega,\ X_2 = j0.3\ \Omega$$

The motor operates at 3 per cent slip and the total losses including copper and constant losses are 2,000 watt. The core effect may be neglected. Determine:

(i) The stator current
(ii) Power factor
(iii) Efficiency of the motor **4 + 6**

9. (a) Explain the following terms:
(i) Electronic field
(ii) Electric field
(iii) Intensity
(iv) Potential difference

(b) A balanced three-phase star-connected load is fed from a 208 volt, three-phase supply. Each leg of the load has a resistance of 35 Ω. Determine the power factor, the total power, the phase currents and line currents of the system. **4 + 6**

Part II (ECE)

Group A

(Multiple Choice Questions)

1. Choose the correct alternatives for any five of the following: **5 × 1 = 5**

(i) The action of JFET in its equivalent circuit can best be represented as a
(a) current-controlled current source
(b) current-controlled voltage source
(c) voltage-controlled voltage source
(d) voltage-controlled current source.

(ii) The effective channel length of a MOSFET in saturation decreases with the increase in
(a) gate voltage
(b) drain voltage
(c) source voltage
(d) body voltage.

(iii) A source follower using an FET usually has a voltage gain which is
(a) greater than + 100
(b) slightly less than unity but positive
(c) exactly unity but negative
(d) about –10.

(iv) The ideal op-amp has which of the following characteristics?
(a) $R_i = \infty, A = \infty, R_o = 0$
(b) $R_i = 0, A = \infty, R_o = 0$
(c) $R_i = \infty, A = \infty, R_o = \infty$
(d) $R_i = 0, A = \infty, R_o = \infty$.

(v) For self-sustained oscillations the condition is
(a) $A\beta < 1$
(b) $A\beta > 1$

(c) $A\beta = 1$
(d) both (b) and (c).

(vi) Which one of the following feedback topologies offers high input impedance?
(a) Voltage series
(b) Voltage shunt
(c) Current series
(d) Current shunt

Group B

(Answer any two of the following) **2 × 5 = 10**

2. Identify the circuit and find out the output voltage V_o for the circuits given below:
(a)

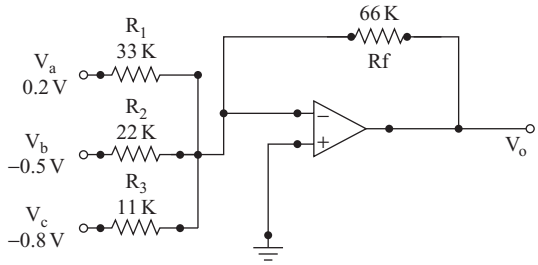

Figure 1

(b)

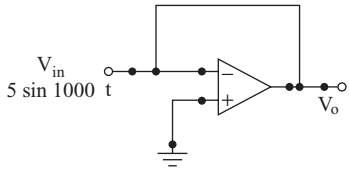

Figure 2

3 + 2

3. (a) Perform the following number conversions: **2 + 3**
(i) $(11101.1011)_2 = (?)_2$
(ii) $(275.46)_{10} = (?)_2$.
(b) Realize the Boolean expression using basic gates:

$$y = \left(\bar{A}BC\bar{E}\right) + \left(A\bar{B}CDE\right) + \left(\bar{A}\bar{C}\bar{D}\right)$$

4. Deriving proper expressions, explain the effect of positive feedback gain, input impedance, output impedance, stability and bandwidth. **5 × 1**

5. (a) State the Barkhausen criterion of sinusoidal oscillations.
(b) Compare and contrast BJT with FET. **2 + 3**

Group C

(Answer any two of the following) **2 × 10 = 20**

6. (a) What do you mean by negative feedback?
   (b) Why is it used in designing an amplifier?
   (c) The variation of the open-loop gain of an amplifier having internal gain 1,000 is 10 per cent, but for a specific use only one per cent gain variation is allowed. Design a feedback amplifier for this purpose and find the corresponding feedback fraction and overall gain. **2 + 3 + 5**
7. (a) What is the relation between JFET parameters? **3 + 4 + 3**
   (b) Draw and explain the drain characteristics of an n-channel enhancement-type MOSFET.
   (c) Calculate the drain current at a gate voltage of – 2 V for an n-channel JFET, which has saturation drain current at zero gate, bias of 10 mA and pinch-off voltage of – 3.5 V.
8. (a) What is an integrated circuit? What are its advantages?
   (b) Draw and explain the principle of operation of an op-amp as an adder. **(2 + 2) + 6**
9. Write short notes on any two of the following: **2 × 5**
   (a) Comparator
   (b) Universal gates
   (c) Pinch-off condition of JFET
   (d) CMRR

Solutions (EE Part)

Group A

1. (i) The three-phase currents will be equal but have a phase difference of 120°. When represented as phasors, the sum of the three equal phasors displaced at 120° will be zero. So, the answer is (a).
   (ii) The speed of a dc motor is expressed as

$$N = \frac{V - I_a R_a}{K\varphi}$$

If the shunt field is opened, ϕ will tend to zero and hence the speed of the motor will become very high. So, the answer is (b).

   (iii) When the secondary load current is increased, the transformer primary will draw additional current from the mains so that $I_2N_2 = I_1N_1$. There will not be any change in the magnitude of mutual flux. So, the answer is (c).
   (iv) The power drawn in both the cases is $\sqrt{3}\ V_L I_L \cos\varphi$. Hence, the correct answer is (a).
   (v) At standstill, the frequency of induced EMF in the rotor will be the same as that of the stator supply frequency. So, the answer is (a).
   (vi) The answer is (d).

Group B

2.

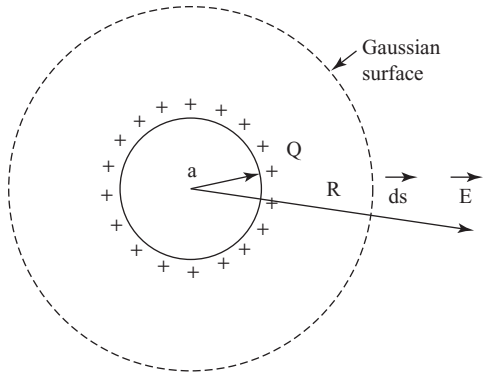

Figure 3

For outside the surface:

$$\oint_s \vec{E} \cdot \vec{ds} = \frac{Q}{\varepsilon_o}$$

$$\oint_s Eds\cos 0^o = \frac{Q}{\varepsilon_o}$$

$$E\oint_s ds = \frac{Q}{\varepsilon_o}$$

$$E(4\pi R^2) = \frac{Q}{\varepsilon_o}$$

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{Q}{R^2}$$

Let ρ_s be the surface change density.

$$\rho_s = \frac{Q}{4\pi a^2} \text{ or } Q = 4\pi a^2 \rho_s$$

$$E = \frac{1}{4\pi\varepsilon_o} \times \frac{4\pi a^2 \rho_s}{R^2} = \frac{\rho_s}{\varepsilon_o} \times \frac{a^2}{R^2}$$

For field intensity on the surface, put R = a. Therefore,

$$E = \frac{\rho_s}{\varepsilon_o}$$

Field intensity inside the sphere is zero because no charge will be enclosed by any Gaussian surface if considered inside the sphere.

3. We know that the torque developed in a dc motor is due to the interaction of a current-carrying conductor in a magnetic field. As such torque, T, is expressed as

$$T = K\phi I_a$$

where ϕ is the flux and I_a is the current. In a series motor the field winding and the armature winding are connected in series. So, $I_f = I_a$ since I_f produces ϕ, we can write, $\phi \propto I_f \propto I_a$.

Therefore,

$$T = K K_1 I_a^2$$

$$T \propto I_a^2$$

It can be observed from Fig. 4 that the series motor attains a dangerously high speed at low or no-load. For this reason, a series motor is always coupled with the load so that the motor can never be allowed to run on no-load.

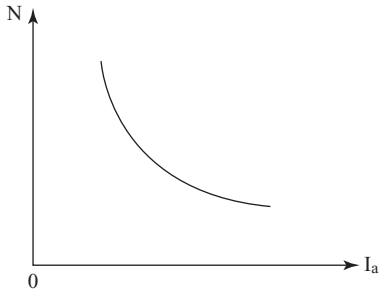

Figure 4

The load–torque characteristic is shown in Fig. 5(a).

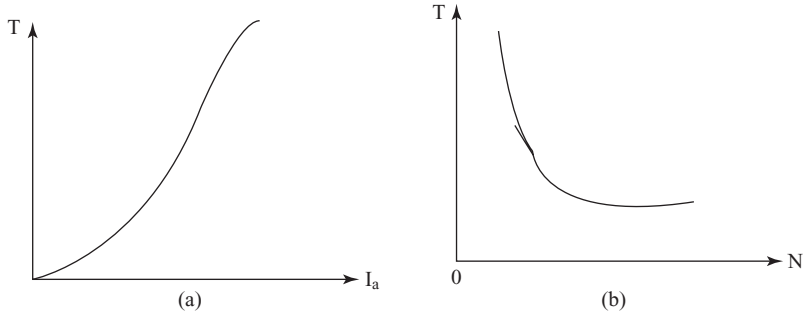

Figure 5

Figure 5(a) shows that the torque produced is proportional to the square of the armature current. The characteristic is parabolic in nature. The curve becomes some what linear after the magnetic saturation. Using N versus I_a characteristic and T versus I_a characteristic, the torque versus speed characteristic is drawn as shown in Figure 5(b).

It can be observed from Fig. 5(b) that a series motor develops high starting torque. Series motors are therefore suitable for applications where high starting torque is required to quickly accelerate heavy loads like in electric traction, cranes, elevators, etc.

4. A rotating magnetic field is produced when three-phase supply is applied to the three phases of the stator. The speed of this field is given as $N_s = \frac{120\,f}{p}$. The rotor is a closed winding. An EMF is induced in the rotor due to electromagnetic induction like in a transformer. Since the motor circuit is a closed winding, current will flow through the rotor winding and a magnetic field is produced.

The two fields are stationary with respect to each other. Torque is developed on the rotor due to interaction of these two fields

The rotor tends to rotate in the direction of rotation of the stator field. Now, according to Lenz's law, the developed torque must oppose the cause to which it is due, i.e. the cutting of flux lines by the rotor conductors. Therefore, the developed torque makes the rotor rotate in the same direction as the rotating stator field so as to reduce the relative speed between the stator flux and the rotor conductors. This way there is a reduction in the rate of the cutting of flux lines by the rotor conductors. The rotor speed can never be equal to speed N_s because at that speed the rotor conductors will be stationary with respect to the stator field. There will be no cutting of flux and no induced EMF in the rotor. So, the developed torque will be zero. The rotor speed of an induction motor, N_r, is always somewhat less than synchronous speed, N_s. The difference between N_s and N_r expressed in percentage of N_s is called the slip of the motor.

5. The equivalent circuit of a transformer and the phasor representation of all the quantities are shown in Fig. 6(a) and (b), respectively.

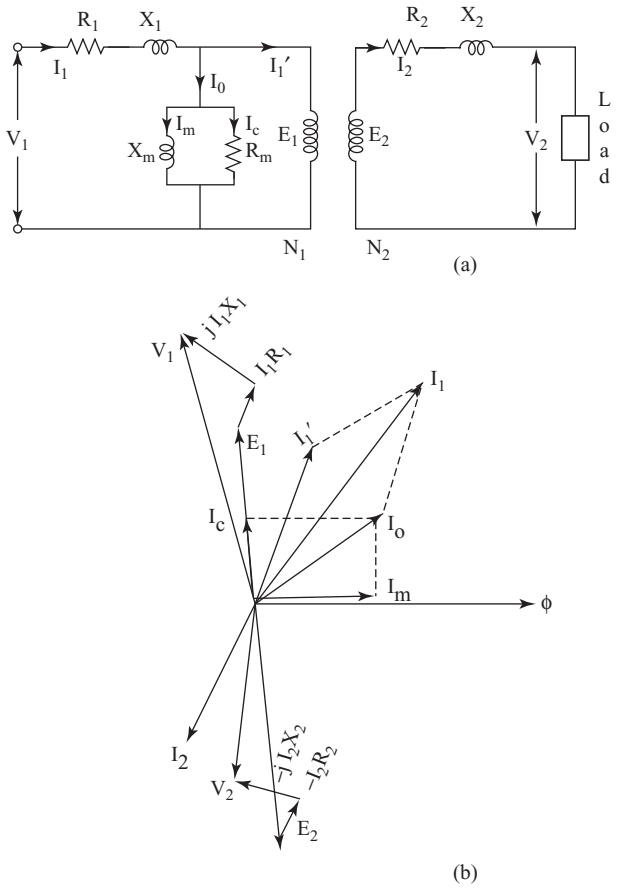

Figure 6

Explanation: Flux ϕ has been drawn on the reference axis. Flux is produced by the magnetizing current I_m. I_m has been drawn in phase with ϕ. I_c has been drawn in phase with E_1. The sum of I_m and I_c is shown as I_o. We have assumed the direction of I_2 as lagging E_2. E_2 is lagging flux ϕ by 90°. I_1' is the additional current drawn from the supply where $I_1' N_1 = I_2 N_2$ (ampere–turn balance). The sum of I_1' and I_o is I_1. Using the relation, $E_1 + I_1 R_1 + jI_1 X_1 = V_1$, the phasor V_1 has been drawn. $I_1 R_1$ has been drawn along the direction of I_1 and $j\, I_1\, X_1$ has been drawn in the perpendicular direction of I_1. To draw V_2, the relation $E_2 - I_2 R_2 - j\, I_2 X_2 = V_2$ has been used.

Group C

6. (a)

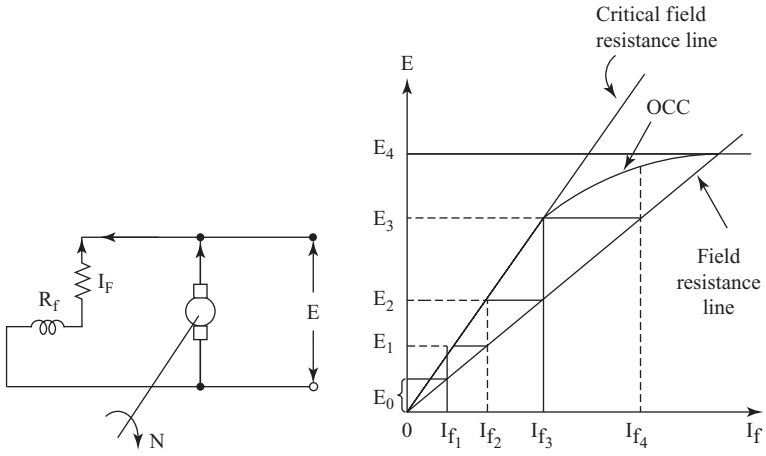

Figure 7

The mechanism of self-excitation of a dc generator has been illustrated in Fig. 7. E_o is the EMF induced due to residual magnetism. The voltage E_o will send a current I_{f1} through the field winding. This field current will produce more flux and an increased induced EMF of E_1. E_1 will cause current of I_{f2}. I_{f2} will produce a voltage of E_2 and this way voltage will build up to E_4 at which the OCC cuts the field resistance line. The value of field resistance should not be higher than the critical field resistance line. Critical field resistance is that value of the field circuit resistance at which the field resistance line becomes tangential to the open circuit characteristic.

The two essential conditions for self-excitation of a dc generator are:

(i) There must be residual magnetism present in the field pole system;

(ii) The value of field circuit resistance should be less than the critical field resistance.

6. (b)

$$I_f = \frac{V}{R_f} = \frac{250}{50} = 5 \text{ A}$$

$$E = V + I_a R_a$$

$$I_a = I_f + I_L = 5 + 200 = 205 \text{ A}$$

$$E = 250 + 205 \cdot 0.02$$

Or,
$$E = 250 + 4.1$$

$$E = 254.1 \text{ V}$$

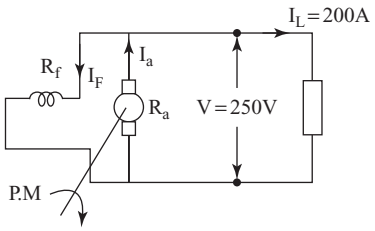

Figure 8

$$\text{Copper losses} = I_a^2 R_a + I_f^2 R_f$$
$$= (205)^2 \cdot 0.02 + 5^2 \cdot 50$$
$$= 840 \cdot 5 + 1250$$
$$= 2090.5\,\text{W}$$

Input to the generator = Output + losses

$$= V \cdot I_L + \text{Copper losses} + \text{Iron and friction losses}$$
$$= 250 \cdot 200 + 2090.5 + 950$$
$$= 53040.5\,\text{W}$$

$$\text{Output} = 250 \cdot 200 = 50000\,\text{W}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{50,000}{53,040.5} \cdot 100 = 94.26\%$$

Output of the prime mover (P.M) = Input to the generator

$$= 53040.5\,\text{W}$$
$$= \frac{53040.5}{735.5} = 72.1\ \text{HP}$$

7. (a)

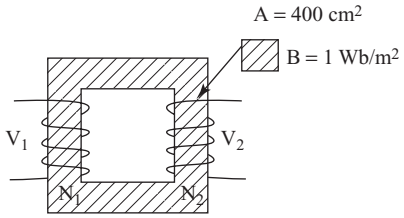

Figure 9

$$E_1 = 4.44\,\phi_m f\,N_1$$
$$E_1 = 4.44\,B_m \times A\,f\,N_1$$
$$E_2 = 4.44\,B_m \times A\,f\,N_2$$

Assuming $E_1 = V_1$ and $E_2 = V_2$

$$3,000 = 4.44\,B_m \cdot Af\,N_1$$

and $$220 = 4.44\, B_m \cdot\ Af\, N_2$$

$$N_1 = \frac{3,000}{4.44 \times 1 \times 400 \times 10^{-4} \times 50} = 338$$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1}$$

$$N_2 = \frac{220}{3,000} \cdot\ 338 = 25$$

7. (b) Transformer output = 4 KVA.

Output in KW = $4\,\text{KVA}\eta \times \cos\phi = 4\,\text{KVA}\eta \times 1 = 4\,\text{KW} = 4{,}000$ W.

$$\text{Efficiency}, \eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{losses}}$$

From OC test, core loss = 70 W

From SC test, at 9 A load, copper loss = 50 W.

Full-load current on HV side $= \dfrac{4 \cdot\ 1000}{400} = 10$ A.

So, the full-load copper loss at 10 A $= 50 \cdot\ (10/9)^2 = 61.6$ W

$$\eta = \frac{4,000 \times 100}{4,000 + 70 + 61.6} = \frac{4,000 \times 100}{4,131.6} = 96.8\%$$

8. (a)

$$P = 6$$
$$f = 50$$

$$N_s = \frac{120\ f}{\rho} = \frac{120 \times 50}{6} = 1,000\ \text{rpm}$$

Rotor current makes 120 cycles / min or 2 cycles / sec.

Therefore, $f_r = 2$

We know, $f_r = s.f$

$$S = \frac{f_r}{f} = \frac{2}{50} = 0.04$$

or, $$S = \frac{N_s - N_r}{N_s}$$

$$0.04 = \frac{1,000 - N_r}{1,000}$$

$$N_r = 960\,\text{rpm} = \frac{960}{60}\,\text{rps} = 16\,\text{rps}$$

$$\text{Shaft power output} = \frac{2\pi T N_r}{60}$$
$$= 6.28 \times 1.50 \times 16 \text{ W}$$
$$= 15{,}072 \text{ W}$$
$$= 15.072 \text{ kW}.$$

8. (b)

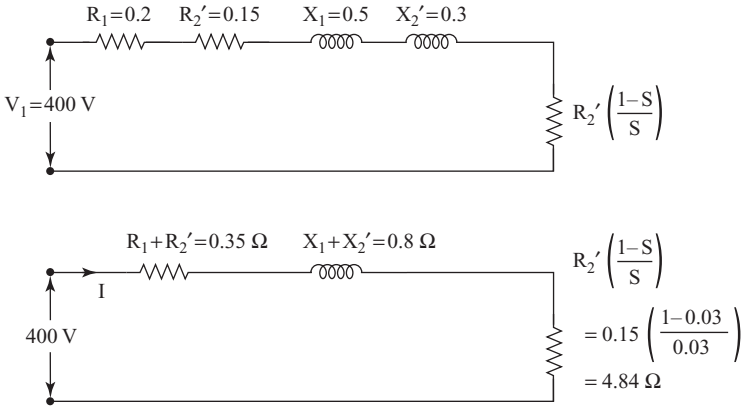

Figure 10

$$I_{ph} = \frac{V_{ph}}{z} = \frac{450}{\sqrt{3}\left[(0.35 + 4.84) + j0.8\right]}$$
$$= \frac{400}{\sqrt{3}(5.19 + j0.8)}$$
$$= \frac{400}{\sqrt{3}\sqrt{(5.19)^2 + (0.8)^2}\,\underline{\tan^{-1}\frac{0.8}{5.19}}}$$
$$= \frac{1}{\sqrt{3}} \times \frac{400}{5.25 \tan^{-1} 0.154}$$
$$= \frac{76}{\sqrt{3}}\underline{|-9^\circ} \text{ Amps.}$$
$$= 43.9\underline{|-9^\circ} \text{Amp}$$

Power factor, $\cos\varphi = \cos 9^\circ = 0.96$ lagging

For calculation of efficiency,

Power input $= 3 V_{ph} I_{ph} \cos\varphi$

$$= 3 \cdot \frac{400}{\sqrt{3}} \cdot 43.9 \cdot 0.96 = 692.8 \cdot 42.14$$
$$= 29{,}134 \text{ W}$$

Total losses = 2,000 W

Efficiency, $$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}}$$

$$= \frac{29{,}134 - 2000}{29{,}134} = \frac{27{,}134}{29{,}134} = 0.93$$

Percentage efficiency $= 0.93 \cdot 100 = 93$ per cent.

9. (a) **Electric field:** Electric field is defined as the region where forces act on charges. Let Q_1 and Q_2 be the two charges. The electric field around charge Q_1 is shown in Fig. 9. When Q_2 is moved closer to Q_1, force is acted on Q_2.

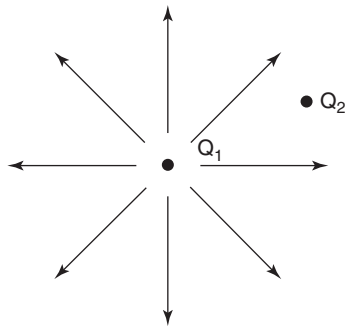

Figure 11

Electric field intensity: Electric field intensity is denoted by $\bar{E}$. It is defined as the force per unit charge. Let $\bar{F}_{12}$ be the vector force acting on Q_2 by Q_1. Q_2 is a positive test charge.

$$\bar{E} = \frac{\bar{F}_{12}}{Q_2} = \frac{K Q_1 Q_2}{Q_2 R^2} . \hat{a}_R = \frac{K Q_1}{R^2} \hat{a}_R ,$$

where $$K = \frac{1}{4 \pi \in_0}$$

Potential difference: When a body is charged, either electrons are supplied to it or they are removed from it. Work is done in both the cases. The work done is stored in the body in the form of electric potential. A charged body has the capacity to do work by exerting a force of attraction or repulsion on other charged particles. The capacity of a charged body to do work is called electric potential.

$$\text{Electric potential} = \frac{\text{Work done}}{\text{Charge}}$$

$$V = \frac{W}{Q}$$

The unit of work is joules, unit of charge is coulomb and the unit of potential is volt.

$$\text{Volt} = \frac{\text{Joules}}{\text{Coloumb}}$$

A body is said to have an electric potential of one volt if one joule of work is done to charge the body by one coulomb.

9. (b)

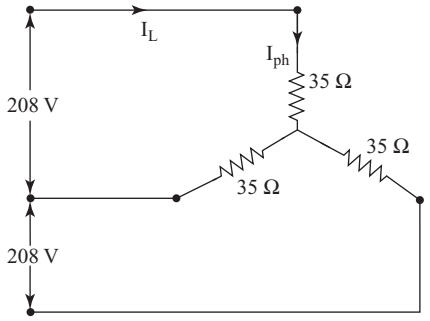

Figure 12

$$V_L = 208 \text{ V}, \; V_{ph} = \frac{208}{\sqrt{3}} = 120 \text{ V}$$

Phase current, $$I_{ph} = \frac{V_{ph}}{R} = \frac{120}{35} = 3.43 \text{ A}.$$

In a star-connected system, $I_L = I_{ph} = 3.43\text{A}$

Since the load is purely resistive, the power factor is unity (i.e equal to 1)

Total power consumed $= 3 V_{ph} I_{ph} \cos\varphi$

$= 3 \cdot 120 \cdot 3.43 \cdot 1$

$= 1234.8 \text{ W}$

Solutions (ECE Part)

Group A

1. The correct alternatives are as follows:
   (i) (d)
   (ii) (a)
   (iii) (b)
   (iv) (a)
   (v) (c)
   (vi) (a)

Group B

2. (a)

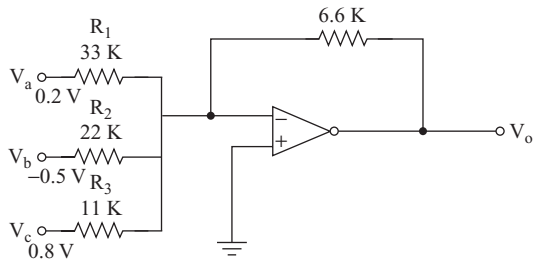

Figure 13

Figure 13 shows the circuit for a scaling adder. Different weights have been assigned to each input.

The output voltage V_0 is given as

$$V_0 = -\left[\frac{R_f}{R_1}(V_a) + \frac{R_f}{R_2}(-V_b) + \frac{R_f}{R_3}(V_c)\right]$$
$$= -\left[\frac{6.6}{33}\times 0.2 + \frac{6.6}{22}(-0.5) + \frac{6.6}{11}\times 0.8\right]$$
$$= -\left[0.2\times 0.2 + 0.3\times(-0.5) + 0.6\times 0.8\right]$$
$$= -\left[0.04 - 0.15 + 0.48\right]$$
$$= -0.37\,\text{V}$$

2. (b)

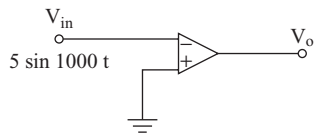

Figure 14

This is a non-inverting op-amp where the closed-loop gain is,

$$A_{vf} = \frac{V_0}{V_{in}} = 1 + \frac{R_f}{R}$$

If $R_f = 0$, i.e. if R_f is short-circuited,

$$V_0 = V_{in}$$

The output exactly follows the input and hence the circuit is called a voltage follower.

3. (a) (i) $(11101.1011)_2 = (?)_{10}$

$$\left[1\times 2^4 + 1\times 2^3 \times 1\times 2^2 + 0\times 2^1 + 1\times 2^0\right].\left[1\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3} + 1\times 2^{-4}\right]$$
$$= \left[16 + 8 + 4 + 0 + 1\right].\left[\frac{1}{2} + 0 + \frac{1}{8} + \frac{1}{16}\right]$$
$$= 29.\frac{8+2+1}{16}$$
$$= (29.6875)_{10}.$$

(ii) $(275.46)_{10} = (?)_2$

2	275	
2	137	1
2	68	1
2	34	0
2	17	0
2	8	1
2	4	0
2	2	0
	1	0

$0.46 \cdot 2 = 0.92$ 0
$0.92 \cdot 2 = 1.84$ 1
$0.84 \cdot 2 = 1.68$ 1
$0.68 \cdot 2 = 1.36$ 1
$0.36 \cdot 2 = 0.72$ 0
$0.72 \cdot 2 = 1.44$ 1

Therefore,

$$(275.46)_{10} = (100010011.011101)_2.$$

3. (b)

$$Y = (\bar{A}BC\bar{E} + (A\bar{B}CDE) + (\bar{A}\bar{C}\bar{D})$$

The above Boolean expression is realized using basic gates as

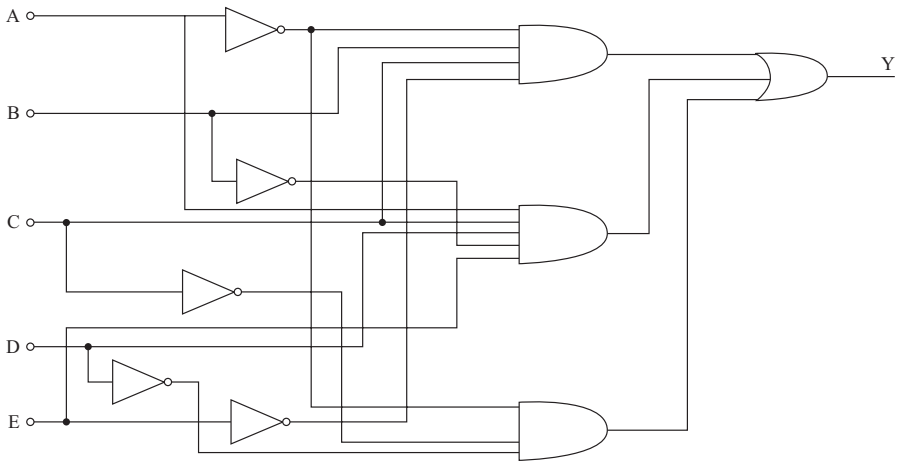

Figure 15

4. Effect of positive feedback on gain, input impedance, output impedance, stability and bandwidth

(i) **Effect on gain:**

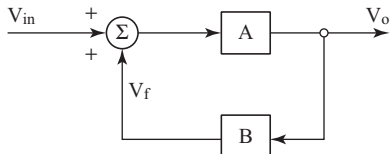

Figure 16

Closed-loop voltage gain:

$$A_f = \frac{V_0}{V_{in}}$$

$$\beta = \frac{R_1}{R_1 + R_f}$$

$$V_f = \beta V_o$$

$$= \frac{R_1 V_o}{R_1 + R_f}$$

$$= V_o = A(V_{in} + V_f)$$

or
$$V_o = A\left(V_{in} + \frac{R_1 V_o}{R_1 + R_f}\right)$$

or,
$$V_0(R_1 + R_f) = A(V_{in})(R_1 + R_f) + AR_1V_0$$

$$V_{in} = \frac{A(R_1 + R_f)}{R_1 + R_f}$$

$$A_f = \frac{V_0}{V_{in}} = \frac{\dfrac{A(R_1 + R_f)}{R_1 + R_f}}{\dfrac{R_1 + R_f}{R_1 + R_f} - \dfrac{AR_1}{R_1 + R_f}} = \frac{A}{1 - A\beta}$$

Where β is the gain of feedback circuit, $A\beta$ is the loop gain, A_f is the closed-loop voltage gain and A is the open-loop voltage gain.

(ii) **Effect on input impedance**:
The input impedance of an op-amp with positive feedback is $\frac{1}{(1 + A\beta)}$ times its value without feedback.

(iii) **Effect on output impedance**:
The output impedance with feedback is $(1 + A\beta)$ times the output impedance of the op-amp.

(iv) **Effect on bandwidth:**

f_0 with feedback is $\frac{1}{1 + A\beta}$ times its value without feedback.

(v) **Effect on stability:**

Overall gain of the amplifier is increased by positive feedback. However, positive feedback causes instability in the gain of the amplifier.

$$A_f = \frac{A}{1 - A\beta} \text{ or } \frac{dA_f}{A_f} = \frac{1}{1 - A\beta}\frac{dA}{A}$$

$$\text{Sensitivity, } S = \left(\frac{dA_f}{A_f}\right) \Big/ \frac{dA}{A} = \frac{1}{1 - A\beta} \text{ ; D (Desensitivity)}$$

$$= \frac{1}{S} = 1 - A\beta$$

D is less than 1 for positive feedback. Overall gain stability is decreased.

5. (a) In an oscillator the oscillations will not be sustained if, at the oscillator frequency, the magnitude of the product of the transfer gain of the amplifier and the magnitude of the feedback factor of the feedback network, i.e. the magnitude of the loop gain is less than unity.

The condition of unity loop gain $A\beta = 1$ is called the Barkhausen Criterion. This condition implies that $|A\beta| = 1$ and that the phase of $-A\beta$ is zero.

The conditions of Barkhausen Criterion for oscillation are (i) positive feedback; (ii) loop gain is unity, $A\beta = 1$ and feedback gain is $A_f = \infty$ and (iii) phase variation is zero.

5. (b) BJTs can be compared with FETs as follows:

1. Only the carriers of one type, i.e. electrons or holes are involved in the operation of FETs. However, in BJT, both types of carriers are involved.
2. In FETs, the carriers are transported through the channel by the drift process, whereas in bipolar transistors, the carriers move using the base by the process of diffusion.
3. FETs are thermally more stable than BJTs.
4. The FET is a voltage-controlled device, whereas BJT is a current-controlled device.
5. FETs offer high input impedances when compared to BJTs.
6. FETs are less noisy than BJTs.
7. FETs are simple to fabricate and require less space in the IC configurations.
8. At audio frequencies, FETs offer a large power gain than BJTs.

Group C

6. (a) **Negative feedback:** If the feedback signal and the original input signal are 180° out of phase with respect to each other, then it is called negative feedback. Negative feedback is used in all amplifiers as shown in Fig. 17.

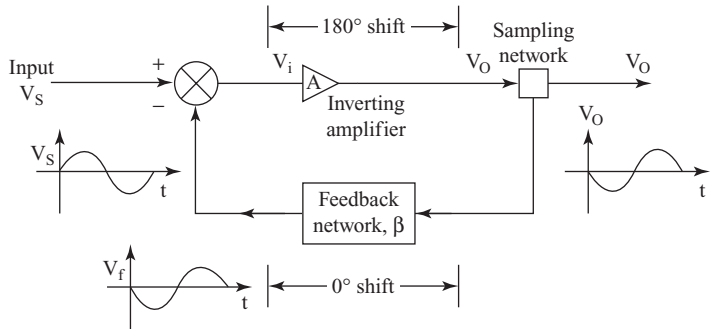

Figure 17

6. (b) Negative feedback is used in designing an amplifier because of the following reasons:

(i) **Gain stability:** The voltage gain of an amplifier with negative feedback is given as $A_f = \frac{A}{1+A\beta}$

If $A\beta >> 1$, then the expression becomes $A_f = \frac{1}{\beta}$, i.e. the overall gain of the feedback amplifier A_f becomes independent of the internal gain and depends only on the feedback ratio β. The value of β depends on passive elements such as resistors. The value of resistors remains more or less constant and hence the gain is stabilized.

(ii) **Reduced non-linear distortion:** For large signals non-linear distortion is reduced by a factor $(1+A\beta)$ when negative feedback is used.

(iii) **Reduced noise:** There is always a noise voltage in the amplified signal, which gets reduced by a factor $(1+A\beta)$ when negative feedback is used.

(iv) **Increased input impedance:** The input impedance with negative feedback is increased by the factor $(1+A\beta)$, which is desirable.

(v) **Reduced output impedance:** The output impedance is reduced by the factor $(1+A\beta)$which is desirable.

6. (c) Given open-loop gain of the amplifier, A = 1,000

Gain of a feedback amplifier, $A_f = \frac{A}{1+A\beta}$

$$\frac{dA_f}{A_f} = \frac{1}{1+A\beta}\ \frac{dA}{A},$$

or, $$1+A\beta = \frac{dA/A}{dA_f/A_f}$$

given, $$\frac{dA}{A} = 10\% \text{ and } \frac{dA_f}{A_f} = 1\%$$

Therefore, $$1+A\beta = \frac{10}{1} = 10$$

$$\text{or, } A\beta = 10-1 = 9$$

$$\beta = \frac{9}{A} = \frac{9}{1,000} = 0.009$$

and A_f = gain with feedback or overall gain.

$$A_f = \frac{A}{1+A\beta} = \frac{1,000}{1+1,000\times 0.009}$$

or, $$A_f = \frac{1,000}{10} = 100.$$

7. (a) **JFET parameters:** For small-signal applications, the ac component of drain current i_d can be expressed as a linear combination of the ac component of the gate voltage v_{gs} and the ac component of the drain voltage v_{ds}. So,

$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_d} \qquad \text{(i)}$$

The parameter g_m is referred to as the mutual conductance or the transconductance. g_m is expressed in siemens (s) or mho (Ω^{-1}) or A/V. It is defined as the ratio of the change in drain current to the corresponding change in the gate source voltage for a constant drain-source voltage.

$$g_m = \frac{i_d}{v_{gs}} \text{ at constant } V_{ds}$$

g_m is given by the slope of the transfer characteristic.

The parameter r_d is known as the drain or channel or output resistance and is expressed in ohm.

The parameter r_d is expressed as

$$r_d = \frac{v_{ds}}{i_d}\Big|v_{gs} = 0.$$

Amplification factor μ is the third parameter. It is expressed as

$$\mu = -\frac{v_{ds}}{v_{gs}}\Big|_{i_d=0}.$$

The negative sign arises because v_{ds} and v_{gs} are of opposite signs for $i_d = 0$.

The equation (i) with $i_d = 0$ becomes

$$0 = g_m v_{gs} + \frac{v_{ds}}{r_d}$$

or, $$-\frac{v_{ds}}{r_d} = g_m v_{gs}$$

or, $$-\frac{v_{ds}}{v_{gs}} = r_d g_m$$

or, $$\propto = r_d g_m$$

This is the relationship between the JFET parameters.

7. (b)

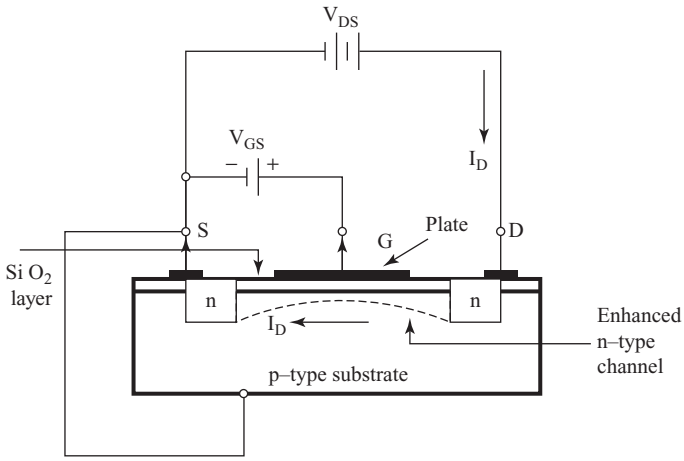

Figure 18

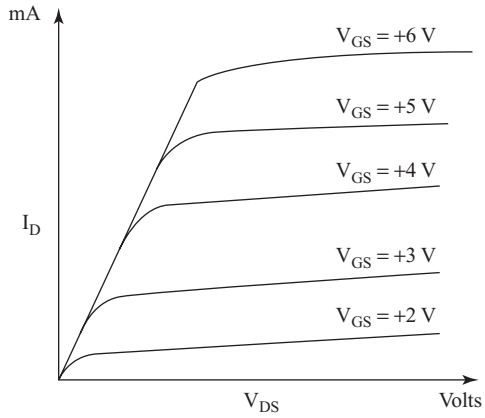

Figure 19

Let us assume that the source terminal is connected to the substrate and a positive gate voltage is applied. The negative charge carriers are attracted to the plate and they accumulate close to the gate but cannot cross the silicon dioxide layer. The minority charge carriers constitute an n-type channel between the drain and the source. As the gate-source voltage V_{GS} is increased, more electrons gather on the surface and the channel resistance is decreased. The channel resistance is controlled by the V_{GS} and hence drain current is also controlled by V_{GS}. The channel conductivity is said to be enhanced by the increase of positive gate voltage. The typical drain characteristics between V_{DS} and I_D have been shown for different values of positive V_{ES} in Fig. 19. It is seen that the drain current increases with increase of V_{GS} for a particular value of V_{DS}.

7. (c)

$$V_{GS} = -2 \text{ V}$$
$$I_{dss} = 10 \text{ mA}$$
$$V_p = -3.5 \text{ V}$$
$$I_d = ?$$

$$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_p}\right]^2$$

$$= 10 \times 10^{-3}\left[1 - \frac{-2}{-3.5}\right]^2$$

$$= 10^{-2}\left[1 - \frac{2}{3.5}\right]^2$$

$$= 10^{-2}\left(0.43\right)^2$$

$$= 1.85 \times 10^{-3} \text{ A}$$

$$= 1.85 \text{ mA}$$

8. (a) An integrated circuit (IC) is a complete electronic circuit in miniature form grown into a single chip of silicon. ICs are classified into monolithic circuits or hybrid circuits from the fabrication point of view. A monolithic circuit is a complete circuit having active and passive circuit components and their interconnections made on a single chip. Integrated circuit technology has grown from small-scale integration to very large-scale integration (VLSI). Hybrid or multichip integrated circuits are made by interconnecting a number of monolithic ICs.

Advantages of ICS are:

(i) **Small size:** The size of an IC is thousand times smaller than an equivalent discrete circuit. For example, one chip of size 4 mm^2 may contain 40 transistors, 35 resistors, a number of capacitors, and other such components along with their interconnections.

(ii) **Low cost:** Since ICs are produced on a large scale (mass production), their cost is much lower as compared to discrete component circuits.

(iii) **Less weight:** Since the circuits are in miniature form, their weight is less and they occupy smaller space.

(iv) **Highly reliable:** ICs are highly reliable as the connections made do not give any trouble during long operating life.

8. (b)

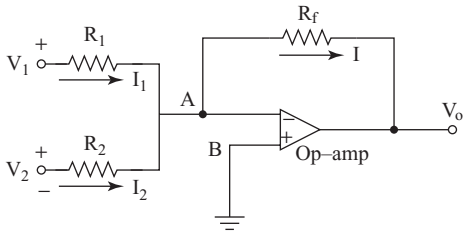

Figure 20

All the inputs to be added are applied to the inverting input terminals of the op-amp. As the terminal B of the op-amp is grounded, from the concept of virtual ground, node A can also be considered at ground potential, i.e. $V_A = 0$.

Looking from the input side,

$$I_1 = \frac{V_1 - V_A}{R_1} = \frac{V_1}{R_1}$$

$$I_2 = \frac{V_2 - V_A}{R_2} = \frac{V_2}{R_2}$$

From the output side,

$$I = \frac{V_A - V_0}{R_f} = -\frac{V_0}{R_f}$$

As no current flows through the amplifier,

$$I = I_1 + I_2$$

or,

$$-\frac{V_0}{R_f} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

or, $$V_0 = -\left[\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2\right]$$

If $R_1 = R_2 = R_f$,

$$V_0 = -(V_1 + V_2)$$

9. (a) **Comparator:** An op-amp in open-loop configuration can be used as a basic comparator. When an op-amp is provided with two inputs, it compares the two inputs and accordingly produces an output voltage, which is either positive saturation voltage $(+V_{sat})$ or negative saturation voltage $(-V_{sat})$.

A comparator compares a signal voltage applied at one input with a known reference voltage available at the other input and produces either a high or a low output voltage. The output of the comparator is linearly proportional to the input voltage.

A non-inverting comparator is shown in Fig. 21.

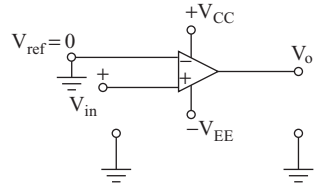

Figure 21

In this comparator, the input voltage is applied to the non-inverting terminal and no reference voltage is applied to the other terminal. The inverting terminal is shown grounded. The input voltage V_{in} is compared with the reference input voltage. In a basic comparator, $V_{ref} = 0$

If $V_{in} > V_{ref}$, then output is $+V_{sat}$ which is almost equal to V_{cc}.

If $V_{in} < V_{ref}$, then output is $-V_{sat}$ which is almost equal to $-V_{EE}$.

With $V_{ref} = 0V$, when V_{in} is positive, $V_0 = V_{sat} \simeq +V_{cc}$.

When V_{in} is negative, $V_0 = -V_{sat} \simeq -V_{EE}$.

As the open-loop gain of the op-amp is very high, even for very small V_{in} the op-amp output gets saturated.

The two possible output levels of the comparator are $+V_{sat}$ and $-V_{sat}$. This type of comparator is called saturating type comparator.

9. (b) **Universal gates**: The NAND or NOR gates are called universal gates because it is possible to implement any Boolean expression with the help of only NAND or only NOR gates.

NAND gate as universal gate: NAND gate is expressed mathematically as $Y = \overline{A.B}$

(i) NOT gate using NAND

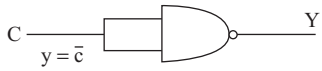

Figure 22

(ii) AND gate using NAND

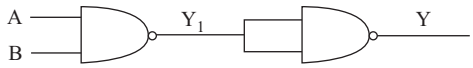

Figure 23

$$y = \overline{y}_1 = \overline{A}.\overline{B}$$
$$y = A.B$$

(iii) OR operation using NAND

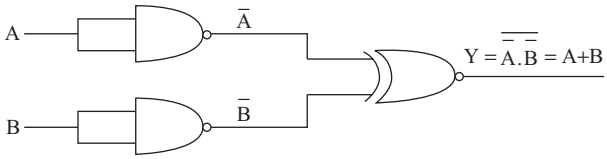

Figure 24

NOR gate as universal gate:

(i) NOT gate using NOR gate

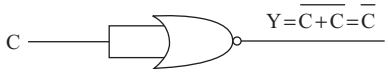

Figure 25

(ii) AND gate using NOR gate

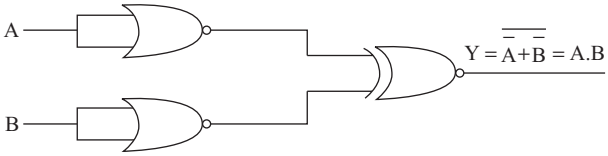

Figure 26

(iii) NAND gate using NOR gate

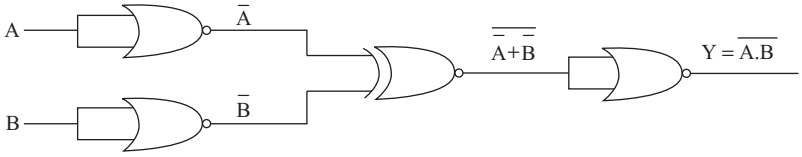

Figure 27

(iv) OR gate using NOR gate

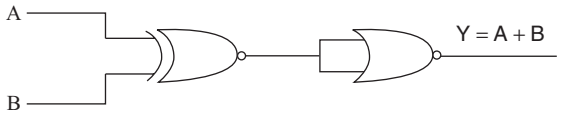

Figure 28

9. (c) **Pinch-off condition of JFET**: If the negative gate voltage is increased such that the depletion layer of the JFET meets at the centre as has been shown in Fig. 29, the drain current I_D is totally cut-off. This negative gate voltage at which the drain current I_D is cut-off is called the pinch-off voltage V_p.

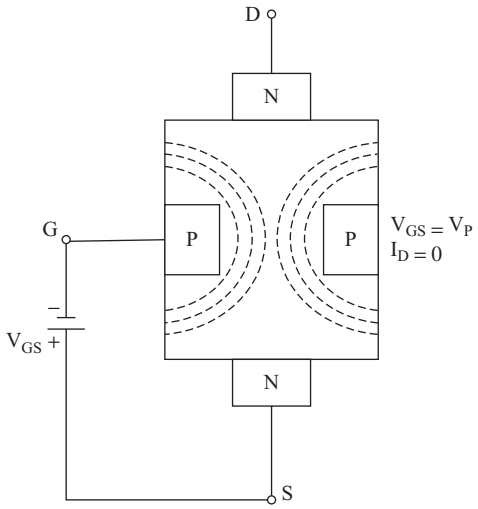

Figure 29

9. (d) **CMMR (Common Mode Rejection Ratio):** When equal amount of voltage is applied to both the input terminals of a differential amplifier, it is said to be operated in a common mode configuration. Many disturbance signals such as noise signals appear as common input signals to both the terminals of a differential amplifier. Such common signals should be rejected by the differential amplifier.

The ability of a differential amplifier to reject a common mode signal is expressed by a ratio called CMMR. It is defined as the ratio of the differential voltage gain A_d to common mode voltage gain A_c.

$$\text{CMMR} = \left|\frac{A_d}{A_c}\right|$$

$$\text{CMMR in dB} = 20 \log \left|\frac{A_d}{A_c}\right|.$$

The ideal value of CMMR is infinite.